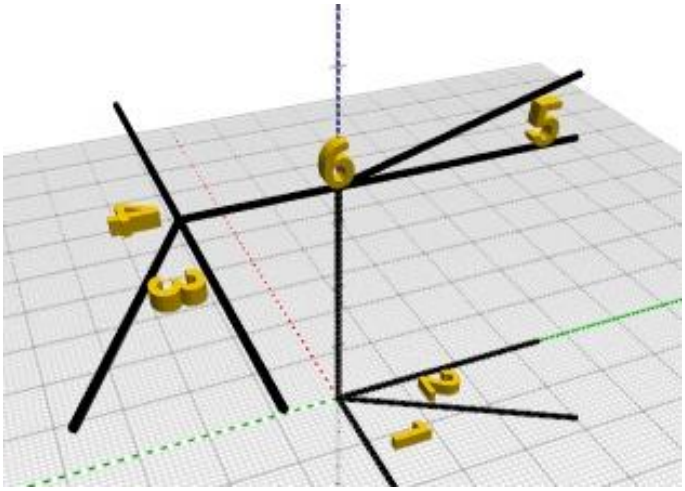
“Angle Deduction”



Grade 7 Math Alignment

Domain: Geometry
Cluster: Draw, construct, and describe geometrical figures and describe the relationships between them
Standard: 7.G.5: Use facts about supplementary, complementary, vertical and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Description

Students will practice **deductive reasoning** to solve for unknown angles in multi-step 3D problems. They will write pseudo-proofs, first filling in the blanks for given statements, premises about **supplementary, complementary, and vertical angles**, and sub-conclusions, before writing their own arguments.

Learning Objectives

- Students will practice using angle properties to determine unknown variables, including work in three dimensions
- Students will use vocabulary important for writing and understanding proofs

Videos

- Working with Code: https://www.youtube.com/watch?v=YBRaaPGuO2E
- Translation: https://www.youtube.com/watch?v=6ulQa_sO8ms
- Rotation: https://www.youtube.com/watch?v=bwzv1bTPmUU

Review/Prior Knowledge

Students will need to apply properties of angles like supplementary, complementary, and vertical angles. One challenge will require use of the parallel lines postulate, but students don't necessarily need to know this ahead of time. Some experience with flowchart proofs might be helpful so that students understand the difference between givens, premises, and conclusions, but this activity can also serve as an introduction to these concepts.

Resources

- Student Worksheet: Angle Deduction → Student guide for activity
- Angle Deduction Student File.xml → Starting blocks for students

Teacher Notes

General Introduction: Introduce students to the difference between colloquial use of "prove" and the math use of "prove". In their daily lives, proving a statement might mean finding evidence that supports it. To prove why dolphins are the best animal in an argument with a friend, for example, you might cite cool facts about dolphins and research that shows that most people prefer dolphins to other animals. Even in science, "proving" the theory of gravity might involve dropping something many, many times until you are left without a reasonable doubt. In math, however, proofs guarantee the validity of the conclusion as long as the premises are true. For example, if you had to prove the area of a given square was bigger than the area of a given circle, you could use agreed upon calculations for area and conceptions of size to show that one area is undoubtedly larger.

Students may also need to be refreshed on their angle theorems and vocabulary before looking at the diagrams and trying to write their mini-proofs. They will likely need a simple calculator for two brief fraction calculations during the activity.

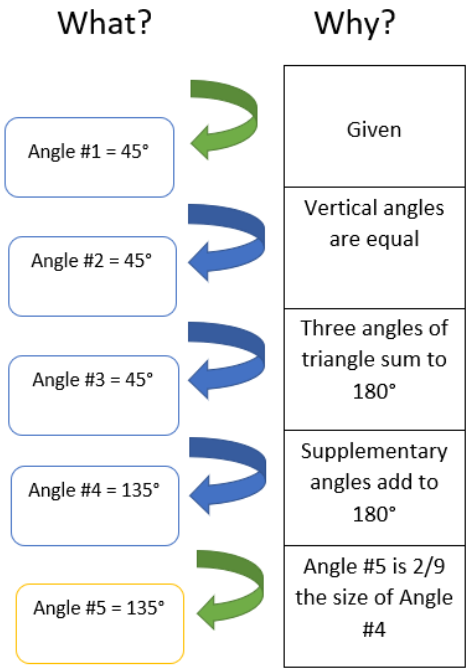
BlocksCAD Introduction/Refresher: This activity requires very little prior experience with BlocksCAD. Students will only need to know how to enable and disable blocks and then zoom and spin around the display on the render screen.

Exploration: It may be worthwhile to complete **Part I** together as a group to make sure students understand the difference between “givens”, “premises” and “conclusions”. Give some examples of premises other than the angle sum premise that generalizations that are either proven, true by definition, or generally accepted as true (squares have equal sides, the circumference of a circle is pi times its diameter, etc.).

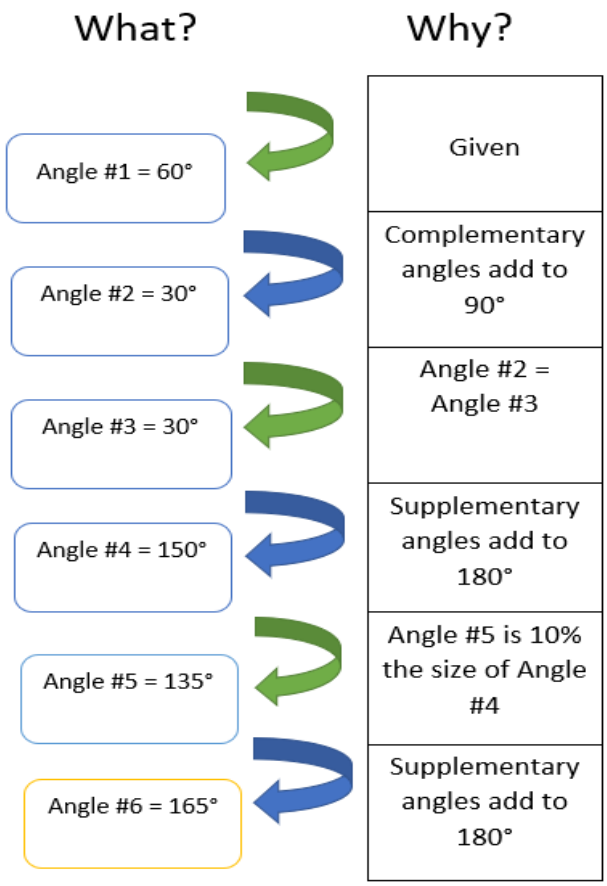
Their proof should look like this when complete:

Premise: The three angle measures in a triangle always add up to 180°.
Given: This triangle has angles that measure 43° and 90°, which add up to 133°.
Conclusion: This means that the third angle, x, must be equal to 47°.

Have students open the BlocksCAD file “Angle Deduction.xml (Assign from your Classroom page or send them the .xml and do New Project → Project → Load Blocks from my computer). Challenge #1 in **Part II** gives students an outline to help them. An example solution is shown below. The proof is not meant to be as detailed as those students might write later in their math careers, but the outline should start to teach good habits.



Challenge #2 gives students an outline, but leaves the premises blank for students to fill in:



For Challenge #3, you can either have students use the same layout as before for their proofs, or you can encourage them to find a different way to represent their argument visually. Be sure to assure them that the lines that appear parallel are indeed parallel so that they can use Alternate Exterior/Corresponding Angles reasoning. Their reasoning should follow this format:

Angle #1 = 150° → Angle #2 = 30° because supplementary angles sum to 180°
→ Angle #3 = 60° because complementary angles sum to 90°
→ Angle #4 = 40° because ⅔ x 60 = 40
→ Angle #5 = 140° by the Alternate Exterior Angles postulate

(If students are unfamiliar with the Alternate Exterior Angles, they can also use another argument that uses supplementary angles and corresponding angles that is more intuitive.)

Reflection Questions and Activities

1. Knowing two angles is enough information to prove the measure of the third in a triangle. How many angles would you need to know to determine the others in a four-sided shape? 5? n?
2. Does every angle have a supplementary angle? A complementary angle? (Sketches or protractors may be helpful.)
3. Other BlocksCAD activities in the angles and triangles progression: Tessellations, Chessboard Triangles, Pythagoras on TV.