QCD critical point and thermal photons

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Introduction

- Beam energy scans: **exploration of QCD phase diagram**

  - RHIC (BNL)
    Phase II (2017-20?): 3.0 GeV?

  - FAIR (GSI), NICA (JINR), SPS (CERN), J-PARC etc.
    + LHC (CERN): 5.5 TeV

We use fluid dynamics to:

- Look for signals of a **QCD critical point**
- Determine the QGP properties at finite $T, \mu_B$
- Understand the origin of “fluidity”
Introduction

- Observable: Elliptic flow ($v_2$)

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2v_1 \cos(\phi - \Psi_1) + 2v_2 \cos(2\phi - 2\Psi_2) + 2v_3 \cos(3\phi - 3\Psi_3) + \ldots \right]$$

Hadron $v_2$ is found to be large

- It follows fluid dynamic description
- An “evidence” for strongly-coupled QGP early equilibration of bulk medium ($\tau < 1$ fm/c)?
Overview of a collision

- Hadronic point of view

Hadronic transport (> 10 fm/c)
- Freeze-out

Fluid dynamic stage (~1-10 fm/c)
- Equilibration

Glasma (~0-1 fm/c)
- Little bang

Color glass condensate (< 0 fm/c)

- Color opaque
  Most information before freeze-out is lost

Colliding nuclei

Graphics by AM
Overview of a collision

- Photonic point of view

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  - Most information before freeze-out is lost

- Electroweak transparent
  - Photons retain information during time-evolution
Observables

- Photonic point of view

- Decay photons
  - from hadronic decay

- Thermal photons (hadronic)
  - from black-body radiation

- Thermal photons (QGP)
  - from hard processes

- Prompt photons
  - from hard processes

- Direct photons

- Color opaque
  Most information before freeze-out is lost

- Electroweak transparent
  Photons retain information during time-evolution

Graphics by AM

Colliding nuclei
Observable?

- QCD critical point (QCP) vs. Thermal freeze-out

PART 1

- QCD medium is thermalized; colored objects (hadrons) are scattered

Signals can be washed away unless
1. QCP is near enough to freeze-out
2. Its effect on evolution is large enough

PART 2

- Thermal photons penetrate through the medium

Can the QCP signals be clear?
Hydrodynamic model for BES

A path we have been through

- Integrated $v_2$ becomes small at lower $E$
- "HYDRO limits" estimated with:
  - Analytical Glauber model
  - EoS with 1\textsuperscript{st} order PT
  - Ideal hydro
  - Kolb et al., PRC62, 054909 (2000)
- Once thought hydro is only for AA at top energies (which may still be true)

Applicability tests

- Differential $v_2$ stays large
- We should see if the state-of-art hydrodynamic interpretations work
Equations to solve

- Relativistic formalism

  Energy-momentum conservation \( \partial_\mu T^{\mu \nu} = 0 \)

  Baryon conservation \( \partial_\mu N_B^{\mu} = 0 \)

  Equation of state \( P = P(e, n_B) \)

  Shear viscosity \( \pi^{\mu \nu} = 2\eta \nabla^{\langle \mu} u^{\nu \rangle} - \tau_\pi D\pi^{\langle \mu \nu \rangle} + ... \)

  Bulk viscosity \( \Pi = -\zeta \nabla_\mu u^\mu - \tau_\Pi D\Pi + ... \)

  Baryon diffusion \( V_B^\mu = \kappa_{VB} \nabla^\mu \frac{u_B}{T} - \tau_{VB} \Delta^{\mu \nu} D V_\nu + ... \)

- Ideal hydrodynamics

- Dissipative hydrodynamics

  response to deformation
  response to expansion
  response to chemical gradients
Near the QCD critical point

- Bulk viscosity becomes dominant
  - Shear viscosity: \( \eta = \xi^{(4-d)/19} \)
  - Bulk viscosity: \( \zeta = \xi^3 \)
  - Baryon diffusion: \( D_B = \xi^{-1} \)
- Relaxation time

\[
\tau_\Pi = \tau_{\Pi,0} \left( \frac{\xi_{\text{eq}}}{\xi_0} \right)^3 \quad \text{as causality suggests}
\]

\[
\lim_{k \to \infty} \frac{d\omega}{dk} = \sqrt{c_s^2 + \frac{\zeta}{\tau_\Pi (\epsilon + P)}} < 1
\]

- 2\textsuperscript{nd} order theory can applicable because \( \Pi \) is “frozen” for large \( \tau_\Pi \)
- We use \( \zeta_0 = 2 \left( \frac{1}{3} - c_s^2 \right) \frac{e + P}{4\pi T} \), \( \tau_{\Pi,0} = C_\Pi \frac{18 - (9 \ln 3 - \sqrt{3}\pi)}{24\pi T} \) based on AdS/CFT

AM, Y. Yin and S. Mukherjee, arXiv:1606.00771

We consider bulk viscosity

\[
\zeta = \zeta_0 \left( \frac{\xi_{\text{eq}}}{\xi_0} \right)^3
\]

but 1\textsuperscript{st} order theory is unstable
Initial conditions

- Longitudinal distribution

  ▶ Color glass models extrapolated to lower energies for the shapes of energy and net baryon distribution

Energy density peaks at $\eta=0$, while net baryon density at finite $\eta$

▷ Chemical potential is larger at forward rapidity $\eta$

* $\eta_s = \frac{1}{2} \ln \frac{t + z}{t - z}$ is the “angle” of hyperbolic coordinate

H. J. Drescher and Y. Nara, PRC 75, 034905; 76, 041903
Y. Mehtar-Tani and G. Wolschin, PRL 102, 182301; PRC 80, 054905
Equation of state

- Hadron resonance gas + lattice QCD  

Lattice QCD has a sign problem at finite density

- Taylor expansion up to the 4\textsuperscript{th} order is used for QGP phase

$$\frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2} \chi_B^{(2)} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{4!} \chi_B^{(4)} \left( \frac{\mu_B}{T} \right)^4 + \mathcal{O} \left( \frac{\mu_B}{T} \right)^6$$

$$\frac{P}{T^4} = \frac{1}{2} \left[ 1 - \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{HRS}}(T)}{T^4} + \frac{1}{2} \left[ 1 + \tanh \frac{T - T_c(\mu_B)}{\Delta T_c} \right] \frac{P_{\text{lat}}(T_s)}{T_s^4}$$

where

- $T_c = 0.166 - c(0.139\mu_B^2 + 0.053\mu_B^4)$
- $T_s = T + d[T_c(0) - T_c(\mu_B)]$

*Currently no QCP because its effect on the EoS is limited*
Trajectories on $\mu_B$-T plane

- 1+1 dimensional hydrodynamic demonstration

**Diagram:**

- Critical point is placed by hand at $(\mu_B, T) = (0.22 \text{ GeV}, 0.16 \text{ GeV})$ by mapping the critical region of Ising model onto the $\mu_B$-T plane.

- If the QCP exists, the trajectory is pushed away from it on the lower $\mu_B$ side because of **bulk viscous entropy production**.
Rapidity distributions

- 1+1 dimensional hydrodynamic demonstration

- Charged particle and net baryon distributions are deformed if the critical point is contacted

- dN_{ch}/dy deformation is caused by entropy production and enhanced flow convection due to the reduction in effective pressure P - \Pi

- dN_{B}/dy deformation is by convection only

AM, Y. Yin and S. Mukherjee, arXiv:1606.00771
Comments on applicability

- Is the model valid near QCP?
  - Viscous correction should satisfy
    \[
    \frac{|\Pi|}{c + P} < 1
    \]
  - When \( |\Pi| \sim P \),
    \[
    \frac{|\Pi|}{c + P} \sim \frac{1}{1 + c_s^{-2}}
    \]

But the sound velocity is \( c_s^2 \sim 0.1 \) near the QCP

- Cavitation could be an issue; bulk viscous relaxation time from AdS/CFT approach (\( C_\Pi = 1 \)) seems to avoid the issue.
Thermal photons

- Does emission rate contain a signal of QCP?

Few studies on the emission rate at finite density in the vicinity

Linear sigma model suggests no dramatic enhancement at QCP


Bulk viscosity can change the emission rate via the distortion of the phase-space distribution

$$E \frac{dR_i}{d^3 p} = \int \frac{d^3 p_1}{2E_1(2\pi)^3} \frac{d^3 p_2}{2E_2(2\pi)^3} \frac{d^3 p_3}{2E_3(2\pi)^3} (2\pi)^4 \delta(p_1^\mu + p_2^\mu - p_3^\mu - p^\mu) |M_i|^2 f_1(E_1) f_2(E_2) [1 \pm f_3(E_3)]$$
Bulk viscous corrections

How to determine $\delta f_{\text{bulk}}$

1. Expand the exponent $y^i$ in $f^i = \frac{1}{\exp(y^i) + 1}$ around equilibrium in terms of $\Pi$

   The tensor structure allowed in Israel-Stewart theory is
   
   $$\delta y^i = [b_i D_{\Pi} u_{\mu} p^\mu_i + B_{\Pi} g_{\mu\nu} p^\mu_i p^\nu_i + (\tilde{B}_{\Pi} - B_{\Pi}) p^\mu_i p^\nu_i ] \Pi$$

2. Have it satisfy the self-consistency conditions

   $$\delta T^{\mu\nu} = \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^{\mu} p_i^{\nu} \delta f^i \quad \delta N^\mu_J = \sum_i \int \frac{q_i^J g_i d^3 p}{(2\pi)^3 E_i} p_i^{\mu} \delta f^i$$

We have the coefficients

$$D_{\Pi} = 3( J_{40} J_{31}^B - J_{41} J_{30}^B ) J_3^{-1} \quad J_3 = 5 J_{42} J_{30}^B J_{30}^B + 3 J_{31}^B J_{40} J_{31}^B + 3 J_{41} J_{41} J_{20}^{BB}$$

$$B_{\Pi} = ( J_{30}^B J_{30}^B - J_{40} J_{20}^{BB} ) J_3^{-1} \quad - 3 J_{31}^B J_{41} J_{30}^B - 3 J_{41} J_{30}^B J_{31}^B - 5 J_{42} J_{40} J_{20}^{BB}$$

$$\tilde{B}_{\Pi} = 3 ( J_{41} J_{20}^{BB} - J_{30}^B J_{31}^B ) J_3^{-1} \quad J_{mn} : \text{momentum integrals of } f_0^i$$
Critical enhancement

- (2+1)-D hydrodynamic tests with

\[ E \frac{dR}{d^3p} = \left[ 1 + 0.1(\xi/\xi_0)^3 \right] \times E \frac{dR}{d^3p} \]

- The magnitude and sign of correction is sensitive to the shape and location of the critical region

- Early emission leads to small momentum anisotropy \( v_2 \)

- Work in progress – stay tuned
Summary and outlook

- QCD critical point is a hot topic in heavy-ion collisions
  - Bulk viscosity can become dominant near QCP
  - Medium evolution itself can be affected if the system came across QCP
    - Trajectories and rapidity distributions are warped by entropy production and enhanced convection
  - Thermal photons can be a good signal of QCP
    - Bulk viscous enhancement is a key
- Full estimation of off-equilibrium and finite-density photon emission rate is important (work in progress)
- Estimations for BNL-RHIC, CERN-SPS, FAIR, NICA are necessary
The end

Thank you!
Takk for at dere hørte på