Viscous Hydrodynamic Evolution with Non-Boost Invariant Flow for the Color Glass Condensate

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Introduction

- Quark-gluon plasma (QGP) at relativistic heavy ion collisions

**RHIC experiments (2000-)**

\[ \sqrt{s_{NN}} = 200 \text{ (GeV)} \]

- The QGP quantified as a nearly-perfect fluid

- Viscosity is important for “improved” inputs (initial conditions, equation of state, etc.)

- Consistent modeling is necessary to extract physical properties from experimental data
Introduction

■ Modeling a high-energy heavy ion collision

Color glass condensate (CGC)
Description of saturated gluons in the nuclei before a collision ($\tau < 0$ fm/c)

Relativistic hydrodynamics
Description of collective motion of the QGP ($\tau \sim 1-10$ fm/c)
First Results from LHC

- **LHC experiments (2010-)**
  
  \[ \sqrt{s_{NN}} = 2.76, 5.5 \text{(TeV)} \]

  Heavy ion collisions of higher energies

  \[ \Rightarrow \] Will the RHIC modeling of heavy ion collisions be working intact at LHC?

- **Mid-rapidity multiplicity**

  Pb+Pb, 2.76 TeV at \( \eta = 0 \)

  **ALICE**
  
  Busza [4]
  HIJING 2.0 [5]
  DPMJET III [6]
  UrQMD [7]
  Albacete [8]
  Levin et al. [9]
  Kharzeev et al. [10]
  Kharzeev et al. [10]
  Kharzeev et al. [11]
  Armesto et al. [12]
  Eskola et al. [13]
  Bozek et al. [14]
  Sarksyan et al. [15]
  Humanic [16]

  **CGC**

  K. Aamodt *et al.* PRL105 252301

  **ALICE data (most central 0-5%)**
  
  \[ \frac{dN_{ch}}{d\eta} = 1584 \pm 4 \text{(stat)} \pm 76 \text{(phys)} \]

  \[ \Rightarrow \] CGC; fit to RHIC data but no longer valid at LHC?
CGC in Heavy Ion Collisions

- Saturation scale in MC-KLN model
  \[ Q^2_{s,A}(x; x_\perp) = 2 \text{ GeV}^2 \frac{T_A(x_\perp)}{1.53 \text{ fm}^{-2}} \left( \frac{0.01}{x} \right) \]
  Fixed via direct comparison with data
  \( dN_{\text{ch}}/d\eta \) gets steeper with increasing \( \lambda \);
  RHIC data suggest \( \lambda \approx 0.28 \)

- CGC + Hydrodynamic Model
  
  Initial condition from the CGC \( \rightarrow \) Hydrodynamic evolution \( \rightarrow \) Observed particle distribution
  
  a missing piece!

- We need to estimate hydrodynamic effects with
  (i) non-boost invariant expansion
  (ii) viscous corrections

D. Kharzeev et al., NPA 730, 448
Hydrodynamic Model

- Decomposition of the energy-momentum tensor by flow $u^\mu$

\[
T^{\mu\nu} = (e_0 + \delta e)u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}
\]

where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projection operator.

- 2 equilibrium quantities
  - Energy density: $e_0$
  - Hydrostatic pressure: $P_0$

  related in equation of state

  $P_0 = P_0(e_0)$

- Stability condition + frame fixing

  Thermodynamic stability demands $\delta e = 0$

  Identify the flow as local energy flux $W^\mu = 0$

  This leaves $\Pi$ and $\pi^{\mu\nu}$

- 10 dissipative currents
  - Energy density deviation: $\delta e$
  - Bulk pressure: $\Pi$
  - Energy current: $W^\mu$
  - Shear stress tensor: $\pi^{\mu\nu}$
Hydrodynamic Model

- **Full 2\textsuperscript{nd} order viscous hydrodynamic equations**

Energy-momentum conservation \( \partial_{\mu} T^{\mu\nu} = 0 \)

\begin{align*}
D \Pi &= \frac{1}{\tau_{\Pi}} \left( - \Pi - \zeta_{\Pi\Pi} \frac{1}{T} \nabla_{\mu} u^{\mu} - \zeta_{\Pi\delta} e D \frac{1}{T} \
+ \chi_{\Pi\Pi}^{b} D \frac{1}{T} + \chi_{\Pi\Pi}^{c} \Pi \nabla_{\mu} u^{\mu} + \chi_{\Pi\Pi}^{\pi\pi} \pi^{\mu\nu} \nabla_{\langle \mu} u_{\nu \rangle} \right) \\
D \pi^{\mu\nu} &= \frac{1}{\tau_{\pi}} \left( - \pi^{\mu\nu} + 2\eta \nabla_{\langle \mu} u_{\nu \rangle} + \chi_{\pi\pi}^{b} \pi^{\mu\nu} D \frac{1}{T} \
+ \chi_{\pi\pi}^{c} \pi^{\mu\nu} \nabla_{\rho} u^{\rho} + \chi_{\pi\pi}^{d} \pi^{\rho} \nabla_{\rho} u^{\langle \mu} u_{\nu \rangle} + \chi_{\Pi\Pi}^{\pi\Pi} \Pi \nabla_{\langle \mu} u_{\nu \rangle} \right)
\end{align*}

Note: (2+1)-D viscous hydro assumes boost-invariant flow

\[
D = u^{\mu} \partial_{\mu} \\
\nabla_{\mu} = \Delta_{\mu\nu} \partial_{\nu}
\]
Model Input for Hydro

- **Equation of state and transport coefficients**
  
  - **Equation of State:** Lattice QCD
  - **Shear viscosity:** \( \eta = s/4\pi \)
  - **Bulk viscosity:** \( \zeta_{\text{eff}} = (5/2)[(1/3) - c_s^2]\eta \)
  - **Relaxation times:** Kinetic theory & \( \eta, \zeta \)
  - **2nd order coefficients:** Kinetic theory & \( \eta, \zeta \)

  - **Initial conditions**

  - **Initial flow:** Bjorken flow (i.e. flow rapidity \( Y_f = \eta_s \))
  - **Energy distribution:** MC-KLN type CGC model
  - **Dissipative currents:** \( \delta T^{\mu\nu} = 0 \)
  - **Initial time:** \( \tau = 1 \text{ fm/c} \)

S. Borsanyi *et al.*, JHEP 1011, 077

P. Kovtun *et al.*, PRL 94, 111601

A. Hosoya *et al.*, AP 154, 229

H. J. Drescher and Y. Nara, PRC 75, 034905; 76, 041903

AM and T. Hirano, NPA 847, 283
Results

- CGC initial distributions + longitudinal viscous hydro

If the true \( \lambda \) is larger at RHIC, it enhances \( dN/dy \) at LHC; Hydro effect is a candidate for explaining the “gap” at LHC.
Results

- Deviation from boost-invariant flow

Flow rapidity: \( Y_f = \ln \frac{u^0 + u_z}{u^0 - u_z} \)

\( \tau = 30 \text{ fm/c} \)

\( \tau = 50 \text{ fm/c} \)

The systems are far from boost invariant at RHIC and LHC

Ideal flow \( \approx \) viscous flow due to competition between

- deceleration by suppression of total pressure \( P_0 - \Pi + \pi \) at early stage and
- acceleration by enhancement of hydrostatic pressure \( P_0 \) at late stage
Summary and Outlook

- We solved full 2\textsuperscript{nd} order viscous hydro in (1+1)-dimensions for the “shattered” color glass condensate

\[\text{Non-trivial deformation of CGC rapidity distribution due to:}\]

\[\begin{align*}
(\text{i}) \text{ outward entropy flux} & \quad \text{(non-boost invariant effect)} \\
(\text{ii}) \text{ entropy production} & \quad \text{(viscous effect)}
\end{align*}\]

\[\text{Viscous hydrodynamic effect may play an important role in understanding the seemingly large multiplicity at LHC}\]

- Future prospect includes:
  
  - Detailed analyses on parameter dependences, rcBK, etc...
  - A (3+1)-dimensional viscous hydrodynamic model, etc...

\[\text{AM & T. Hirano, in preparation}\]
The End

- Thank you for listening!
- Website: http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/index.html
Results

- Parameter dependences

\[(i) \frac{\eta}{s} = 0, \frac{\zeta_{\text{eff}}}{s} = 0\]

\[(ii) \frac{\eta}{s} = \frac{1}{4\pi}, \frac{\zeta_{\text{eff}}}{s} = \frac{(5/2)[(1/3) - c_s^2]}{4\pi}\]

\[(iii) \frac{\eta}{s} = \frac{3}{4\pi}, \frac{\zeta_{\text{eff}}}{s} = \frac{(15/2)[(1/3) - c_s^2]}{4\pi}\]

- Comparison to boost-invariant flow

Larger entropy production for more viscous systems

Longitudinal viscous hydro expansion is essential
Results

- Time evolution for LHC settings

![Graphs](image)

- Rapidity distribution: no sizable modification after 20 fm/c
  It can be accidental; needs further investigation on parameter dependence

- Flow rapidity: visible change even after 20 fm/c
  Rise-and-dip at 5 fm/c is due to reduction in effective pressure $P_0 - \Pi + \pi$
Thermodynamic Stability

Maximum entropy state condition

\[ s^{\mu} u_{\mu} = s_0 + \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\mu u_\mu y_i^0 \delta f_i - \frac{1}{2} \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\mu u_\mu \frac{\delta f_i^2}{f_0^i (1 \pm f_0^i)} + \mathcal{O}(\delta f^3) \]

\[ = 0 \]

\[ \leq 0 \]

- Stability condition (1\textsuperscript{st} order)

\[ \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\mu y_i^0 \delta f_i \]

\[ = \frac{1}{T} u_\mu \delta T^{\mu\nu} u_\nu - \sum_J \frac{\mu_J}{T} u_\mu \delta N_J^\mu = 0 \]

\[ \Rightarrow \delta e = \delta n_J = 0 \]

- Stability condition (2\textsuperscript{nd} order) \( \xrightarrow{\text{automatically satisfied in kinetic theory}} \)

*Stability conditions are NOT the same as the law of increasing entropy*
Introduction

- Properties of the QCD matter

**Equation of state**: relation among thermodynamic variables sensitive to *degrees of freedom* in the system

**Transport coefficients**: responses to thermodynamic forces sensitive to *interaction* in the system

Naïve interpretation of dissipative processes

- Shear viscosity = response to deformation
- Bulk viscosity = response to expansion
- Energy dissipation = response to thermal gradient
- Charge dissipation = response to chemical gradients
Introduction

- Geometrical setup of colliding nuclei

![Diagram showing longitudinal and transverse expansions](image)

- Relativistic coordinates
  
  \[
  \tau = \sqrt{t^2 - z^2}
  \]
  
  Space-time rapidity: \( \eta_s = \frac{1}{2} \ln \frac{t+z}{t-z} \)

  - Transverse mass: \( m_T = \sqrt{E^2 - p_z^2} \)
  - Rapidity: \( y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z} \)

- Centrality

  Determined by groups (20-30%, etc.) of “events per number of participants”