Pre-equilibrium dynamics of QCD matter by gluon splitting

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Overview

1. Introduction
   - The quark-gluon plasma
   - High-energy heavy ion collisions

2. Thermalization of gluons
   - Collinear gluon splitting and recombination
   - Fokker-Planck type diffusion
   - Numerical estimations: gluon distribution

3. Thermal & chemical equilibration of quarks
   - Quark and gluon splitting/recombination processes
   - Numerical estimations: quark and gluon distributions

4. Summary and outlook
1. Introduction

Next: 2. Gluon splitting in early thermalization
Introduction

- **Quark-gluon plasma (QGP):** many-body system of deconfined quarks and gluons

The QGP is supposed to have filled the early universe; It can be produced in heavy ion experiments at RHIC & LHC

- Heavy ion QGP is found to exhibit near-perfect fluidity
- Early local equilibration ($\tau < 0.6-1 \text{ fm}$) is implied
Introduction

■ Modeling a “Little Bang”

\[ \tau > 10 \text{ fm/c: hadronic gas} \]
(weakly coupled)

\[ \tau \sim 1-10 \text{ fm/c: QGP/hadronic fluid} \]
(strongly coupled)

\[ \tau \sim 0-1 \text{ fm/c: CGC -> QGP ?} \]
(pre-equilibrated)

\[ \tau < 0 \text{ fm/c: color glass condensate} \]
(saturated gluons)

Pre-equilibrated QCD medium is a high-energy frontier in hadron physics

Dynamical description is required for understanding QCD matter
Motivation

- **Color glass condensate (CGC)**
  - Gluons emitted from gluons emit gluons in a fast-travelling nucleon
  - They start to overlap and saturated
  - QCD matter at the initial stage of heavy ion collisions is dominated by gluons

- **QGP fluid**
  - Azimuthal momentum anisotropy $\nu_2$ is large compared with spatial one $\varepsilon_2$
  - QCD matter is locally equilibrated at some point and behaves as a fluid
Motivation

- Early (local) equilibration

- One of the long standing issues in heavy ion physics
  - Naïve CGC and glasma lead to negative longitudinal pressure
  - CGC has large fraction of high-momentum gluons

Equilibration requires

We focus on thermalization and provide a model based on collinear splitting to introduce low momentum partons

Cf: parton cascade picture
bottom-up approaches
Bose condensate

K. Geiger and B. Müller (1992)
R. Baier et. al. (2001), P. Arnold et al. (2003)
2. Thermalization of gluons

Next: 3. Thermal and chemical equilibration of quarks
Collinear splitting model

**Gluon splitting**

- A splitting modifies the parton distribution $f$ as

$$ f(p_i) \to z^{-d} f\left(\frac{p_i}{z}\right) + (1 - z)^{-d} f\left(\frac{p_i}{1 - z}\right) $$

as number is **doubled** while momentum is **conserved**:

$$ 2 \int \frac{dp_i}{(2\pi)^d} f(p_i) = \int \frac{dp_i}{(2\pi)^d} \left[ z^{-d} f\left(\frac{p_i}{z}\right) + (1 - z)^{-d} f\left(\frac{p_i}{1 - z}\right) \right] $$

$$ \int \frac{dp_i}{(2\pi)^d} p_j f(p_i) = \int \frac{dp_i}{(2\pi)^d} p_j \left[ z^{-d} f\left(\frac{p_i}{z}\right) + (1 - z)^{-d} f\left(\frac{p_i}{1 - z}\right) \right] $$

- **Time evolution of gluon distribution by collinear splitting**

$$ \left. \frac{\partial f(p_i)}{\partial t} \right|_{sp} = \frac{1}{2} \int_0^1 dz \ r(z)\left[ f^z(p_i) + f^{1-z}(p_i) - f(p_i) \right] \equiv C_{sp}(p_i) $$

where $f^z(p_i) \equiv z^{-d} f\left(\frac{p_i}{z}\right)$, $f^{1-z}(p_i) = (1 - z)^{-d} f\left(\frac{p_i}{1 - z}\right)$ and

$$ r(z) = \Gamma R_{gg}(z) = 6\Gamma \frac{[1 - z(1 - z)]^2}{z(1 - z)} $$

is the splitting rate
Collinear splitting model

- Gluon splitting

- Rough estimation of parton emission rate $\Gamma$
  
  In static frame  \[ \Gamma \sim \alpha_s Q \]
  
  In boosted frame  \[ \Gamma \sim \alpha_s Q(Q/p) \sim \alpha_s \hat{q}L/p \]

  $\Rightarrow \Gamma \sim \alpha_s^{1/2}(\hat{q}/p)^{1/2}$ is implied

  (In thermal system $\Gamma_{th} \sim \alpha_s^{3/2}T$)

- The time evolution can violate the second law of thermodynamics if splitting continues on, i.e., not stable at thermal equilibrium

  $\Rightarrow$ A mechanism that prevents “over-shrinking” in phase space should be present
Collinear splitting model

■ Gluon recombination

Time evolution should stop at local equilibrium (2nd law of thermodynamics)

⇒ Recombination should occur when density Is large (in analogy with Balitsky-Kovshegov equation)

Parameterization of recombination process

\[
\frac{\partial f(p_i)}{\partial t} \bigg|_{rc} = -\frac{1}{2} \int_0^1 dz \, \tilde{r}(p_i, z) \frac{f_{eq}^z(p_i) + f_{eq}^{1-z}(p_i) - f_{eq}(p_i)}{f_{eq}(zp_i) f_{eq}((1 - z)p_i)} f(zp_i) f((1 - z)p_i)
\]

\[
\equiv C_{rc}(p_i)
\]

where \( f_{eq}(p) = \frac{d_g}{\exp(\sqrt{p^2 + m_{th}^2/T}) - 1} \) is equilibrium distribution in medium

Degeneracy \( d_g = 16 \)

Effective thermal mass \( m_{th} \sim \alpha_s^{1/2} T \)
Collinear splitting model

- **Momentum smearing effect**

  - Splitting occurs when a parton becomes **off-shell** by interacting with medium
  - Elastic scattering is taken into account by relativistic **Fokker-Planck equation**

\[
\frac{\partial f}{\partial t}|_{\text{FP}} = \frac{\partial}{\partial p_i} \left[ A^i f + \frac{\partial}{\partial p_j} ( B^{ij} f ) \right] = C_{FP}
\]

where \( A^i \sim \nu p^i \) and \( B^{ij} \sim D \delta^{ij} \) characterizes drag and diffusion effects
Collinear splitting model

- Momentum smearing effect
  
  ▶ Rough estimation of drag and diffusion coefficients
  
  Analytic solution of a diffusion equation for a delta function $\delta(p)$
  
  $$\frac{1}{\sqrt{4\pi Dt}} e^{-p^2/4Dt} \quad \Rightarrow \quad \text{Standard deviation } \sigma = \sqrt{2Dt}$$
  
  $$\quad \Leftarrow \quad \text{Longitudinal momentum modification } \sigma \sim Q^2/2p$$
  
  $$\Rightarrow \quad D \sim q^2/\Gamma p^2 \quad \text{and} \quad D_{\text{th}} \sim \alpha_s^{5/2} T^3 \quad \text{are implied}$$
  
  Relativistic Einstein relation $D = \nu ET$ yields the drag coefficient

- CGC-like initial condition
  
  ▶ Gluon distribution
  
  $$f_g(p < Q_s) \sim 1/\alpha_s \quad \text{and} \quad f_g(p > Q_s) \sim 0 \quad Q_s \sim 2 \text{ GeV}$$
Numerical analyses

- Time evolution in one-dimensional non-expanding system
Numerical analyses

- Time evolution in one-dimensional non-expanding system

![Graphs showing time evolution of gluon distribution functions](image-url)
Numerical analyses

- Time evolution in one-dimensional non-expanding system
Numerical analyses

- Time evolution in one-dimensional non-expanding system

- Collinear gluon splitting contributes visibly to **quick thermalization**
- Entropy production is confirmed positive

- Recombination is a key; Is the dynamics in dense region strong enough in 3D to enforce isotropization?
3. Thermal & chemical equilibration of quarks

Next: 4. Summary and outlook
Motivation

- Early (local) equilibration

  Equilibration requires more

  Gluon medium eventually becomes *quark*-gluon plasma
  Is quark thermalization fast enough?
  Is the QGP chemical equilibrated?
Parton splitting model

Gluons + quarks

Time evolution of parton distributions by collinear processes

\[
\frac{\partial f_g(p_i)}{\partial t} \bigg|_{sp} = \frac{1}{2} \int_0^1 dz \, r_{gg}(z) \left[ f_g^z(p_i) + f_g^{1-z}(p_i) - f_g(p_i) \right] + \int_0^1 dz \, r_{gg}(z) f_g(z) p_i - \int_0^1 dz \, r_{gg}(z) f_g(p_i) \equiv C_{sp}^g(p_i)
\]

\[
\frac{\partial f_q(p_i)}{\partial t} \bigg|_{sp} = \int_0^1 dz \, r_{qq}(z) \left[ f_q^{1-z}(p_i) - f_q(p_i) \right] + \int_0^1 dz \, r_{qq}(z) \left[ f_q^z(p_i) + f_q^{1-z}(p_i) \right] \equiv C_{sp}^q(p_i).
\]

Splitting functions

\[
\begin{align*}
    r_{gg}(z) &= \Gamma R_{gg}(z) = 6 \frac{[1 - z(1 - z)]^2}{z(1 - z)} \\
    r_{qq}(z) &= \Gamma R_{qq}(z) = 4 \frac{1 + (1 - z)^2}{z} \\
    r_{qg}(z) &= \Gamma R_{qg}(z) = 2N_f \Gamma R_{qg}(z) = 3 \frac{z^2 + (1 - z)^2}{z} \Gamma \quad (N_f = 3)
\end{align*}
\]

Quark-antiquark symmetry assumed

Gluon splitting
Gluon emission from quark
Quark pair production
Parton splitting model

- **Gluons + quarks**

Parameterization of recombination processes

\[
\frac{\partial f_g(p_i)}{\partial t}_{rc} = -\frac{1}{2} \int_0^1 dz \tilde{r}_{gg}(p, z) \frac{f_{geq}^z(p_i) + f_{geq}^{1-z}(p_i) - f_{geq}(p_i)}{f_{geq}(zp_i) f_{geq}((1 - z)p_i)} f_g(zp_i) f_g((1 - z)p_i) \\
- \int_0^1 dz \tilde{r}_{qg}(p, z) \frac{f_{geq}^z(p_i)}{f_{geq}(zp_i) f_{eq}(1 - z)p_i)} f_g(zp_i) f_q((1 - z)p_i) \\
+ \int_0^1 dz \tilde{r}_{qg}(p, z) \frac{f_{geq}(p_i)}{f_{geq}(zp_i) f_{eq}(1 - z)p_i)} f_q(zp_i) f_q((1 - z)p_i) \equiv C_{rc}^g(p_i)
\]

\[
\frac{\partial f_q(p_i)}{\partial t}_{rc} = -\int_0^1 dz \tilde{r}_{qg}(p, z) \frac{f_{eq}^{1-z}(p_i) - f_{eq}(p_i)}{f_{eq}(zp_i) f_{eq}(1 - z)p_i)} f_q(zp_i) f_q((1 - z)p_i) \\
- \int_0^1 dz \tilde{r}_{qg}(p, z) \frac{f_{geq}^z(p_i) + f_{geq}^{1-z}(p_i)}{f_{geq}(zp_i) f_{eq}(1 - z)p_i)} f_q(zp_i) f_q((1 - z)p_i) \equiv C_{rc}^q(p_i)
\]

Equilibrium distributions

\[
f_{geq}(p) = \frac{d_g}{\exp(\sqrt{p^2 + m_{th}^2}/T) - 1}
\]

\[
f_{eq}(p) = \frac{d_q}{\exp(\sqrt{p^2 + m_{th}^2}/T) + 1}
\]

- \(d_g = 16\)
- \(d_q = 31.5 \ (N_f = 3)\)
- \(m_{th} \sim \alpha_s^{1/2} T\)

\(T\) : fixed from momentum conservation
Parton splitting model

- Momentum smearing effect

  \[ \frac{\partial f}{\partial t} \bigg|_{\text{FP}} = \frac{\partial}{\partial p_i} \left[ A^i f + \frac{\partial}{\partial p_{ij}} (B^{ij} f) \right] = C_{\text{FP}} \]

  where \( f = \{ f_g, f_q \} \)

- Initial condition (CGC-like)

  - Gluon distribution
    \[ f_g(p < Q_s) \sim 1/\alpha_s \quad \text{and} \quad f_g(p > Q_s) \sim 0 \quad Q_s \sim 2 \text{ GeV} \]

  - Quark distribution
    \[ f_q(p, t_0) = 0 \]
Numerical analyses

- Time evolutions ($t = 0.2 \text{ fm/c}$)

Gluon distribution

Quark distribution

Numerical analyses

Gluon distribution

Quark distribution
Numerical analyses

- Time evolutions ($t = 0.4 \text{ fm/c}$)

![Gluon distribution](image1.png)

![Quark distribution](image2.png)
Numerical analyses

- Time evolutions (t = 0.6 fm/c)

**Gluon distribution**
- $f^g_{\text{init}}$ (t = 0.2 fm/c)
- $f^g$ (t = 0.6 fm/c)
- $f^g_{\text{eq}}$

**Quark distribution**
- $f^q_{\text{init}}$ (t = 0.2 fm/c)
- $f^q$ (t = 0.6 fm/c)
- $f^q_{\text{eq}}$
Numerical analyses

- Time evolutions ($t = 0.8 \text{ fm/c}$)

Gluon distribution

Quark distribution

- $f^g_{\text{init}}$ ($t = 0.2 \text{ fm/c}$)
- $f^g_t$ ($t = 0.8 \text{ fm/c}$)
- $f^g_{\text{eq}}$
Numerical analyses

- Time evolutions ($t = 1.0 \text{ fm/c}$)

- Distributions approach the thermal ones; chemical equilibration is relatively fast but would be slower than thermalization ($\sim 1.5$-$2.0 \text{ fm}$)
- Shape of quark distribution reflects that of gluon distribution as quarks are pair-created from gluons
- “Fermi pressure” would not develop as # of quarks are not enough
Numerical analyses

- Splitting with no recombination (logarithmic scale)

- The shape of quark distribution follows that of gluon distribution
- Quark number becomes *too large* for fermions near $p = 0$; splitting should be suppressed, leading to slower chemical equilibration
Numerical analyses

Comparison of $f_g$ (pure gauge), $f_g$ and $f_q$ ($N_f = 3$)

- Quark-gluon equilibration may be less “efficient” but more realistic
Summary and outlook

- Collinear quark and gluon splitting in early thermalization
  - Low momentum gluons are quickly produced
  - Quark production is reasonable fast; recombination would be important for fermions
  - Describes transition from CGC to QGP
  - Thermalization might be faster than chemical equilibration

- Future prospects include
  - Three dimensional modeling for analyses on effects of expansion and isotropization
  - Non-thermalized partons lead to off-equilibrium energy-momentum tensor at the initial time of hydrodynamic stage
The end

- Thank you for listening!
- Website: http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/