Bulk Viscous Effects on Relativistic Hydrodynamic Models of the Quark-Gluon Plasma

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Outline

- **Introduction**
  Relativistic viscous hydrodynamics

- **Distortion of Distribution**
  How to express $\delta f^i$ in terms of dissipative currents

- **Numerical Estimation**
  Effects of $\delta f^i$ on observables

- **Summary and Outlook**
  Summary and constitutive equations
Introduction

**Success of ideal hydrodynamic models**
at relativistic heavy ion collisions

**Development of viscous hydrodynamic models**
to correctly extract information from experimental data

**How does bulk viscosity affects observables?**
It has almost been neglected, BUT bulk viscosity is not so small near $T_c$

Bulk viscosity = response of pressure to volume change

- Mizutani et al. (‘88)
- Paech & Pratt (‘06)
- Kharzeev & Tuchin (’08) ...
Introduction

How does bulk viscosity affect observables?

- One needs a translator of flow field into particles at freezeout

**Cooper-Frye formula**

\[
\frac{d^2 N_i}{d^2 p_T dy} = \frac{g_i}{(2\pi)^3} \int \sum p_i^\mu \delta \sigma_\mu (f_0^i + \delta f^i)
\]

Modification of the distribution

Express \( \delta f^i \) with dissipative currents in a multi-component system
Macroscopic to Microscopic

Express $\delta f^i$ in terms of dissipative currents

**Macroscopic quantities**

**Dissipative currents** (given from hydro)

**Microscopic quantities**

**Distortion of distribution** (unknown)

14 “bridges” from Relativistic Kinetic Theory

\[
\Pi = -\frac{1}{3} \Delta_{\mu \nu} \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i
\]

\[
W^\mu = \Delta^\mu_\nu u_\rho \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^\nu p_i^\rho \delta f^i
\]

\[
V^\mu = \Delta^\mu_\nu \sum_i \int \frac{b_i g_i d^3 p}{(2\pi)^3 E_i} p_i^\nu \delta f^i
\]

\[
\pi^{\mu \nu} = \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^{\langle \mu} p_i^{\nu \rangle} \delta f^i
\]

\[
0 = u_\mu \sum_i \int \frac{b_i g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu \delta f^i
\]

\[
0 = u_\mu u_\nu \sum_i \int \frac{g_i d^3 p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i
\]
\[ \delta f^i \text{ in Multi-Component System} \]

- Grad’s 14-moment method $\leftrightarrow$ 14 unknowns $\varepsilon_{\mu}^{i}$, $\varepsilon_{\mu\nu}^{i}$

\[ \delta f^i = -f^i_0 (1 \pm f^i_0) [p_{i\mu}^{\mu} \varepsilon_{\mu}^{i} + p_{i\mu}^{\mu} p_{i\nu}^{\nu} \varepsilon_{\mu\nu}^{i}] \]

No scalar, but non-zero trace tensor

\[ \partial_{\mu} s^\mu = \varepsilon_{\mu\nu} \partial_{\alpha} I_{\mu\nu\alpha} \geq 0 : 2^{\text{nd}} \text{ law of thermodynamics} \]

+ constitutive equation $\partial_{\alpha} I_{\mu}^{\mu\alpha} \neq 0 \Rightarrow \varepsilon_{\mu}^{i} \neq 0$

- The distortion is uniquely obtained:

\[ \varepsilon_{\mu}^{i} = D_0 \Pi u_{\mu} + D_1 W_{\mu} + \tilde{D}_1 V_{\mu} \]

\[ \varepsilon_{\mu\nu}^{i} = (B_0 \Delta_{\mu\nu} + \tilde{B}_0 u_{\mu} u_{\nu}) \Pi + 2B_1 u_{(\mu} W_{\nu)} + 2\tilde{B}_1 u_{(\mu} V_{\nu)} + B_2 \pi_{\mu\nu} \]

where $D_i$ and $B_i$ are calculated in kinetic theory.
Models Inputs

- Estimation of particle spectra (with bulk viscosity in $\delta f^i$):

\[
\frac{d^2 N_i}{d^2 p_T dy} = \frac{g_i}{(2\pi)^3} \int \frac{1}{\Sigma} p^\mu d\sigma_\mu \left[ f^i_0 - f^i_0(1 \pm f^i_0)(D_0 \Pi_{\mu\nu}p^\mu + (B_0 \Delta_{\mu\nu} + \tilde{B}_0 u_{\mu}u_{\nu})p^\mu p^\nu)\right]
\]

Flow $u^\mu$, freezeout hypersurface $d\sigma_\mu$:
(3+1)-D ideal hydrodynamic model
- Hirano et al. (’06)

Equation of State: 16-component hadron resonance gas
(hadrons up to $\Delta(1232)$, under $\mu \to 0$)

Freezeout temperature: $T_f = 0.16$ GeV

Bulk pressure: $\Pi = -\zeta \nabla_{\mu} u^\mu$
- Navier-Stokes limit

Transport coefficients:
\[
\zeta = \alpha \left( \frac{1}{3} - c_s^2 \right)^2 \eta, \quad \eta = \frac{1}{4\pi} s
\]
where $c_s \equiv \sqrt{\frac{\partial p}{\partial e}}$: sound velocity
$s$: entropy density
- Weinberg (’71)
- Kovtun et al. (’05)
Bulk Viscosity and Particle Spectra

- \( Au+Au, \sqrt{s_{NN}} = 200(\text{GeV}), b = 7.2(\text{fm}), p_T\)-spectra and \( v_2(p_T) \) of \( \pi^- \)

\[ p_T\)-spectra \quad \rightarrow \quad \text{suppressed} \]

\[ v_2(p_T) \quad \rightarrow \quad \text{enhanced} \]

*Possible overestimations due to... (i) Navier-Stokes limit (no relaxation effects) (ii) ideal hydro flow (derivatives are larger)
Summary and Outlook

- Determination of $\delta f^i$ in a multi-component system
  - Viscous correction $\varepsilon_{\mu\nu}$ has non-zero trace.
- Visible effects of $\delta f_{\text{bulk}}$ on particle spectra
  - $p_T$-spectra is *suppressed*; $v_2(p_T)$ is *enhanced*

- Bulk viscosity can be important in extracting information (e.g. transport coefficients) from experimental data.
- **Full Viscous** hydrodynamic models need to be developed to see more realistic behavior of the particle spectra.
Estimation of Dissipative Currents

- 2nd order Israel-Stewart theory

Naïve generalization to a multi-component system does NOT work

Constitutive equations in a multi-component system:

**Bulk pressure**

\[
\Pi = -\zeta \theta \\
- \tau_\Pi D \Pi + \chi^{a}_{\Pi W} W_{\mu} D u^{\mu} + \chi^{a}_{\Pi V} V_{\mu} D u^{\mu} \\
+ \chi^{b}_{\Pi W} \nabla^{\mu} W_{\mu} + \chi^{b}_{\Pi V} \nabla^{\mu} V_{\mu} \\
+ \chi^{c}_{\Pi \Pi} + \chi^{c}_{\Pi \Pi} \Pi \theta + \chi^{c}_{\Pi W} W_{\mu} \nabla^{\mu} \phi_{\Pi W} \\
+ \chi^{c}_{\Pi V} V_{\mu} \nabla^{\mu} \phi_{\Pi V} + \chi_{\Pi \pi} \pi_{\mu \nu} \sigma^{\mu \nu}
\]

Shear tensor \( \pi^{\mu \nu} \) in conformal limit reduces to AdS/CFT result (Baier et al. ’08)

Navier-Stokes term

Israel-Stewart

2nd order terms

Post Israel-Stewart

2nd order terms
Thank You

- The numerical code will become available at

  http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/distributions.html
Appendix
Shear Viscosity and Particle Spectra

- $p_T$-spectra and $v_2(p_T)$ of $\pi^-$ with shear viscous correction

Non-triviality of shear viscosity; both $p_T$-spectra and $v_2(p_T)$ suppressed


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Shear & Bulk Viscosity on Spectra

- $p_T$-spectra and $v_2(p_T)$ of $\pi^-$ with corrections from shear and bulk viscosity

Accidental cancellation in viscous corrections in $v_2(p_T)$
Quadratic Ansatz

- $p_T$-spectra and $v_2(p_T)$ of $\pi^-$ when $\varepsilon_{\mu\nu} = C_1 \pi_{\mu\nu} + C_2 \Delta_{\mu\nu} \Pi$

Effects of the bulk viscosity is underestimated in the quadratic ansatz.
Bjorken Model

- $p_T$-spectra and $v_2(p_T)$ of $\pi^-$ in Bjorken model with cylindrical geometry: $R_0 = 10.0\text{fm}, \tau = 7.5\text{fm}$
  
  $u^\tau = 1$, $u^r = u^\phi = u^\eta = 0$
  
  $d\sigma_\tau = \tau d\eta r dr d\phi$, $d\sigma_\tau = d\sigma_\phi = d\sigma_\eta = 0$

**Bulk viscosity suppresses $p_T$-spectra**

**Shear viscosity enhances $p_T$-spectra**
Blast wave model

\[ u^r = u_0 \frac{r}{R_0} \left[ 1 + u_2 \cos(2\phi) \right] \Theta(R_0 - r) \]
\[ u^\tau = \sqrt{1 + (u^r)^2} \]
\[ u^\phi = u^{\eta_s} = 0 \]

Shear viscosity *enhances* \( p_T \)-spectra and suppresses \( v_2(p_T) \).

\[ R_0 = 7.5 \text{ fm}, \quad \tau = 5.25 \text{ fm} \]
\[ u_0 = 0.55, \quad u_2 = 0.2 \]