Dissipative hydrodynamic evolution of hot quark matter at finite baryon density

Akihiko Monnai

Department of Physics, The University of Tokyo
Theoretical Research Division, Nishina Center, RIKEN

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Introduction

- **Quark-gluon plasma (QGP):** many-body system of deconfined quarks and gluons

The QGP created in high-energy heavy ion collisions is quantified as a **relativistic fluid** with extremely small viscosity.

- **Net baryon density** has been neglected in viscous hydrodynamic analyses.

- **This talk:** viscous hydro + net baryon.
Introduction

- Observables of the hot QCD matter

Electromagnetic probes:
- Jets, heavy quarks:
- Hadronic spectra:

EM transparent
- color opaque
- hydrodynamic

This talk

Confinement X, 8th October 2012, Technische Universität München, Germany
Introduction

“Standard model” of high-energy heavy ion collisions

- Color glass condensate (CGC): saturated gluons ($\tau < 0 \text{ fm/c}$)
  - Local thermalization (instabilities in glasma?)
- Relativistic hydrodynamics: QCD fluid ($\tau \sim 1-10 \text{ fm/c}$)
  - Freezeout (flow to particle)
- Hadronic cascade, UrQMD: hadron gas ($\tau > 10 \text{ fm/c}$)
**Motivation**

- “CP asymmetry” of heavy ion collisions

Net baryon is conserved at forward rapidity
- Precision physics including particle identification (p/\bar{p} ratio, etc.)
- Finite-density transport properties

- Baryon stopping

**Baryon stopping** can quantify kinetic energy available for QGP production

mean rapidity loss \(<\delta y>\)
- rapidity of incoming projectile \(y_p\)
- mean rapidity of net baryon \(<y>\)
Motivation

- Exploring the **QCD phase diagram**
  - Finite baryon density is a *difficult* issue in first-principle calculations
  - Hydrodynamics can be a help in the exploration

**Aim of this work**

- Estimate dissipative hydro evolution of net baryon rapidity distribution with viscosities *and* baryon diffusion
  - (1+1)-D expansion is considered because dependence on transverse geometry is small
Dissipative hydrodynamics

- Decomposition of *energy-momentum tensor* and *net baryon current* in terms of flow $u^\mu$ in energy frame

\[ T^{\mu\nu} = (e_0 + P_0 + \Pi) u^\mu u^\nu - (P_0 + \Pi) g^{\mu\nu} + \pi^{\mu\nu} \]
\[ N^\mu_B = n_{B0} u^\mu + V^\mu \]

3 equilibrium quantities

- Energy density: $e_0$
- Hydrostatic pressure: $P_0$
- Net baryon density: $n_{B0}$

9 dissipative currents

- Bulk pressure: $\Pi$
- Shear stress tensor: $\pi^{\mu\nu}$
- Baryon dissipation current: $V^\mu$

- Naive interpretation of the dissipative processes

  - **bulk viscosity**
    - response to expansion
  - **shear viscosity**
    - response to deformation
  - **baryon dissipation**
    - response to chemical gradients
Dissipative hydrodynamics

- Relativistic hydrodynamic equations

Conservation laws
\[ \partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu N_B^{\mu} = 0 \]

The law of increasing entropy
\[ \partial_\mu s^{\mu} \geq 0 \]
Dissipative hydrodynamics

Relativistic hydrodynamic equations

Conservation laws

\[ \partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu N_B^{\mu} = 0 \]

\[ D = u^\mu \partial_\mu \]

\[ \nabla^\mu = \partial^\mu - u^\mu D \]

The law of increasing entropy \( \rightarrow \) Constitutive equations

\[
\Pi = -\varsigma \nabla_\mu u_\mu - \zeta \Pi \delta \epsilon D \frac{1}{T} + \zeta \Pi \delta n_b D \frac{\mu B}{T} - \tau \Pi D \Pi + \chi_a^{\Pi \Pi} \Pi D \frac{\mu B}{T} + \chi_b^{\Pi \Pi} \Pi D \frac{1}{T} + \chi_c^{\Pi \Pi} \Pi \nabla_\mu u_\mu \\
+ \chi_a^{\Pi V} V_\mu \nabla_\mu \frac{\mu B}{T} + \chi_b^{\Pi V} V_\mu \nabla_\mu \frac{1}{T} + \chi_c^{\Pi V} V_\mu D u_\mu + \chi_d^{\Pi V} V_\mu \nabla_\mu V_\mu + \chi^{\Pi \pi \pi \mu \nu} \nabla^{(\mu \nu)}
\]

\[
V_\mu = \kappa_V \nabla_\mu \frac{\mu B}{T} - \kappa_V W \left( \frac{1}{T} D u_\mu + \nabla_\mu \frac{1}{T} \right) - \tau_V \Delta^{\mu \nu} D V_\nu + \chi_a^{\Pi V} V_\mu D \frac{\mu B}{T} + \chi_b^{\Pi V} V_\mu D \frac{1}{T} \\
+ \chi_c^{\Pi V} \nabla_\mu u_\nu + \chi_d^{\Pi V} \nabla_\mu \nabla_\nu u_\mu + \chi_c^{\Pi V} V_\nu \nabla_\mu u_\nu + \chi_a^{\Pi \pi \mu \nu} \nabla_\nu \frac{\mu B}{T} + \chi_b^{\Pi \pi \mu \nu} \nabla_\nu \frac{1}{T} + \chi_c^{\Pi \pi \mu \nu} \nabla_\nu \frac{1}{T}
\]

\[
\tau^{\mu \nu} = 2\eta \nabla^{(\mu \nu)} - \tau \Pi D \tau^{(\mu \nu)} + \chi^{\Pi \Pi} \Pi \nabla^{(\mu \nu)} + \chi_a^{\Pi \pi \mu \nu} D \frac{\mu B}{T} + \chi_b^{\Pi \pi \mu \nu} D \frac{1}{T} + \chi_c^{\Pi \pi \mu \nu} \nabla_\rho u_\rho \\
+ \chi_d^{\Pi \pi \mu \nu} \nabla_\rho u_\nu + \chi_c^{\Pi V} V^{(\mu \nu)} \frac{\mu B}{T} + \chi_d^{\Pi V} V^{(\mu \nu)} \frac{1}{T} + \chi_c^{\Pi V} V^{(\mu \nu)} D u_\nu + \chi_d^{\Pi V} V^{(\mu \nu)}
\]
**Model input for hydro**

- **Equation of state:** Lattice QCD with Taylor expansion
  \[ \frac{P(T, \mu_B)}{T^4} = \frac{P(T, 0)}{T^4} + \frac{\chi_B^{(2)}(T, 0)}{2} \left( \frac{\mu_B}{T} \right)^2 + \mathcal{O} \left( \frac{\mu_B}{T} \right)^4 \]
  \[ P(T, 0) : \text{Equation of state at vanishing } \mu_B \]
  \[ \chi_B^{(2)}(T, 0) : \text{2nd order baryon fluctuation} \]

- **Transport coefficients:** AdS/CFT + phenomenology
  - Shear viscosity: \[ \eta = \frac{s}{4\pi} \]
  - Bulk viscosity: \[ \zeta = 5 \left( \frac{1}{3} - c_s^2 \right) \eta \]
  - Baryon dissipation: \[ \kappa_V = \frac{c_V}{2\pi} \left( \frac{\partial \mu_B}{\partial n_B} \right) \frac{1}{T} \]

- **Initial conditions:** *Color glass theory*
  - Energy density: MC-KLN
  - Net baryon density: Valence quark dist.

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S. Borsanyi et al., JHEP 1011, 077
S. Borsanyi et al., JHEP 1201, 138
P. Kovtun et al., PRL 94, 111601
A. Hosoya et al., AP 154, 229
M. Natsuume and T. Okamura, PRD 77, 066014
H. J. Drescher and Y. Nara, PRC 75, 034905; 76, 041903
Y. Mehtar-Tani and G. Wolschin, PRL 102, 182301; PRC 80, 054905
Results

- Net baryon rapidity distributions at RHIC and LHC

- Hydrodynamic evolution carries the net baryon number to forward rapidity
- Effects of viscosities and dissipation are visible at RHIC
Results

Mean rapidity loss at RHIC

Mean rapidity loss \( \langle \delta y \rangle = y_p - \langle y \rangle \)

\[
\langle y \rangle = \int_0^{y_p} y \frac{dN_{B-\bar{B}}(y)}{dy} dy / \int_0^{y_p} \frac{dN_{B-\bar{B}}(y)}{dy} dy
\]

Initial loss (RHIC): \( \langle \delta y \rangle = 2.67 \)

Ideal hydro: \( \langle \delta y \rangle = 2.09 \)

Viscous hydro: \( \langle \delta y \rangle = 2.16 \)

Dissipative hydro: \( \langle \delta y \rangle = 2.26 \)

- Transparency of the collision is effectively enhanced in hydrodynamic evolution

- More kinetic energy is available for QGP production

BRHAMS, PLB 677, 267 (2009)
Discussion

New result from LHC (QM 2012)

CMS PRELIMINARY

More transparent initial conditions are preferred

Note: a different observable

\[
\langle \delta y \rangle_E = \frac{2}{E_N N_{\text{part}}} \int_{-y_{\text{beam}}}^{y_{\text{beam}}} y' \frac{dE}{dy'} dy'
\]
Cross-coupling effects

- Linear response theory and cross terms

**Bulk pressure (w/o charges)**

\[
\Pi = -\zeta_{\Pi} \frac{1}{T} \nabla_{\mu} u^{\mu} - \zeta_{\Pi} \delta e D \frac{1}{T} = - \left( \frac{\zeta_{\Pi}}{T} + \frac{\zeta_{\Pi} \delta e}{T} c_s^2 \right) \nabla_{\mu} u^{\mu}
\]

- Response to expansion
- Response to cooling

- Bulk viscosity \( \zeta \)

- Response to expansion itself can be as large as shear viscosity
- Cancelled by the cross term except for crossover where \( c_s^2 \approx 0 \)

- A reason for general smallness of bulk viscosity

**Baryon dissipation current**

\[
V^{\mu} = \kappa_V \nabla^{\mu} \frac{\mu_B}{T} - \kappa_{VW} \left( \nabla^{\mu} \frac{1}{T} + \frac{1}{T} D u^{\mu} \right)
\]

- Baryon dissipation can be induced by thermal gradient + acceleration
Results

- Thermo-diffusion effect (a.k.a. Soret effect)

Numerical estimation of the vector cross-coupling where

\[ V^\mu = \kappa_V \nabla^\mu \frac{\mu B}{T} - \kappa_{VW} \left( \frac{\nabla^\mu}{T} + \frac{1}{T} Du^\mu \right) \]

\[ \kappa_{VW} = \frac{c_{VW} n_{B0}}{e_0 + P_0} \sqrt{\kappa_W \kappa_V} \]

- The cross coefficient can be either positive or negative

- The effect of the cross coupling is likely to be small in high-energy collisions because of the matter-antimatter symmetry

\[ V^\mu(\mu_B) = -V^\mu(-\mu_B) \] which leads to \( \kappa_{VW}(\mu_B = 0) = 0 \)
Summary and outlook

- Dissipative hydrodynamic model is developed at finite baryon density for the first time
  - Net baryon distribution is widened in hydrodynamic evolution
  - Transparency of the collision is effectively enhanced
  - More kinetic energy may be available at QGP production in early stage
- Results are sensitive to baryon diffusion coefficient
  - Ambiguities remain in the initial condition, but the distribution has important information

Future prospects include:
- Estimation of transverse expansion, inclusion of more realistic transport coefficients, etc.
The end

- Danke für Ihre Aufmerksamkeit!
- Website: http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/