Collinear parton splittings for early thermal and chemical equilibration

Akihiko Monnai
RIKEN BNL Research Center
In collaboration with: Berndt Müller (Duke U./BNL)

Brain Workshop 2014 “Frontiers of Hadronic Physics: Brains Recirculate Two”
13th March 2014, Brookhaven National Laboratory, Upton NY, USA
Introduction

- **Quark-gluon plasma (QGP):** many-body system of deconfined quarks and gluons

The QGP is supposed to have filled the early universe; it can be produced in heavy ion experiments at RHIC & LHC.

- Heavy ion QGP is found to exhibit near-perfect fluidity.
- **Early local equilibration** $(\tau < 0.4-1 \text{ fm})$ is implied.
Introduction

- Modeling a “Little Bang”

- \( \tau > 10 \text{ fm/c}: \) hadronic gas (weakly coupled)
- \( \tau \sim 1-10 \text{ fm/c}: \) QGP/hadronic fluid (strongly coupled)
- \( \tau \sim 0-1 \text{ fm/c}: \) CGC \( \rightarrow \) QGP ? (pre-equilibrated)
- \( \tau < 0 \text{ fm/c}: \) color glass condensate (saturated gluons)

Pre-equilibrated QCD medium is a high-energy frontier in hadron physics

Dynamical description is required for understanding QCD matter
Introduction

- **Color glass condensate (CGC)**
  - Gluons emitted from gluons emit gluons in a fast-travelling nucleon
  - They start to overlap and saturate
  - QCD matter at the initial stage of heavy ion collisions is **dominated by gluons**

- **QGP fluid**
  - Azimuthal momentum anisotropy $v_2$ is large compared with spatial one $\varepsilon_2$
  - QCD matter is **locally equilibrated** at some point and behaves as a fluid
Motivation (I)

- Early (local) equilibration

  One of the long standing issues in heavy ion physics
  - Naïve CGC and glasma lead to negative longitudinal pressure
  - CGC has large fraction of high-momentum gluons

- Equilibration requires

  We focus on thermalization and provide a model based on collinear splitting to introduce low momentum partons

Cf: parton cascade picture
- K. Geiger and B. Müller (1992)

Isotropization

\[ p_T \rightarrow \mathbf{p}_L \]

\&

Thermalization

\[ f \rightarrow p \]
Motivation (II)

- Early (local) equilibration

- Equilibration requires *more*

  ![Graphs showing Isotropization and Thermalization](#)

  ▶ Gluon medium eventually becomes *quark*-gluon plasma
  
  Is quark thermalization fast enough?
  
  Is the QGP chemical equilibrated?

Next slide: Collinear splitting model
Collinear splitting model

Gluon splitting

- A splitting modifies the parton distribution $f$ as

$$f(p_i) \rightarrow z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)$$

as number is **doubled** while momentum is **conserved**:

$$2 \int \frac{dp^d}{(2\pi)^d} f(p_i) = \int \frac{dp^d}{(2\pi)^d} \left[z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)\right]$$

$$\int \frac{dp^d}{(2\pi)^d} p_j f(p_i) = \int \frac{dp^d}{(2\pi)^d} p_j \left[z^{-d} f\left(\frac{p_i}{z}\right) + (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)\right]$$

- Time evolution of gluon distribution by collinear splitting

$$\frac{\partial f(p_i)}{\partial t} \bigg|_{sp} = \frac{1}{2} \int_0^1 dz \; r(z) \left[f^z(p_i) + f^{1-z}(p_i) - f(p_i)\right] \equiv C_{sp}(p_i)$$

where $f^z(p_i) = z^{-d} f\left(\frac{p_i}{z}\right)$, $f^{1-z}(p_i) = (1-z)^{-d} f\left(\frac{p_i}{1-z}\right)$ and

$$r(z) = \Gamma R_{gg}(z) = 6\Gamma \frac{[1 - z(1-z)]^2}{z(1-z)}$$

is the splitting rate
Collinear splitting model

- **Gluon + quark splittings**

**Time evolution of parton distributions by collinear processes**

\[
\frac{\partial f_g(p_i)}{\partial t} \bigg|_{sp} = \frac{1}{2} \int_0^1 dz \ r_{gg}(z)[f_g^z(p_i) + f_g^{1-z}(p_i) - f_g(p_i)] \\
+ \int_0^1 dz \ r_{gg}(z)f_q^z(p_i) - \int_0^1 dz \ r_{gg}(z)f_g(p_i) \equiv C_{sp}^g(p_i)
\]

\[
\frac{\partial f_q(p_i)}{\partial t} \bigg|_{sp} = \int_0^1 dz \ r_{gg}(z)[f_q^{1-z}(p_i) - f_q(p_i)] \\
+ \int_0^1 dz \ r_{gg}(z)[f_g^z(p_i) + f_g^{1-z}(p_i)] \equiv C_{sp}^q(p_i).
\]

**Splitting functions**

\[
r_{gg}(z) = \Gamma R_{gg}(z) = 6 \frac{[1 - z(1 - z)]^2}{z(1 - z)} \Gamma
\]

\[
r_{gq}(z) = \Gamma R_{gq}(z) = \frac{4}{3} \frac{1 + (1 - z)^2}{z} \Gamma
\]

\[
r_{qg}(z) = 2N_f \Gamma R_{qg}(z) = 3[z^2 + (1 - z)^2] \Gamma \quad (N_f = 3)
\]

**Quark-antiquark symmetry assumed**
Collinear splitting model

- **Gluon recombination**

  Time evolution should stop at local equilibrium (otherwise 2nd law of thermodynamics is violated)

  \[ \text{Recombination should occur when density is large (similar but different from Balitsky-Kovshegov equation)} \]

  Parameterization of recombination process

  \[
  \left. \frac{\partial f(p_i)}{\partial t} \right|_{rc} = -\frac{1}{2} \int_0^1 dz \tilde{r}(p_i, z) \frac{f_{eq}(z p_i) + f_{eq}^{1-z}(p_i) - f_{eq}(p_i)}{f_{eq}(z p_i) f_{eq}((1 - z) p_i)} f(z p_i) f((1 - z) p_i) \\
  \equiv c_{rc}(p_i)
  \]

  where \( f_{eq}(p) = \frac{d_g}{\exp\left( \sqrt{p^2 + m_{th}^2/T} \right) - 1} \) is equilibrium distribution in medium

  Degeneracy \( d_g = 16 \)

  Effective thermal mass \( m_{th} \sim \alpha_s^{1/2} T \)
Collinear splitting model

- Gluon + quark recombination

For quark-gluon mixture system

\[
\frac{\partial f_g(p_i)}{\partial t} \bigg|_{rc} = -\frac{1}{2} \int_0^1 dz \, \tilde{r}_{gg}(p, z) \frac{f_{geq}(p_i) + f_{geq}^{1-z}(p_i) - f_{geq}(p_i)}{f_{geq}(zp_i) f_{geq}((1-z)p_i)} f_g(zp_i) f_g((1-z)p_i)
- \int_0^1 dz \, \tilde{r}_{qg}(p, z) \frac{f_{geq}(p_i)}{f_{geq}(zp_i) f_{geq}((1-z)p_i)} f_g(zp_i) f_q((1-z)p_i) = C_{rc}^g(p_i)
+ \int_0^1 dz \, \tilde{r}_{qg}(p, z) \frac{f_{geq}(p_i)}{f_{geq}(zp_i) f_{geq}((1-z)p_i)} f_q(zp_i) f_q((1-z)p_i) = C_{rc}^q(p_i)
\]

Equilibrium distributions

\[
f_{geq}(p) = \frac{d_g}{\exp(\sqrt{p^2 + m_{th}^2}/T) - 1}
f_{eq}(p) = \frac{d_q}{\exp(\sqrt{p^2 + m_{th}^2}/T) + 1}
\]

- \(d_g = 16\)
- \(d_q = 31.5\) (\(N_f = 3\))
- \(m_{th} \sim \alpha_s^{1/2} T\)
- \(T\) : fixed from momentum conservation
Collinear splitting model

- Momentum smearing effect

▶ Splitting occurs when a parton becomes off-shell by interacting with medium

→ Parton-medium interaction is taken into account by relativistic Fokker-Planck equation

\[
\frac{\partial f}{\partial t} \bigg|_{\text{FP}} = \frac{\partial}{\partial p_i} \left[ A^i f + \frac{\partial}{\partial p_j} (B^{ij} f) \right] = C_{\text{FP}}
\]

\[
f = \{ f_g, f_q \}
\]

where \( A^i \sim \nu p^i \) and \( B^{ij} \sim D \delta^{ij} \) characterizes drag and diffusion effects

Relativistic Einstein relation \( D = \nu E T \) holds
Numerical analyses

- Initial conditions (CGC-like)

  - Gluon distribution
    \[ f_g(p < Q_s) \sim 1/\alpha_s \quad \text{and} \quad f_g(p > Q_s) \sim 0 \quad Q_s \sim 2 \text{ GeV} \]
  
  - Quark distribution
    \[ f_q(p, t_0) = 0 \]
  
  - We consider time evolution in a transverse direction for non-expanding systems for \( N_f = 0 \) and 3

- Parameter for splitting process

  - Rough estimation of parton emission rate \( \Gamma \)
    
    In static frame \( \Gamma \sim \alpha_s Q \)
    
    In boosted frame \( \Gamma \sim \alpha_s Q(Q/p) \sim \alpha_s \hat{q}L/p \)

    \[ \Gamma \sim \alpha_s^{1/2} (\hat{q}/p)^{1/2} \] is implied \( \quad \) (In thermal system \( \Gamma_{th} \sim \alpha_s^{3/2}T \))
Numerical analyses

- Parameter for momentum smearing

  ▶ Rough estimation of drag and diffusion coefficients

  Analytic solution of a diffusion equation for a delta function $\delta(p)$

  \[
  \frac{1}{\sqrt{4\pi D t}} e^{-p^2/4Dt} \quad \Rightarrow \quad \text{Standard deviation } \sigma = \sqrt{2Dt}
  \]

  \[
  \Leftrightarrow \quad \text{Longitudinal momentum modification } \sigma \sim Q^2/2p
  \]

  \[
  D \sim \hat{q}^2/\Gamma p^2 \quad \text{and} \quad D_{th} \sim \alpha_s^5/2T^3 \quad \text{are implied}
  \]

  Relativistic Einstein relation $D = \nu ET$ yields the drag coefficient
Gluon system ($N_f = 0$)

- Time evolution ($t = 0.8$ fm/c)

- Collinear gluon splitting contributes visibly to quick thermalization
- Entropy production is confirmed positive

- Recombination is important; Is the dynamics in dense region strong enough in 3D to enforce isotropization?
Quark-gluon system \((N_f = 3)\)

- **Time evolution** \((t = 1.0 \text{ fm/c})\)

- Distributions approach the thermal ones; chemical equilibration is relatively fast but would be slower than thermalization \((\sim 1.5-2.0 \text{ fm})\)

- Shape of quark distribution reflects that of gluon distribution as quarks are pair-created from gluons

- “Fermi pressure” would not develop as # of quarks are not enough
Quark-gluon system ($N_f = 3$)

- Splitting with no recombination (logarithmic scale)

- The shape of quark distribution follows that of gluon distribution
- Quark number becomes too large for fermions near $p = 0$; splitting should be suppressed, leading to slower chemical equilibration
Summary

- Comparison of $f_g$ (pure gauge), $f_g$ and $f_q$ ($N_f = 3$)

- Quark-gluon equilibration may be less “efficient” but more realistic
Summary and outlook

- Collinear quark and gluon splitting in early thermalization
  - Low momentum gluons are quickly produced
  - Quark production is reasonable fast; recombination would be important for fermions
  - Describes transition from CGC to QGP
  - Thermalization might be faster than chemical equilibration

- Future prospects include
  - Three dimensional modeling for analyses on effects of expansion and isotropization
  - Non-thermalized partons lead to off-equilibrium energy-momentum tensor at the initial time of hydrodynamic stage
Chemically-equilibrating QGP

Enhancement of photon $v_2$

Energy-momentum conservation

$$\partial_\mu T^\mu_\nu + \partial_\mu T^\mu_q = 0$$

Particle number evolution

$$\partial_\mu N^\mu_g = (r_a - r_b)n_g - r_a \frac{1}{n_g^{eq}} n_g^2 + r_b \frac{n_g^{eq}}{(n_q^{eq})^2} n_q^2$$

$$+ r_c n_q - r_c \frac{1}{n_g^{eq}} n_q n_g$$

$$\partial_\mu N^\mu_q = 2r_b n_g - 2r_b \frac{n_g^{eq}}{(n_q^{eq})^2} n_q^2$$

- Slow chemical equilibration = late production of quarks

- Photons are emitted from quarks after sizable elliptic flow has been developed
- Increased photon $v_2$; reduces the “large photon $v_2$” problem
In expanding geometry

- Expansion effects

Effects of expanding systems is implemented in drift term

\[
\frac{\partial f}{\partial t} + \frac{p_z}{E} \frac{\partial f}{\partial z} = \frac{\partial f}{\partial t} - \frac{p_z}{t} \frac{\partial f}{\partial p_z} = \mathcal{C}_{sp} + \mathcal{C}_{rc} + \mathcal{C}_{FP}
\]

in boost-invariant geometry

The distribution would be indirectly affected even at \( p_z = 0 \) because the equilibrium distribution changes
The end

- Thank you for listening!
- Website: http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/