Chapter 3: Video 3 - Supplemental Slides

Autocovariance and Autocorrelation

The function γ *(gamma) is called the autocovariance function.*

Note that $\gamma(h) = \gamma(-h)$. *Why?*

Assuming weak stationarity:

Correlation between Y_t *and* Y_{t+h} *is denoted by* $\rho(h)$.

The function ρ *(rho) is called the autocorrelation function.*

Note:

- $\gamma(0) = \sigma^2$ *(variance)*
- $\gamma(h) = \sigma^2 \rho(h)$ *(autocovariance)*
- $\rho(h) = \gamma(h)/\sigma^2 = \gamma(h)/\gamma(0)$ *(autocorrelation)*

Estimating Parameters of a Stationary Process

Suppose we observe $Y_1, \ldots, Y_n$ from a weakly stationary process.

Estimate the mean μ and variance σ^2 using:

- the sample mean $\overline{y}$ and sample variance s^2.

Estimate the autocovariance function using

- the sample autocovariance function

$$\widehat{\gamma}(h) = n^{-1} \sum_{t=1}^{n-h} (Y_{t+h} - \overline{y})(Y_t - \overline{y}) = n^{-1} \sum_{t=h+1}^{n} (Y_t - \overline{y})(Y_{t-h} - \overline{y}).$$

Some define $\widehat{\gamma}(h)$ with the factor n^{-1} replaced by $(n-h)^{-1}$

The difference is minor if n is large and h is small relative to n

Estimating Autocorrelations of a Stationary Process

To estimate $\rho(\cdot)$, we use the sample autocorrelation function (sample ACF) defined as

$$\widehat{\rho}(h) = \frac{\widehat{\gamma}(h)}{\widehat{\gamma}(0)},$$

for each lag h.

ACF Plots

R will plot a sample ACF with test bounds.

- Bounds test the null hypothesis that an autocorrelation coefficient is 0.
- The null hypothesis is rejected if the sample autocorrelation is outside the bounds.
- The usual level of the test is $\alpha = 0.05$
- We expect 1 out of 20 sample autocorrelations outside the test bounds simply by chance.

ACF Plots: Example

Inflation rates and changes in the inflation rate—sample ACF plots

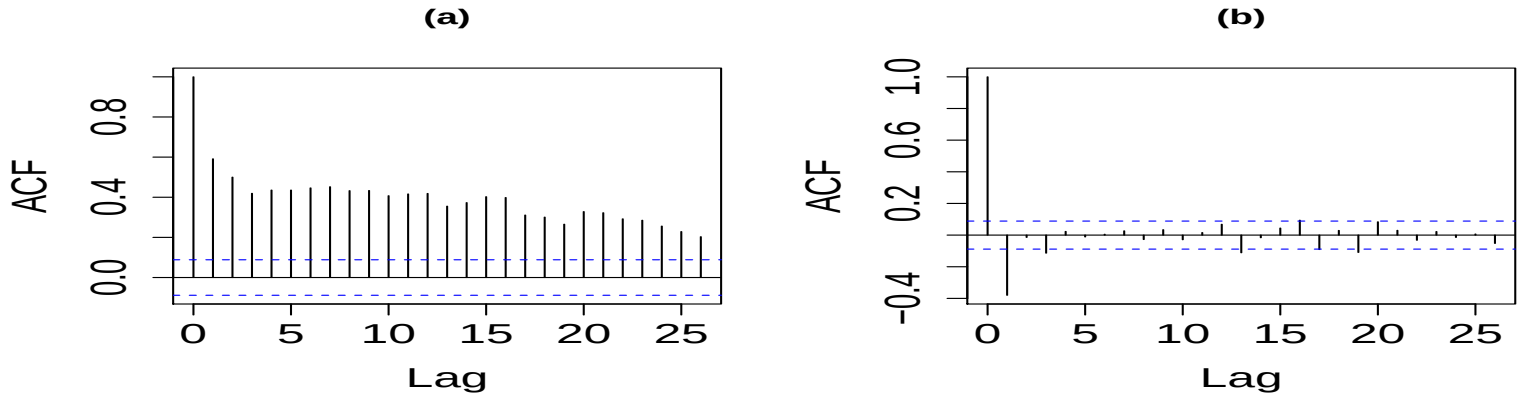

Figure: *Sample ACF plots of the one-month inflation rate (a) and changes in the inflation rate (b).*

```
data(Mishkin, package = "Ecdat")
y = as.vector(Mishkin[,1])
par(mfrow=c(1,2))
acf(y)
acf(diff(y))
```