Fields Medals in Mathematics Won by Four Under Age 40

The prize has gone to a woman for the second time in its history. One recipient didn't start working on complex math problems until he was 23.

By Kenneth Chang

July 5, 2022

Here's what you need to know:

- An Olympic-like honor for younger mathematicians.
- June Huh's monochrome chess puzzle paved the way for chromatic geometry.
- Maryna Viazovska: Second to none in any dimension.
- James Maynard minds the gap in prime numbers.
- Hugo Duminil-Copin and his magnetic math.
- Mark Braverman receives a refreshed computer science medal.

An Olympic-like honor for younger mathematicians.

Four mathematicians whose research covers areas like prime numbers and the packing of eight-dimensional spheres are the latest recipients of the Fields Medals, which are given out once every four years to some of the most accomplished mathematicians under the age of 40.

At a ceremony in Helsinki on Tuesday, the International Mathematical Union, which administers the awards, bestowed the medals, made of 14-karat gold, to Hugo Duminil-Copin, 36, of the Institut des Hautes Études Scientifiques just south of Paris and the University of Geneva in Switzerland; June Huh, 39, of Princeton University; James Maynard, 35, of the University of Oxford in England; and Maryna Viazovska, 37, of the Swiss Federal Institute of Technology in Lausanne.

Mark Braverman, 38, of Princeton University received the Abacus Medal, a newer award that was modeled after the Fields for young computer scientists.

Dr. Viazovska is just the second woman to receive a Fields Medal, while Dr. Huh defies the stereotype of a math prodigy, having not been drawn into the field until he was already 23 and in his last year of college.

The Fields Medals, first awarded in 1936, were conceived by John Charles Fields, a Canadian mathematician. They and the Abacus Medal are unusual among top academic honors in that they go to people who are still early in their careers — younger than 40 years on Jan. 1 — and honor not just past achievements but also the promise of future breakthroughs.

That the Fields are given only once every four years adds prestige through rarity — something more like gold medals at the Olympics. Another award, the Abel Prize, is modeled more on the Nobel Prize and recognizes mathematicians annually for work over their careers.

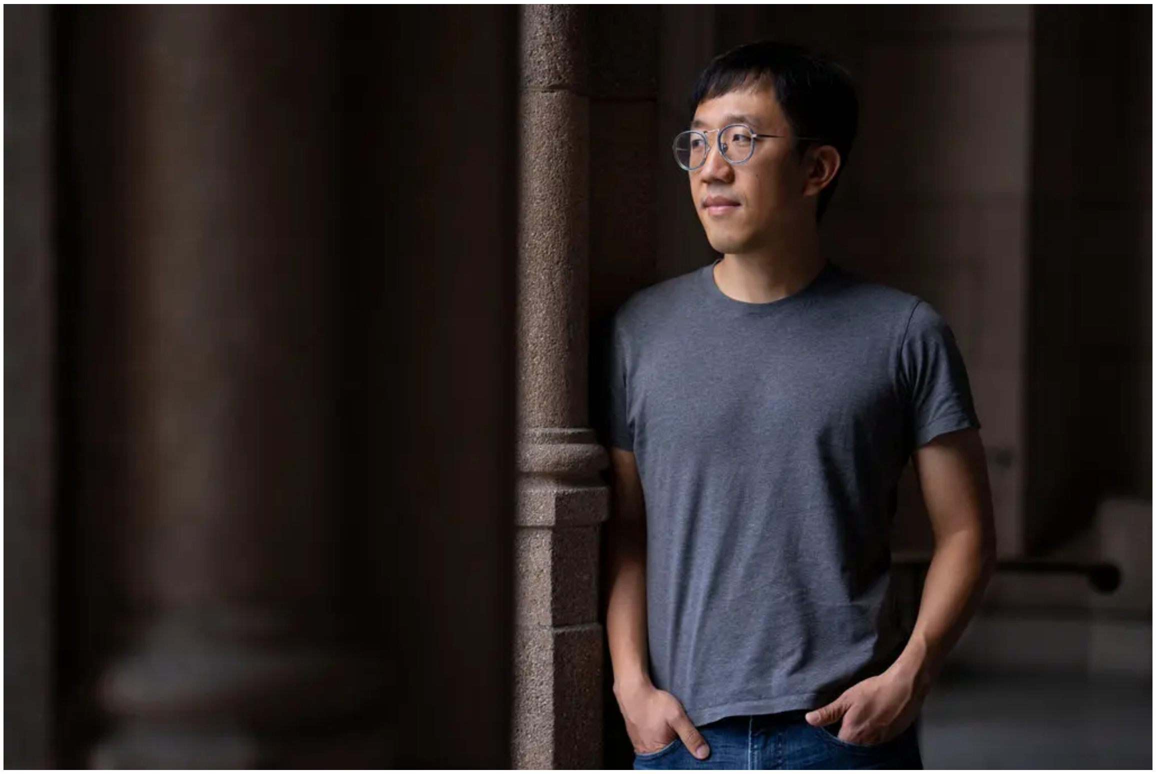
The recipients learned months ago that they had been chosen but were told not to share the news with friends and colleagues. They were not even told who else had been chosen.

“Officially, I don’t know,” Dr. Huh said last week. “I have guesses.”

Usually, the ceremonies for the Fields, the Abacus and other awards take place as part of the International Congress of Mathematics, a quadrennial gathering of mathematicians. This year’s congress was to be held in St. Petersburg, Russia. But after Russia’s invasion of Ukraine, the leadership of the International Mathematical Union decided to hold the congress as an online event outside Russia. The awards ceremony was still held in person but was moved to Helsinki.

— *Kenneth Chang*

June Huh’s monochrome chess puzzle paved the way for chromatic geometry.



Ruth Fremson/The New York Times

Most top mathematicians discovered the subject when they were young, often excelling in international competitions.

By contrast, math was a weakness for June Huh, who was born in California and grew up in South Korea. “I was pretty good at most subjects except math,” he said. “Math was notably mediocre, on average, meaning on some tests I did reasonably OK. But other tests, I nearly failed.”

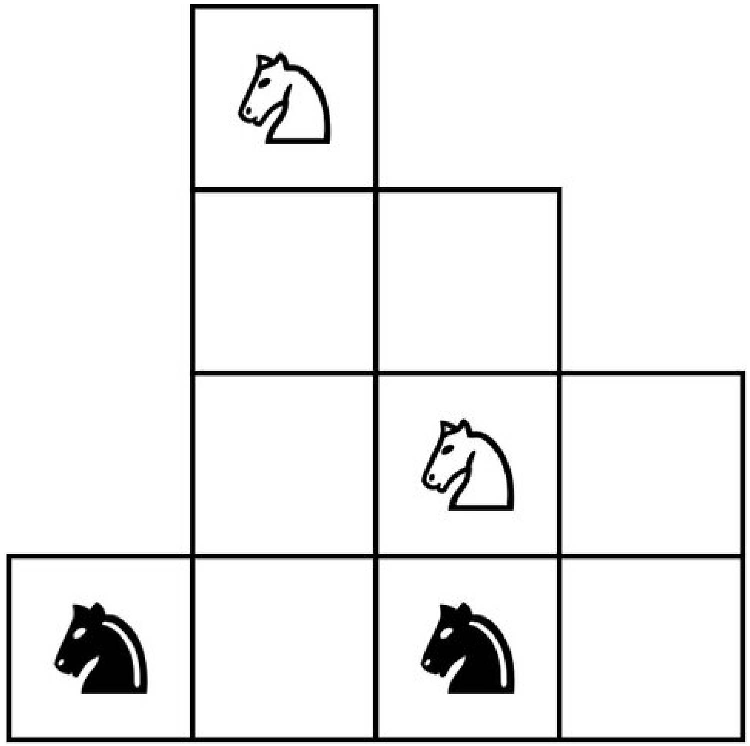
As a teenager, Dr. Huh wanted to be a poet, and he spent a couple of years after high school chasing that creative pursuit. But none of his writings were ever published. When he entered Seoul National University, he studied physics and astronomy and considered a career as a science journalist.

Looking back, he recognizes flashes of mathematical insight. In middle school in the 1990s, he was playing a computer game, “The 11th Hour.” The game included a puzzle of four knights, two black and two white, placed on a small, oddly shaped chess board.

The task was to exchange the positions of the black and white knights. He spent more than a week flailing before he realized the key to the solution was to find which squares the knights could move to. The chess puzzle could be recast as a graph where each knight can move to a neighboring unoccupied space, and a solution could be seen more easily.

Recasting math problems by simplifying them and translating them in a way that makes a solution more obvious has been the key to many breakthroughs. “The two formulations are logically indistinguishable, but our intuition works in only one of them,” Dr. Huh said.

Here is **the puzzle that June Huh beat**:



The New York Times

Goal: Exchange the positions of the black and white knights. →

It was only in his last year of college, when he was 23, that he discovered math again. That year, Heisuke Hironaka, a Japanese mathematician who had won a Fields Medal in 1970, was a visiting professor at Seoul National.

Dr. Hironaka was teaching a class about algebraic geometry, and Dr. Huh, long before receiving a Ph.D., thinking he could write an article about Dr. Hironaka, attended. “He’s like a superstar in most of East Asia,” Dr. Huh said of Dr. Hironaka.

Initially, the course attracted more than 100 students, Dr. Huh said. But most of the students quickly found the material incomprehensible and dropped the class. Dr. Huh continued.

“After like three lectures, there were like five of us,” he said.

Dr. Huh started getting lunch with Dr. Hironaka to discuss math.

“It was mostly him talking to me,” Dr. Huh said, “and my goal was to pretend to understand something and react in the right way so that the conversation kept going. It was a challenging task because I really didn’t know what was going on.”

Dr. Huh graduated and started working on a master’s degree with Dr. Hironaka. In 2009, Dr. Huh applied to about a dozen graduate schools in the United States to pursue a doctoral degree.

“I was fairly confident that despite all my failed math courses in my undergrad transcript, I had an enthusiastic letter from a Fields Medalist, so I would be accepted from many, many grad schools.”

All but one rejected him — the University of Illinois Urbana-Champaign put him on a waiting list before finally accepting him.

“It was a very suspenseful few weeks,” Dr. Huh said.

At Illinois, he started the work that brought him to prominence in the field of combinatorics, an area of math that figures out the number of ways things can be shuffled. At first glance, it looks like playing with Tinker Toys.

Consider a triangle, a simple geometric object — what mathematicians call a graph — with three edges and three vertices where the edges meet.

One can then start asking questions like, given a certain number of colors, how many ways are there to color the vertices with the rule that the vertices at each end of an edge cannot be the same color? The mathematical expression that gives the answer is called a chromatic polynomial.

More complex chromatic polynomials can be written for more complex geometric objects.

Using tools from his work with Dr. Hironaka, Dr. Huh proved Read’s conjecture, which described the mathematical properties of these chromatic polynomials.

In 2015, Dr. Huh, together with Eric Katz of Ohio State University and Karim Adiprasito of the Hebrew University of Jerusalem, proved the Rota Conjecture, which involved more abstract combinatorial objects known as matroids instead of triangles and other graphs.

For the matroids, there are another set of polynomials, which exhibit behavior similar to chromatic polynomials.

Their proof pulled in an esoteric piece of algebraic geometry known as Hodge theory, named after William Vallance Douglas Hodge, a British mathematician.

But what Hodge had developed, “was just one instance of this mysterious, ubiquitous appearance of the same pattern across all of the mathematical disciplines,” Dr. Huh said. “The truth is that we, even the top experts in the field, don’t know what it really is.”

Correction: July 5, 2022
Because of an editing error, this article misstated how chromatic polynomials are calculated. The two vertices at the end of an edge in a graph must be different colors, not that all of the vertices of the graph must be different colors.

— Kenneth Chang

Maryna Viazovska: Second to none in any dimension.

Fred Merz/EPFL

Maryna Viazovska, a Ukrainian who is now a professor at the Swiss Federal Institute of Technology in Lausanne, is known for proofs for higher-dimensional equivalents of the stacking of equal- sized spheres. She is also only the second woman ever to win the Fields Medal.

Of the 60 mathematicians who won Fields Medals before this year, 59 were men. In 2014, a Stanford mathematician, Maryam Mirzakhani, was the first and, until now, the only woman to receive one.

“I feel sad that I’m only the second woman,” Dr. Viazovska said. “But why is that? I don’t know. I hope it will change in the future.”

Dr. Viazovska’s work is a variation of a conjecture by Johannes Kepler more than 400 years ago. Kepler is best known for realizing that the planets move around the sun in elliptical orbits, but he also considered the stacking of cannonballs, asserting that the usual pyramid stacking was the densest way that they could arranged, filling up just under 75 percent of the available space.

Kepler could not prove that statement, however. Neither could anyone else until Thomas Hales, then at the University of Michigan, succeeded in 1998 with a 250-page proof and, controversially, the help of a computer program.

Proving something similar for the packing of equal-size spheres in dimensions higher than three has been impossible so far — with a couple of exceptions.

In 2016, Dr. Viazovska found the answer in eight dimensions, showing that a particularly symmetric packing structure known as E8 was the best possible, filling about one-quarter of the volume. Within a week, she and four other mathematicians showed that a different arrangement known as the Leech lattice was the best possible packing in 24 dimensions. In high dimensions, the filled volume is not very full, with the Leech lattice of 24-dimensional spheres occupying about 0.2 percent of the volume.

What’s so special about eight and 24 dimensions?

“I think that’s a mystery,” Dr. Viazovska said. “It’s just in these dimensions, certain things happen which don’t happen in other dimensions.”

She said that a method that generally gives an upper bound on the packing density turns out to be the exact solution in these cases.

High-dimensional sphere packings are related to the error-correcting techniques used to fix garbles in the transmission of information.

She said that the Russian invasion of Ukraine had taken its toll on her family. “It’s very difficult,” she said.

Her parents still live near Kyiv, Dr. Viazovska said, while her sisters, nephew and niece left and joined her in Switzerland.

Correction: July 5, 2022
An earlier version of this article misstated the packing density of spherical objects in three dimensions. It is just under 75 percent, not just over 75 percent.

— Kenneth Chang

James Maynard minds the gap in prime numbers.

Evan Nedyalkov

A prime number is a whole number that possesses exactly two divisors: 1 and itself. Thus, 2, 3, and 5 are prime; 6 is not, because it can be divided by 2 and 3.

“I personally find them just totally fascinating,” said James Maynard of the University of Oxford. “They feel like some of the most basic and fundamental objects in mathematics.”

More than 2,000 years ago, Euclid proved there are an infinite number of prime numbers, but many more questions about prime numbers remain.

“It’s the tension between being somehow simple and fundamental but mysterious and poorly understood,” Dr. Maynard said. “Which is really why they fascinate me so much.”

One of the unsolved problems is the Twin Prime Conjecture. Other than 2, even numbers are not prime because all even numbers can be divided by two. Thus, other than 2 and 3, the smallest difference between two prime numbers can be 2, and such pairs, like 5 and 7 or 11 and 13, are called “twin primes.”

As numbers get larger, prime numbers become sparser. And twin primes become scarcer still — yet mathematicians had long believed there are an infinite number of twin primes.

That is, the conjecture says that for any pair of primes separated by 2, there will always be a larger pair.

That is the problem Dr. Maynard has worked on for years. “We really don’t understand the gaps between prime numbers,” he said. “Typically the gap gets pretty big as you look at bigger and bigger prime numbers.”

But occasionally, two prime numbers are relatively close to each other.

In 2013, Yitang Zhang, then a little-known mathematician at the University of New Hampshire, published a breakthrough in the problem — that there are an infinite number of prime pairs whose separation is less than 70 million. (That was an arbitrary large number where Dr. Zhang’s equations worked.)

Six months later, Dr. Maynard reproduced Dr. Zhang’s result with a different approach.

Dr. Maynard also narrowed the size of the gap to 600.

Since the ancient Greeks, mathematicians have used what is called a sieve method to find prime numbers. In the simplest version, attributed to the Eratosthenes, one can first eliminate all multiples of 2, then multiples of 3 and so on for all known primes up to a given limit.

Dr. Maynard devised an improved sieve that could find groups of primes. “It could show you that not only do you get pairs of primes coming close together, you get clusters of many primes coming close together,” he said.

Thus, he could show there were triplets of primes within a certain range as well as larger clusters.

A group of mathematicians have since collaborated to push the size of the prime gap down to 246. (That group includes Terence Tao of U.C.L.A., who won a Fields in 2006 and is one of the most influential mathematicians today. Dr. Tao came up with almost the same result as Dr. Maynard but deferred publishing his result to avoid eclipsing Dr. Maynard.)

Another result that Dr. Maynard has proved is that there are an infinite number of primes that do not have the digit 7 in them. The same proof works for any other digit too.

“Although this sounds like a bit of a curiosity, the key point is that it overcame various sorts of technical mathematical hurdles for studying the prime numbers,” Dr. Maynard said.

He said he liked these problems involving prime numbers because someone who remembers some high school math can understand them.

“But then it’s sort of fascinating for me when you have to really go deeply into very complicated and very modern math to actually prove these results,” he said.

— *Kenneth Chang*

Hugo Duminil-Copin and his magnetic math.

Reto Albertalli for The New York Times

The areas of mathematics that Hugo Duminil-Copin work on are not entirely abstract. “Actually, for a long time, I was torn apart between physics and math,” he said. “I loved the idea of physics, that you want to describe the world around you. And at the same time, I loved the beauty of the math solution, where in some sense you have the concrete truth and there is no question about whether it is complete or not. It's just a proof."

The changing of matter from one form to another — for example, ice melting into liquid water — is known as a phase transition. Dr. Duminil-Copin's most noteworthy work involved studying the phase transition in a model of magnetism.

Certain materials like iron, nickel and cobalt are called ferromagnetic because their atoms act like tiny bar magnets. At high enough temperatures, the bar magnets randomly point in different directions, but the material as a whole is not magnetic. But when cooled below what is known as the critical temperature, the little bar magnets undergo a phase transition and start lining up to produce a measurable magnetic field.

Without that phase transition, permanent magnets would remain permanent magnets regardless of temperature.

Physicists have studied ferromagnetism and the phase transition from nonmagnetic to magnetic with what is known as the Ising model, named after Ernst Ising, a German physicist. Ising's thesis adviser, Wilhelm Lenz, came up with the model, and Ising solved the one-dimensional version of it for his doctoral thesis in 1924.

In two dimensions, the Ising model is much more difficult to unravel, with an exact solution found in 1944. The three-dimensional version — the one of most interest to physicists because we live in a three-dimensional universe — is intractable.

"The ability to produce exact formulas just collapses completely," Dr. Duminil-Copin said. "Nobody has any idea how to compute things exactly."

Computer simulations can provide approximate answers in three dimensions, but they do not help in answering certain questions. The exact two-dimensional solution for the Ising model shows that at the phase transition point, there are no sudden jumps in the magnetization as the temperature changes. The curve describing the magnetization does not have any breaks and is, in the language of mathematics, continuous.

That is not true for all phase transitions. When water freezes at 32 degrees Fahrenheit, its volume jumps as its molecules stack into the crystal structure of ice.

Although it is still not possible to solve the three-dimensional Ising model exactly, Dr. Duminil-Copin and his collaborators proved that the phase transition was continuous, just like it was in two dimensions.

The breakthrough came by making an unexpected connection between the Ising model and percolation — how water moves through porous rocks, for instance. The mathematicians had come up with a broader generalization of percolation.

“A large family, if you want, of percolation models,” Dr. Duminil-Copin said. “And once we had done that, then, in some sense, the connections started to click.”

The mathematicians found that the tendency of a region of bar magnets to line up in the same direction corresponded to certain aspects of percolation — a sort of dictionary to translate the math between percolation theory and the Ising model. “Basically, by combining the strengths of the two theories, we could break new ground,” Dr. Duminil-Copin said.

— *Kenneth Chang*

Mark Braverman receives a refreshed computer science medal.

Hannah Yoon for The New York Times

The work of Mark Braverman, a computer scientist at Princeton University who is this year’s recipient of the Abacus Medal, ranges widely, touching on dynamics, information theory, economics and game theory.

One particular topic he has worked on is communication complexity, which “asks about the amount of resources you need to perform some computational task” when the computation requires exchange of information between different parties, Dr. Braverman said.

That builds on the fundamental work of Claude Shannon, who founded the field of information theory by coming up with mathematical descriptions of how much information is contained within a string of 1s and 0s, the basic bits of data used by computers. That gave insight into how much data could be compressed and how to implement error correction to correct glitches during transmission.

While Shannon’s information theory described one-way communication, Dr. Braverman said he was interested in back-and-forth conversation. “There are two parties doing some computation together,” he said.

One simple example would be two people with two lists of names, and they want to know if there’s a name on both lists. “You can ask, how much information do we need to exchange to solve it?” Dr. Braverman said.

Dr. Braverman devised a protocol that minimized how much each party would need to reveal about what they knew.

The International Mathematical Union’s computer science award was established in 1981 as the Rolf Nevanlinna Prize, named after a Finnish mathematician. But in 2018, the general assembly of the International Mathematical Union decided to rename the prize to the Abacus Medal because Nevanlinna had been a collaborator with the Nazis during World War II.

— *Kenneth Chang*