

AMC, vii

American Mathematics Competitions, see AMC American Regions Math League, see ARML area, see probability, with areas arithmetic, 1 ARML, viii Art of Problem Solving, v, 244 average, 165

balls and boxes, 89–91 binomial, 209 binomial coefficients, 212 Binomial Theorem, 212 use in proving identities, 215 Buffon's Needle, 164

casework, 27-34 exclusive options, 29 independent choices, 29 organization using, 33 subcases, 33 Catalan numbers, 95–96 check your answer, 12 chickens counting before they hatch, 27 coefficients of binomials, 211-214 combination, 67 as binomial coefficients, 212 entries of Pascal's Triangle, 180 formula, 70 how to compute, 70, 71 identity, 74, 75 committees, 65-67, 74-75, 84-88 complementary counting, 35–37 when to use, 37

complementary probabilities, 129 constructive counting, 38-41 dealing with restrictions, 39 continuous, 155 convex polygon, 56 counting chickens before they hatch, 27 complementary, see complementary counting constructive, see constructive counting diagonals of a polygon, 56 independent events, 15 lists of numbers, 2–5 multiple events, 13–16 overlapping groups, 7 pairs of objects, 53-57 using Venn diagrams, 6–12 ways to form a committee, 66-67 with restrictions, 42-44 with symmetries, 57–61 democracy, 49 discrete, 155 distinguishable, 88–91 by the mind, not the eye, 81 distributions, 192–199 relationship to Pascal's Triangle, 197 using dividers, 198–199 equally likely outcomes, see outcomes, equally likely exclusive, 28 expectation, 165 and life, 165 expected value, 165, 166 factorial, 18 fair price, 168

INDEX

Fibonacci numbers, 189 Gauss, Carl Friedrich (1777-1855), 55, 174 general problem, 18 geometry, 153 and poetry, 153 areas, 156-161 lengths, 154-155 great rhombicuboctahedron, 224 great rhombicuoctahedron, 226 grid, 14 Harvard-MIT Math Tournament, see HMMT HMMT, viii Hockey Stick Identity, 200-204 definition of, 202 picture of hockey stick, 200 proof of, 202-203 icosahedron, 57 identity, 75 Hockey Stick, 200–204 how to prove, 186 Pascal's, 182 prove in many ways, 183 prove using Binomial Theorem, 215 sum of row in Pascal's Triangle, 185 inclusive, 2, 3 independent events, 15, 132 indistinguishable, 88-91 keychain, 60-61 Khayyám, Omar (c. 1048–1131), 177 Latin square, 79-80 Leibniz Triangle, 190 length, see probability, with lengths lists of numbers, 2–5 lost boarding pass problem, 152 magic square, 79-80 Mandelbrot Competition, vii MATHCOUNTS, vii mean, 165 memorization, 1, 18 mutually exclusive, see outcomes, mutually exclusive

n!, 18 *n*-queens problem, 48 n-rooks problem, 48 organization importance of, 14, 33 outcomes consistently counting, 117 equally likely, 115 mutually exclusive, 124, 126 overcounting, 49-61 Pascal's Identity, 182 proof using algebra, 182 proof using committees, 183 Pascal's Pyramid, 218 Pascal's Triangle, 177 3-D version, 218 coloring, 207-208 construction, 176-177 entries are combinations, 178-180 in Binomial Theorem, 211–212 path walking on, 179 relationship to distributions, 197 sum of entries in a row, 185 Pascal, Blaise (1623–1662), 175, 177 paths on a grid, 82–83 patterns, 40 permutation, 16, 19–21 with repeated elements, 49–52 probability, 111, 113 complementary, 128-130 consistently counting outcomes, 117 dependent events, 136-137 geometric, 153 independent events, 132 maximum and minimum possible, 113-114 mutually exclusive events, 125 notation, 113 using addition, 125 using algebra, 128 using multiplication, 131–135 with areas, 156–161 with lengths, 154-155 problem solving, iii think about it, 145

INDEX

reflective symmetry, *see* symmetry resources, v rotational symmetry, *see* symmetry round-robin tournament, 53, 55, 71 Rubik, Ernő (1944-present), 26 shortcuts, 4 Simpson's Paradox, 144 solving the general problem, 18 strategic overcounting, 51 subcases, 33 Sudoku, 79–80 sum of first *n* positive integers, 55, 62

symmetry, 58–61, 147 reflective, 61

rotational, 59

tree, 14, 16

USA Mathematical Talent Search, see USAMTS USAMTS, viii

variables choosing, 10 Venn diagram, 7–12 strategy for using, 9 Venn, John (1834–1923), 7–8

weighted average, 165

Yanghui (c. 1238–1298), 177