

1.8: Statistics

There are three measures of central tendency (averages) used in most competition problems: the mean, median, and mode for a set of data. The range is also used and is a measure of how spread out a set of data is.

The **arithmetic mean** is commonly referred to as the average for a set of values. The mean is the sum of all values divided by the number of values.

The **median** is the middle number for a set of data. When the values are arranged from least to greatest, the median is the value in the middle for an odd number of values. For an even number of values (where there is no middle term), the median is the mean of the two middle terms.

The **mode** is the term which occurs the most. There can be more than one mode (if more than one value occurs more than the rest), or no mode at all (if every value occurs with the same frequency).

The **range** is the difference between the highest and lowest values in a set.

Example: Find the mean, median, mode, and range for the following set of values: 16, 20, 13, 21, 24, 30, 21, 7.

Reasoning: First, we order the values from least to greatest: 7, 13, 16, 20, 21, 21, 24, 30.

Mean: $(7+13+16+20+21+21+24+30)/8 = 19$.

Median: $(20+21)/2 = 19.5$.

Mode: 21.

Range: $30 - 7 = 23$.

There are many other statistical measures, but these are the ones most commonly used in major competitions.

Statistics: Mean, Median, Mode and Range.

The most commonly used skill required for competition problems involves using the mean to find the sum of the values in a set of data.

Example: Peter averaged a 92 on his first seven quizzes. What score will he need on his 8th quiz to increase his average to a 93?

Reasoning: To average a 92 on seven quizzes, you would need to score $92 \cdot 7 = 644$ points. Make sure you see why: label his first 7 scores $a+b+c+d+e+f+g$.

$$\frac{a+b+c+d+e+f+g}{7} = 92,$$

$$\text{so } a+b+c+d+e+f+g = 92(7).$$

To average 93 on eight quizzes, he would need to score $93 \cdot 8 = 744$ points. This is a difference of 100 points, so Peter will need to score a 100 on his 8th quiz.

Sometimes it is actually quicker to do the math in your head. It doesn't matter what his first seven scores were, so to make it easy we will just assume he scored seven 92's. Another 92 would keep his average at 92, but we want to add a point to his average. One obvious way to do this would be to add a point to each of the eight scores. We can also add those 8 points to the final score, making it a $92 + 8 = 100$.

Example: The mean for the set of data below is 45. What is the value of x ? 35, 40, 42, 42, 46, 47, 50, x .

Reasoning: We could subtract the sum of the seven scores from $45 \cdot 8$ to get 58. There is a method that makes this easy to do in your head. Consider the difference of each number from the mean. The differences must add up to zero, so x is 13 more than the mean.

$$-10, -5, -3, -3, +1, +2, +5, +13 \text{ (58)}.$$

Practice: Mean, Median, Mode and Range.

- 1.051 What is the mean of the first ten positive *odd* integers?
- 1.151 The average for Akhil's first 7 tests this year is 88.
What score will he need to average on his next two tests if he wants to improve his test average to 90?
- 1.251 The mean of a set of seven positive integers is 7, the median is 8, and the distinct mode is 9. What is the greatest possible range for the set of integers?
- 1.351 Four integers are added to the set $\{3, 4, 5, 5, 8\}$, increasing the mean, median, and mode each by 1. What is the greatest integer in the new set?
- 1.451 Micah scores an 80 on his tenth quiz, increasing his quiz average by 2 points. What is his new quiz average?
- 1.551 In a set of ten integers, the average of the two smallest integers is 102, the average of the three smallest integers is 103, the four smallest is 104, and so forth until the average of all ten integers is 110. What is the greatest of these integers?
- 1.651 If Tanya scores 19 points in her last game, she will have averaged exactly 18 points per game for the season. If she can score 35 points, her average will be 20 points per game. How many games are in a season?
- 1.751 Austin has calculated that if he scores a 95 on his next three tests, it will improve his test average to 93. If Austin has taken six tests so far this year, what is his current average?
- 1.851 How many distinct sets of three positive integers have a mean of 6, a median of 7, and no mode?