

When comparing two positive numbers, the larger number always has the larger square.

EXAMPLE | Is $\sqrt{10}$ more or less than 3.5?

Since $\sqrt{10}$ and 3.5 are both positive, we can compare $\sqrt{10}$ and 3.5 by comparing their squares.

$\sqrt{10}$ is the number we square to get 10. So, $(\sqrt{10})^2 = 10$.

We have $3.5^2 = 12.25$.

Since $10 < 12.25$, we know $\sqrt{10}$ is **less than** 3.5.

COMPARING SQUARE ROOTS

PRACTICE

Fill each circle below with $<$ or $>$ to indicate which expression is greater.

51. $2 \bigcirc \sqrt{5}$

52. $\sqrt{3,599} \bigcirc 60$

53. $200 \bigcirc \sqrt{36,000}$

54. $\sqrt{250} \bigcirc 16$

55. $50 \bigcirc \sqrt{49 \cdot 51}$

56. $111 \bigcirc \sqrt{12,345}$

57. $\sqrt{\frac{1}{2}} \bigcirc \frac{2}{3}$

58. $\frac{7}{8} \bigcirc \sqrt{\frac{3}{4}}$

PRACTICE

Answer each question below.

59. For how many integer values of a is $\sqrt{\frac{1}{a}} > \frac{1}{2}$? 59. _____

60. Is $\sqrt{250}$ closer to $\sqrt{100}$ or to $\sqrt{400}$? 60. _____

61. Round $\sqrt{30.3}$ to the nearest whole number. 61. _____

EXAMPLE | Which is greater, $2\sqrt{3}$ or $3\sqrt{2}$?

The expression $2\sqrt{3}$ means $2 \cdot \sqrt{3}$.

To compare $2\sqrt{3}$ to $3\sqrt{2}$, we can compare their squares.

$$\begin{array}{rcl} (2\sqrt{3})^2 & & (3\sqrt{2})^2 \\ = 2 \cdot \sqrt{3} \cdot 2 \cdot \sqrt{3} & & = 3 \cdot \sqrt{2} \cdot 3 \cdot \sqrt{2} \\ = (2 \cdot 2) \cdot (\sqrt{3} \cdot \sqrt{3}) & & = (3 \cdot 3) \cdot (\sqrt{2} \cdot \sqrt{2}) \\ = 4 \cdot 3 & & = 9 \cdot 2 \\ = 12 & & = 18 \end{array}$$

Since 18 is greater than 12, $3\sqrt{2}$ is greater than $2\sqrt{3}$.

We read $3\sqrt{2}$ as "three root two," and $2\sqrt{3}$ as "two root three."



PRACTICE | Answer each question below.

62. What is the area in square centimeters of a square whose sides are $4\sqrt{5}$ centimeters long? **62.** _____

63. Circle every expression below that equals $3\sqrt{8}$.

$$\sqrt{72} \quad 2\sqrt{18} \quad 4\sqrt{6} \quad 5\sqrt{3} \quad 6\sqrt{2} \quad \frac{17}{2}$$

64. Circle every expression below that equals $\sqrt{500}$.

$$25\sqrt{2} \quad 20\sqrt{5} \quad 10\sqrt{5} \quad 8\sqrt{15} \quad 5\sqrt{20} \quad 2\sqrt{125}$$

65. Order 11, $\frac{\sqrt{500}}{2}$, and $2\sqrt{30}$ from least to greatest. **65.** _____ < _____ < _____

66. If n is an integer, and $13 < n\sqrt{11} < 14$, then what is n ? **66.** $n =$ _____
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EXAMPLE | Compute $\sqrt{25^3}$.

We could compute $25^3 = 25 \cdot 25 \cdot 25$ and then look for the square root of the result.

Or, we can use the fact that $25 = 5 \cdot 5$ to help us find the square root of 25^3 .

$$\begin{aligned}\sqrt{25^3} &= \sqrt{25 \cdot 25 \cdot 25} \\ &= \sqrt{(5 \cdot 5) \cdot (5 \cdot 5) \cdot (5 \cdot 5)} \\ &= \sqrt{(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)} \\ &= \sqrt{(5 \cdot 5 \cdot 5)^2} \\ &= 5 \cdot 5 \cdot 5 \\ &= 125\end{aligned}$$

So, the square root of 25^3 is **125**.

Factoring a number can help us find its square root.

**PRACTICE** | Write each expression below as an integer.

67. $\sqrt{11^4} =$ _____

68. $\sqrt{7^6} =$ _____

69. $\sqrt{9^3} =$ _____

70. $\sqrt{4^5} =$ _____

71. $\sqrt{3^2 \cdot 2^8} =$ _____

72. $\sqrt{3^4 \cdot 5^2} =$ _____

73. $\sqrt{6^2 \cdot 15^2} =$ _____

74. $\sqrt{12^3 \cdot 3} =$ _____

75. If $\sqrt{2^n} = 64$, then what is n ?

75. $n =$ _____



76. What is the units digit of $\sqrt{3^{100}}$?

76. _____



EXAMPLE | Compute $\sqrt{6 \cdot 24}$.

We multiply $6 \cdot 24 = 144$, then find the square root.

$$\begin{aligned}\sqrt{6 \cdot 24} &= \sqrt{144} \\ &= 12.\end{aligned}$$

— *or* —

Using prime factorization, we have

$$\begin{aligned}\sqrt{6 \cdot 24} &= \sqrt{(2 \cdot 3) \cdot (2 \cdot 2 \cdot 2 \cdot 3)} \\ &= \sqrt{(2 \cdot 2 \cdot 3) \cdot (2 \cdot 2 \cdot 3)} \\ &= \sqrt{(2 \cdot 2 \cdot 3)^2} \\ &= 2 \cdot 2 \cdot 3 \\ &= 12.\end{aligned}$$

PRACTICE | Solve each problem below.

77. $\sqrt{4 \cdot 9} =$ _____

78. $\sqrt{3 \cdot 27} =$ _____

79. $\sqrt{32 \cdot 8} =$ _____

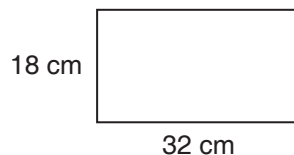
80. $\sqrt{21 \cdot 84} =$ _____

81. $\sqrt{135 \cdot 15} =$ _____

82. $\sqrt{2,916} =$ _____

83. What is the side length in centimeters of a square that has the same area as the rectangle below?

83. _____



84. The prime factorization of 1,382,976 is $2^6 \cdot 3^2 \cdot 7^4$. What is the prime factorization of $\sqrt{1,382,976}$?

84. _____

85. The expression $\sqrt{540 \cdot k}$ is equal to an integer for some positive integer k . What is the smallest possible value of k ?

85. _____