

Ms. Q.  
Estimation

Ms. Q.?

Yes, Grogg?

I know the square roots of perfect squares like 25, 81, and 144.

But, how do I find the square root of other numbers, like 20?

Does 20 even have a square root?

$\sqrt{25} = 5$

$\sqrt{81} = 9$

$\sqrt{144} = 12$

$\sqrt{20} = ?$

The square root of 20 is between 4 and 5.

How can you tell?

Because  $\sqrt{20}$  is between  $\sqrt{16}$  and  $\sqrt{25}$ .

I see.  $\sqrt{20}$  is more than  $\sqrt{16} = 4$ ...

...but less than  $\sqrt{25} = 5$ .

That's right. Is  $\sqrt{20}$  closer to 4 or to 5?

FOR ANY TWO NONNEGATIVE NUMBERS  $a$  AND  $b$ , IF  $a < b$ , THEN  $\sqrt{a} < \sqrt{b}$ . SO,  $\sqrt{16} < \sqrt{20} < \sqrt{25}$ .



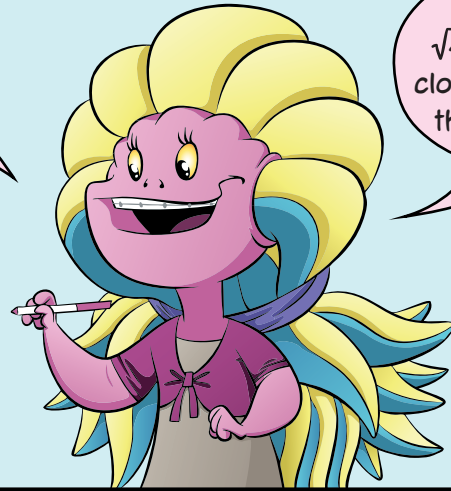
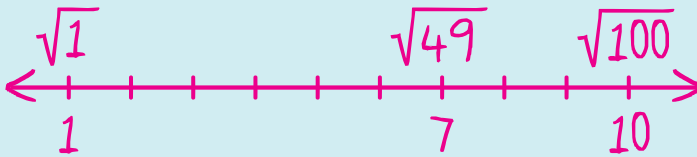
20 is closer to 16 than to 25.

Does that mean that  $\sqrt{20}$  is closer to  $\sqrt{16}$  than to  $\sqrt{25}$ ?

I don't think square roots work like that.

For example, 49 is closer to 1 than to 100...

...but  $\sqrt{49}$  is much closer to  $\sqrt{100}$  than to  $\sqrt{1}$ .



To find out if  $\sqrt{20}$  is closer to 4 or to 5, we need to know if  $\sqrt{20}$  is more or less than 4.5.

$4.5^2$  is 20.25.

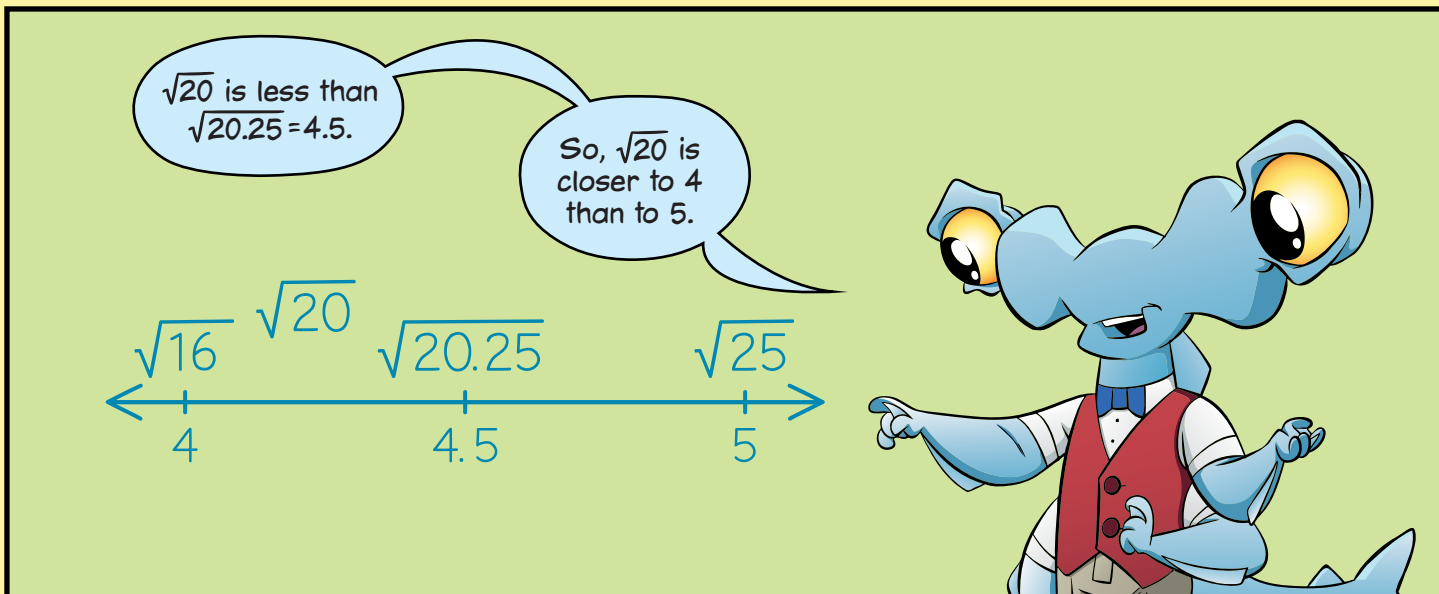
That means  $4.5 = \sqrt{20.25}$ .

$$4.5^2 = 20.25$$

$$4.5 = \sqrt{20.25}$$



How does knowing  $\sqrt{20.25} = 4.5$  help you estimate  $\sqrt{20}$ ?

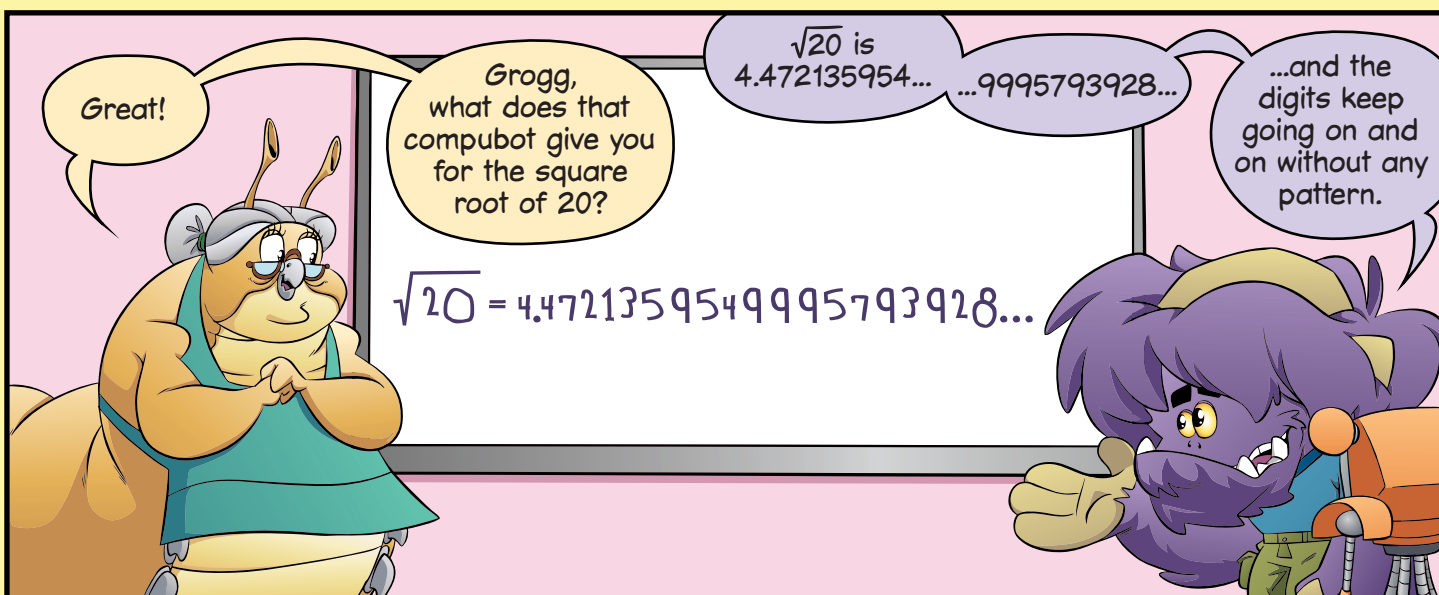


$\sqrt{20}$  is less than  $\sqrt{20.25} = 4.5$ .

So,  $\sqrt{20}$  is closer to 4 than to 5.

$\sqrt{16}$   $\sqrt{20}$   $\sqrt{20.25}$   $\sqrt{25}$

4 4.5 5



Great!

Grogg, what does that computob give you for the square root of 20?

$\sqrt{20}$  is 4.472135954... 9995793928...

...and the digits keep going on and on without any pattern.

$\sqrt{20} = 4.4721359549995793928...$



That's right, Grogg. The digits don't repeat regularly.

So, we can't write a decimal that is *exactly* equal to  $\sqrt{20}$ ...

...we can only estimate.

How could you estimate  $\sqrt{2}$  to the nearest tenth *without* a computob?

$\sqrt{2}$

$\sqrt{20}$   
= 4.472135954  
9995793928...

Find  $\sqrt{2}$  to the nearest tenth.