

DYNAMICAL CLASSIFICATION OF CHAOTIC CISLUNAR TRAJECTORIES VIA RESONANCE STRUCTURE

Amelie M. Lutz*, Dirk M. Luchtenburg†, Anjali Rawat‡, and Shane D. Ross§

Dynamics beyond geosynchronous orbit exhibit strongly nonlinear and chaotic behavior driven by lunar mean-motion resonances (MMRs). These resonances structure phase space through stable and unstable invariant manifolds associated with unstable MMR orbits. This study proposes a dynamical classification framework for chaotic trajectories in the circular restricted three-body problem at a fixed Jacobi constant. A Poincaré surface of section at perigee, expressed in semi-major axis and synodic longitude of perigee, is partitioned using unstable MMR periodic orbits and their invariant manifolds. Chaotic trajectories are then encoded through symbolic sequences representing residence within resonance zones, enabling systematic characterization of cislunar transport structure.

1 INTRODUCTION

The dynamics in the region beyond geosynchronous orbit (xGEO) are governed by strongly nonlinear and chaotic behavior. While this chaotic structure enables low-energy transfer opportunities, it also complicates the systematic identification, prediction, and design of such trajectories. With the rapid growth of planned missions and resident objects in the xGEO regime, systematic characterization of these trajectories is increasingly critical for space domain awareness and mission design.¹

Existing studies have revealed the underlying phase-space structure of these regions through resonance regions of influence and lobe dynamics.^{2,3} However, a general classification framework for chaotic cislunar trajectories that is grounded in these natural dynamical structures remains lacking.

Since the global structure of orbital phase space is strongly governed by resonance structures,¹ leveraging these features offers a robust path forward. In particular, lunar mean-motion resonances play a key role in shaping dynamics and stability by defining stable and unstable regions of influence. A classification framework grounded directly in resonance geometry provides a dynamically consistent alternative to traditional approaches that classify orbits by instantaneous orbital-element ranges or by numerically extracted fundamental frequencies.

To bridge this gap, this study constructs a symbolic, dynamical description of chaotic trajectories based on resonance zone visitation. Specifically, this paper introduces a novel “dynamical barcode” methodology, in the spirit of symbolic dynamics, where chaotic trajectories are described by their itineraries through a partitioned phase space.^{4,5} The barcode compresses complex chaotic orbital

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histories into a discrete, quantifiable sequence based on zone-to-zone transport. The contributions of this work are threefold: a partition of the perigee Poincaré section into resonance zones bounded by invariant manifolds, a transition-matrix quantification of the transport between them, and the barcode encoding itself, demonstrated at two energy levels.

The remainder of this paper is structured as follows: Section 2 introduces the Earth-Moon system, providing an overview of the planar circular restricted three-body problem, its parameterization using the Jacobi constant, and its application to a Poincaré surface of section at perigee. Section 3 details the post-processing of the Poincaré section, starting with a review of mean-motion resonances and their regions of influence, which are subsequently used to partition the section into zones. Section 4 discusses the transport of trajectories between these zones, quantifies the transport through a transition matrix, and, leveraging the zone partitioning and transport characterization, formulates the dynamical barcode, which is then demonstrated on a sample trajectory. Section 5 evaluates a simplified version of the framework at a different Jacobi constant, demonstrating its broader applicability. Finally, Section 6 summarizes the contributions of this work and proposes directions for future research.

2 MATHEMATICAL FRAMEWORK

This section establishes the mathematical model used to simulate spacecraft motion in the Earth-Moon system. The dynamical framework is the planar circular restricted three-body problem (PCR3BP). Using the Jacobi constant and a Poincaré surface of section, the phase space is reduced to two dimensions, enabling identification of the system's underlying orbital structures.

2.1 Planar Circular Restricted Three-Body Problem

To establish the dynamical environment for this study, the Earth-Moon system is modeled using the planar circular restricted three-body problem. This formulation describes the motion of a spacecraft of negligible mass moving under the influence of two primary bodies in mutual circular orbit, a solution of the two-body problem. The system is evaluated in a coplanar, barycentric, rotating reference frame, illustrated in Figure 1.

The system is non-dimensionalized such that the combined mass of primaries, the distance between them, and their angular velocity are all unity, so that the Moon's sidereal period is $T_m = 2\pi$. The system dynamics are governed by the mass ratio $\mu = \frac{m_2}{m_1+m_2}$, with $\mu = 0.012150584270571545$ for the Earth-Moon system considered here. The non-dimensional equations of motion for the spacecraft are,

$$\begin{aligned}\ddot{x} - 2\dot{y} &= x - (1 - \mu)\frac{x + \mu}{r_1^3} - \mu\frac{x - 1 + \mu}{r_2^3} \\ \ddot{y} + 2\dot{x} &= y - (1 - \mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3}\end{aligned}\tag{1}$$

where the Earth ($m_1 = E$) and Moon ($m_2 = M$) are fixed on the x -axis of the rotating frame at $-\mu$ and $1 - \mu$, respectively. The distances from the spacecraft to the Earth and Moon are $r_1 = \sqrt{(x + \mu)^2 + y^2}$ and $r_2 = \sqrt{(x - 1 + \mu)^2 + y^2}$, respectively.

Motion using this model evolves within a 4-dimensional phase space manifold, $\mathcal{M}$, described in Cartesian coordinates $X = (x, y, \dot{x}, \dot{y})$ or in geocentric orbital elements as $X = (a, e, l, \varpi)$, where a is the semi-major axis, e the eccentricity, l the mean anomaly, and ϖ the synodic longitude

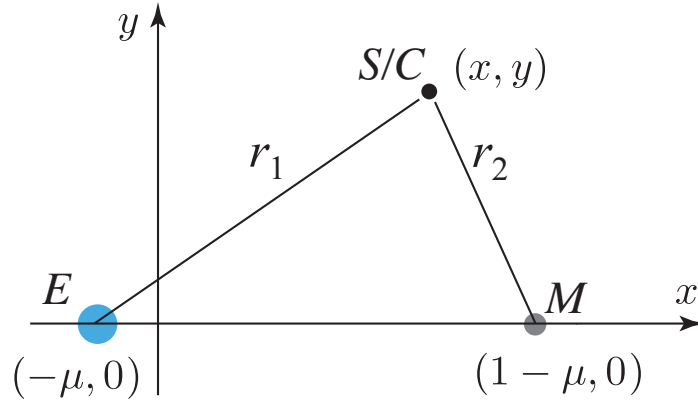


Figure 1: Schematic of the barycentric rotating reference frame of the planar circular restricted three-body problem, with the Earth and Moon fixed on the x -axis at $-\mu$ and $1 - \mu$, respectively.

of perigee. The phase space is restricted to a 3-dimensional energy surface by setting the Jacobi integral, Eq. (2), to a constant value ($\mathcal{C}(X) = C$), herein referred to as the Jacobi constant,

$$\mathcal{C}(x, y, \dot{x}, \dot{y}) = x^2 + y^2 + 2\left(\frac{1 - \mu}{r_1} + \frac{\mu}{r_2}\right) - (\dot{x}^2 + \dot{y}^2) \quad (2)$$

The Jacobi constant is approximated in geocentric orbital elements by the Tisserand parameter, Eq. (3), which constrains the orbital elements of the spacecraft,

$$T_p = \frac{1}{a} + 2\sqrt{a(1 - e^2)} \approx C. \quad (3)$$

To effectively visualize and analyze the complex trajectories within the 3-dimensional energy manifold, the system is further reduced to a discrete map by defining a Poincaré surface of section at perigee.

2.2 Poincaré Surface of Section at Perigee

Following the methodology developed by Rawat et al. 2026,² the surface of section is defined at the perigee events of the spacecraft. To construct the map for a fixed Jacobi constant, a grid of initial states is defined in Cartesian coordinates. These initial states are numerically integrated using MATLAB's `ode45` solver over a non-dimensional time span of $t \in [0, 1000]$. To isolate the perigee crossings, an event-detection algorithm is embedded in the propagation scheme. The Cartesian states are converted to geocentric orbital elements, where the perigee event function, Eq. (4) is triggered when $h(l) = 0$,

$$h(l) = \cos(l) + \frac{1}{4} \sin(l) - 1. \quad (4)$$

This transformation allows the section to be depicted in the orbital element parameter space, where the inherent cylindrical topology is flattened into a 2-dimensional phase space. Specifically, projecting these crossings into the (ϖ, a) space exposes the hidden dynamical features of the phase space,⁶ allowing for the visualization of periodic orbits, quasi-periodic librational tori, quasi-periodic circulating tori, and chaotic regions. Note that from the Jacobi constant and semi-major axis, the eccentricity of a state can be estimated from the Tisserand parameter, Eq. (3).

3 RESONANCE STRUCTURES AND CLASSIFICATION

With the phase space reduced to a 2-dimensional map, distinct dynamical regimes can be geometrically grouped. This section details the methodology for partitioning the phase space into discrete zones governed by the resonance structures. First, the unstable mean-motion resonances are identified and their associated stable and unstable invariant manifolds are computed to define the dynamical skeleton of the framework. These boundaries are then used to formally define distinct regions of influence.

3.1 Mean-Motion Resonances

Mean-motion resonances (MMRs) occur when the orbital periods of the spacecraft and the Moon form coprime, positive integer ratios, resulting in stable and unstable zones of influence. Consider a spacecraft that completes k orbits around the Earth (seen in an inertial frame) with an orbital period T , while the Moon completes k_m orbits with its sidereal period of T_m . In the CR3BP, their orbital ratio can be approximated by $T/T_m \approx k_m/k$ and they are denoted as a $k : k_m$ resonance.

To compute the MMR regions of influence, the unstable resonant periodic points must first be isolated. On a Poincaré section, resonant periodic orbits correspond to period- n points (generalization of fixed points), where a trajectory's crossings map back onto themselves after n iterations. For a $k : k_m$ resonance at a given Jacobi constant, a family of period- k points can typically be identified near a semi-major axis of approximately $a_{k:k_m} = (k_m/k)^{2/3}$, with k crossings per period on the section.

Saddle-type unstable resonant periodic points serve as the dynamical skeleton of this classification framework.⁷ The invariant manifolds associated with these saddle points define the boundaries of the resonance zones and form the basis for subsequent spatial partitioning.⁵ For each saddle-type periodic point, the stable and unstable invariant manifolds are computed by applying a small state displacement from the point along the stable and unstable eigenvectors of the point's monodromy matrix.⁸ These perturbed states are then propagated forward and backward in time. The resulting manifolds provide critical insight into the transport dynamics beyond the stable regions.

As a representative example, the 3:1 periodic orbits are identified at $C = 3.10$, and their corresponding manifolds are computed, as depicted in Figure 2. The stable and unstable manifold branches are propagated until their first intersection. This point, denoted by the shortest arclength between the two branches, is labeled as the boundary intersection point. The scope of this study is limited to interior resonance structures, where $k > k_m$; specifically, the 3:1, 5:2, 7:2, and 4:1 resonances are examined at a single Jacobi constant.

It should be noted that the 2:1 resonance is excluded from this analysis. At $C \geq 3.10$, the upper boundary intersection point for this resonance disappears,² preventing the proper determination of a closed unstable MMR zone. To compensate for this exclusion and ensure a sufficient partition of the phase space, the 7:2 resonance is evaluated.

3.2 Phase Space Partitioning

Building upon the invariant manifolds computed for low-order MMRs at $C = 3.10$, the primary manifold intersections form separatrices that bound the $k : k_m$ unstable resonance regions. The collection of unstable resonance regions associated with a single resonant periodic orbit family is grouped into an overarching “zone.” Since these regions share similar dynamical behaviors, the individual bounded areas comprising a zone are referred to as “sub-zones.” Establishing this

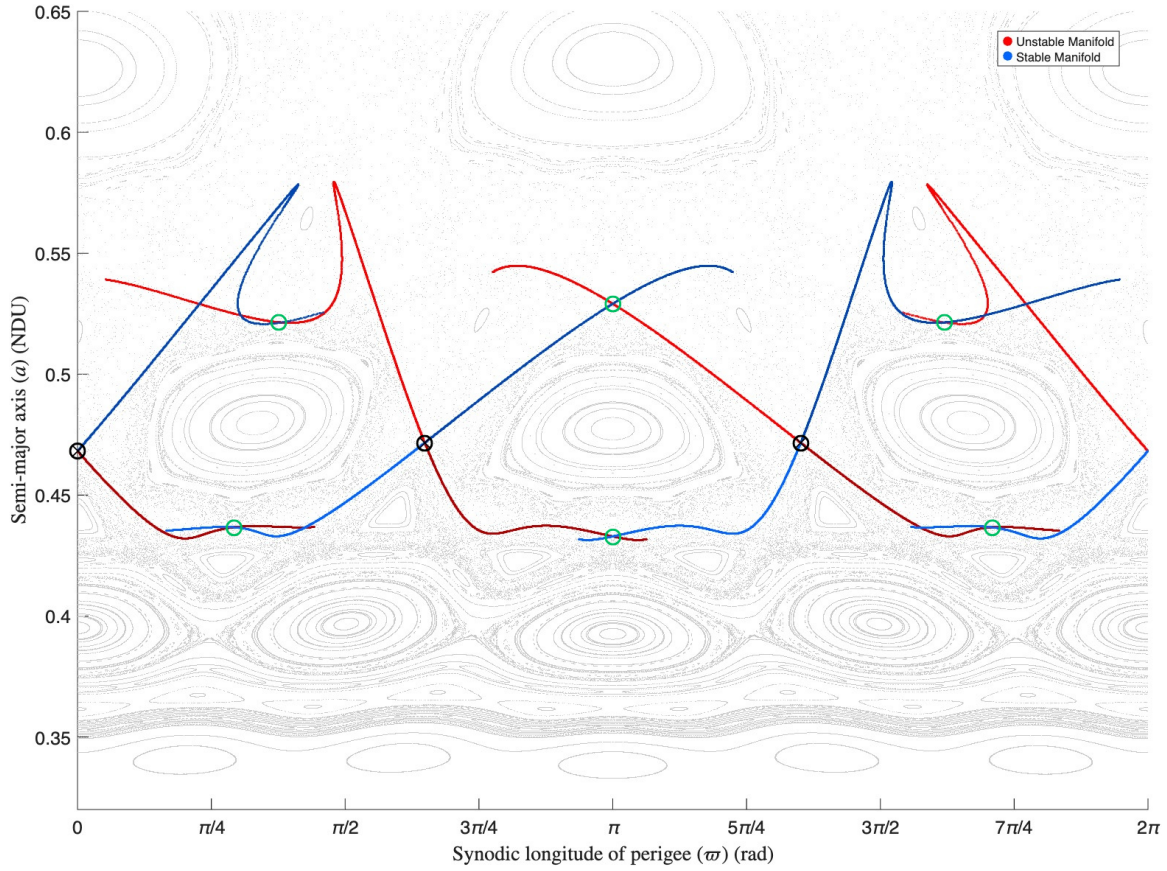


Figure 2: The 3:1 mean-motion resonance at $C = 3.10$ on the perigee surface of section, with saddle-type periodic points (black circles), stable (blue) and unstable (red) invariant manifold branches, and boundary intersection points (green circles) identified.

terminology allows for the systematic delineation of cislunar space, mapping regions which have dynamics similar to the periodic resonant orbits themselves. The resulting partition of the section at $C = 3.10$, including all four MMR zones, is shown in Figure 3.

By partitioning the Poincaré section in this manner, the phase space is divided into discrete zones governed by known underlying dynamics. Note that the zone associated with a particular resonance contains stable sub-zones, visible on the section as concentric closed loops (cross-sections of quasi-periodic librational tori). Outside the stable sub-zones, the dynamics are chaotic: while a chaotic trajectory’s short-term behavior may mimic that of the resonance zone it currently occupies, its long-term behavior may (and likely will) involve crossing into other resonance zones. On the Poincaré section, the chaotic region appears to be connected and is commonly referred to as the “chaotic sea,” a terminology we adopt here. Strictly speaking, however, the resonance zone boundaries subdivide this region, so one might more aptly speak of a chaotic *ocean* partitioned into several smaller chaotic *seas*.

Since the system is conservative, phase space volume is preserved. Consequently, initial states for a non-maneuvering spacecraft that originate in the chaotic sea will remain chaotic. For this reason, the stable MMR regions are dynamically isolated from the chaotic flow; they are therefore neglected

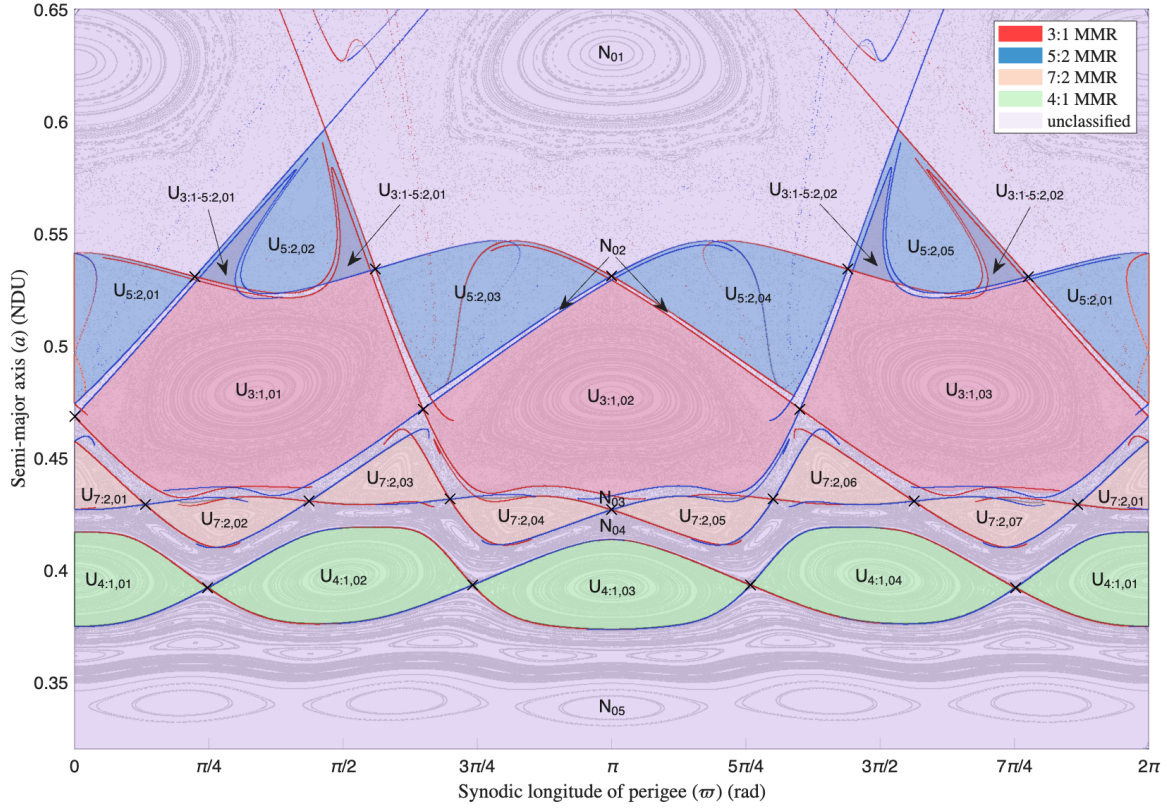


Figure 3: Perigee surface of section at $C = 3.10$ partitioned into unstable resonance zones governed by the 3:1, 5:2, 7:2, and 4:1 mean-motion resonances.

in this partitioning scheme, as they do not contribute to the system-wide transport dynamics.

The zone boundaries constructed in this manner are not invariant barriers to transport. Each boundary consists of stable and unstable manifold segments truncated at their boundary intersection point; beyond this point, the manifolds continue and intersect transversally, forming homoclinic and heteroclinic tangles.⁷ These tangles are the mechanism by which trajectories cross zone boundaries, entering and exiting the unstable MMR zones.⁶ The resulting transport dynamics are detailed in Section 4.

4 TRAJECTORY CHARACTERIZATION

Following the phase space partitioning into distinct resonance zones, the long-term behavior of trajectories can be systematically analyzed. This section discusses the global transport between zones, facilitated by the intersections of invariant manifolds. To quantify and track this chaotic transport, a symbolic classification scheme is introduced, translating complex orbital histories into easily comparative dynamical barcodes.

4.1 Transport Mechanisms

Beyond the boundary intersection point, subsequent intersections of the 1-dimensional stable and unstable manifolds dictate local phase space transport. Homoclinic connections arise when intersecting manifolds originate from the same periodic orbit family, whereas heteroclinic connections

occur between the manifolds of different periodic orbits. These geometric intersections generate transport pathways that organize global phase-space dynamics.⁹

Beyond the separatrix, successive manifold intersections enclose phase space regions into bounded lobes. These lobes act as turnstiles, governing transport into or out of resonance zones at each map iterate.^{3,7} Since the manifolds in this analysis are only propagated up until their first boundary intersection point, lobe-mediated transfers are beyond the scope of this study. Instead, the focus remains on near-heteroclinic transitions, rather than individual turnstile mechanics.

Transport between resonance zones is organized by intersections among the associated invariant manifolds, including heteroclinic pathways between distinct resonant structures. While these transitions can be visually highlighted on the Poincaré section, they are most effectively quantified via a transition matrix.

To demonstrate this zone transport and its quantification, a representative trajectory with an initial state of $X = [0.126167, 0, 0, 3.350792]$ is propagated for 500 perigee crossings at $C = 3.10$, as shown in Figure 4. Its transition matrix is shown in Figure 5, revealing the transport probabilities for this trajectory. By subjecting this trajectory to the established partitioning framework, its behavior, which would otherwise appear random in the chaotic sea, can be methodically characterized. It should be noted that the 4:1 MMR does not host any heteroclinic connections due to dynamical barriers, namely quasi-periodic circulating tori which bound the energy manifold.² However, the 4:1 zones are retained in this analysis to ensure a fully partitioned and visually complete phase space.

The transition matrix admits a direct dynamical interpretation. Its diagonal entries dominate, quantifying the “stickiness” of the resonance zones: once a trajectory enters a zone, it is far more likely to remain there at the next crossing than to escape. The off-diagonal entries provide empirical per-crossing escape probabilities between specific zone pairs. For the representative trajectory, residence in the 3:1 zone totals 436 of the 500 crossings across five visits, with only four zone departures, corresponding to a per-crossing escape probability of less than 1%; the 5:2 zone, visited twice for 28 total crossings, exhibits a per-crossing escape probability roughly an order of magnitude larger. Mean residence times per zone follow directly from the same entries, providing a compact quantitative summary of the trajectory’s transport behavior.

It should be emphasized that a transition matrix constructed from a single trajectory is an empirical estimate of the underlying transport operator, conditioned on the region of the chaotic sea explored by that trajectory. A natural extension is to estimate transition probabilities from an ensemble of initial conditions, in the spirit of set-oriented transfer operator methods,⁷ which would yield zone-to-zone transport probabilities characteristic of the energy surface as a whole rather than of an individual trajectory.

4.2 Dynamical Barcode Representation

Each trajectory is encoded as a symbolic sequence of resonance-zone labels, forming a dynamical “barcode.” This formulation records both the sequential order of zone visitations and the residence time spent within each region. Within this symbolic framework, unstable MMR regions of influence are designated by the identifier U , while the unclassified regions of the phase space remain denoted as N . A subscript is applied to indicate the specific MMR zone that was visited (e.g. $U_{3:1}$), and a superscript is used to denote the residency time within that zone, measured by the number of crossings (e.g. $U_{3:1}^n$).

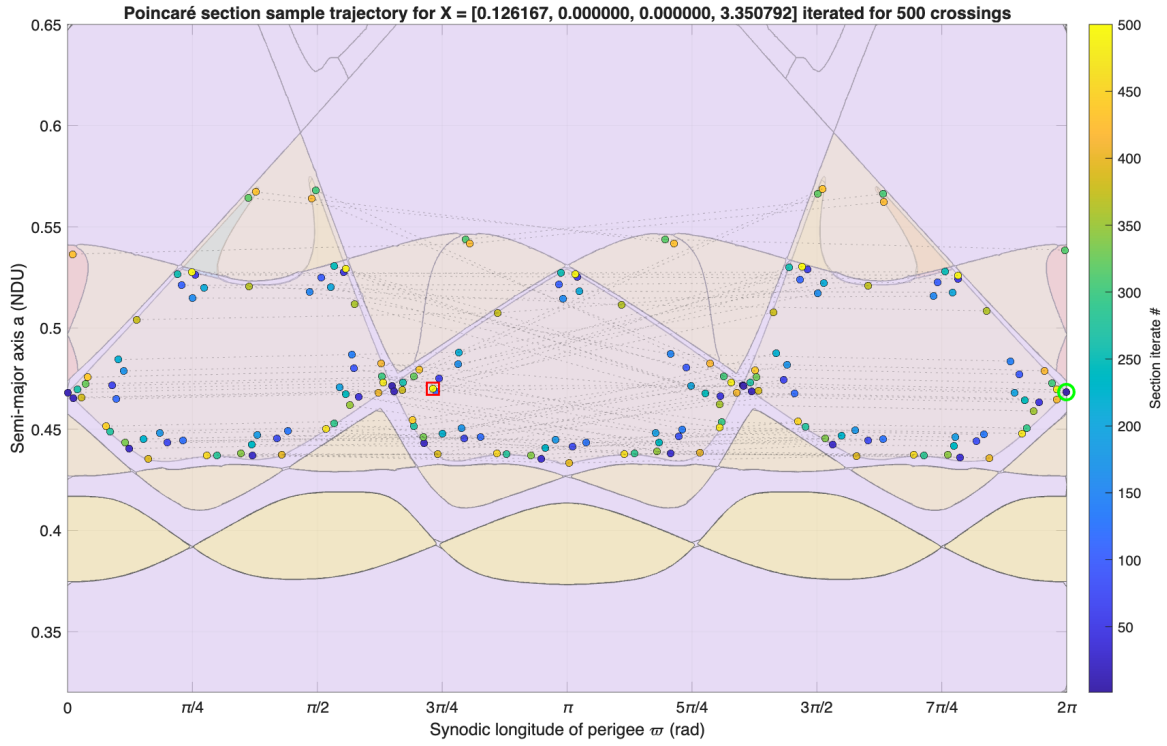


Figure 4: Sample trajectory with initial state $X = [0.126167, 0, 0, 3.350792]$ iterated for 500 perigee crossings, shown on a perigee surface of section at $C = 3.10$.

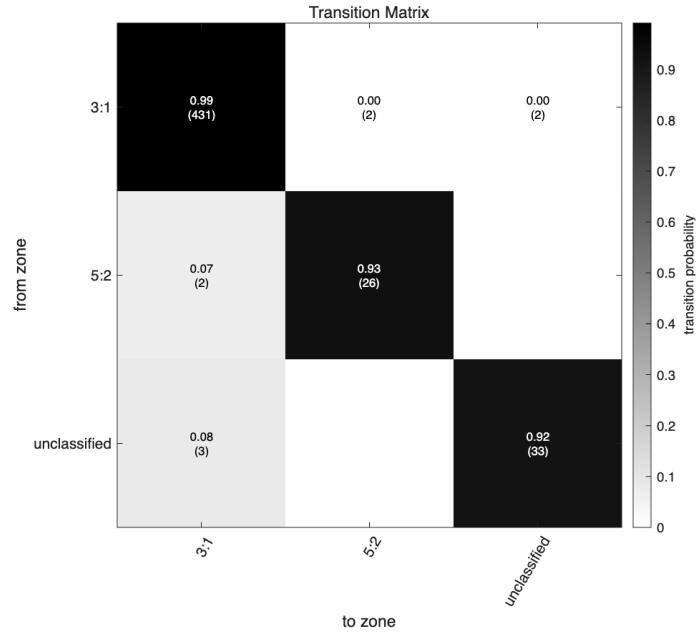


Figure 5: Transition matrix for sample trajectory with initial state $X = [0.126167, 0, 0, 3.350792]$ iterated for 500 perigee crossings at $C = 3.10$.

Figure 6 provides an application of this tool for the representative trajectory discussed previously, displaying the MMR zonings, the trajectory’s transport history, and the resulting barcode in a color-mapped format. This specific trajectory demonstrates a prolonged residence within the 3:1 MMR family before transitioning to the 5:2 family, with a symbolic barcode of $N^{29}U_{3:1}^{222}N^4U_{3:1}^{50}U_{5:2}^{14}U_{3:1}^{98}U_{5:2}^{14}U_{3:1}^{58}N^3U_{3:1}^8$.

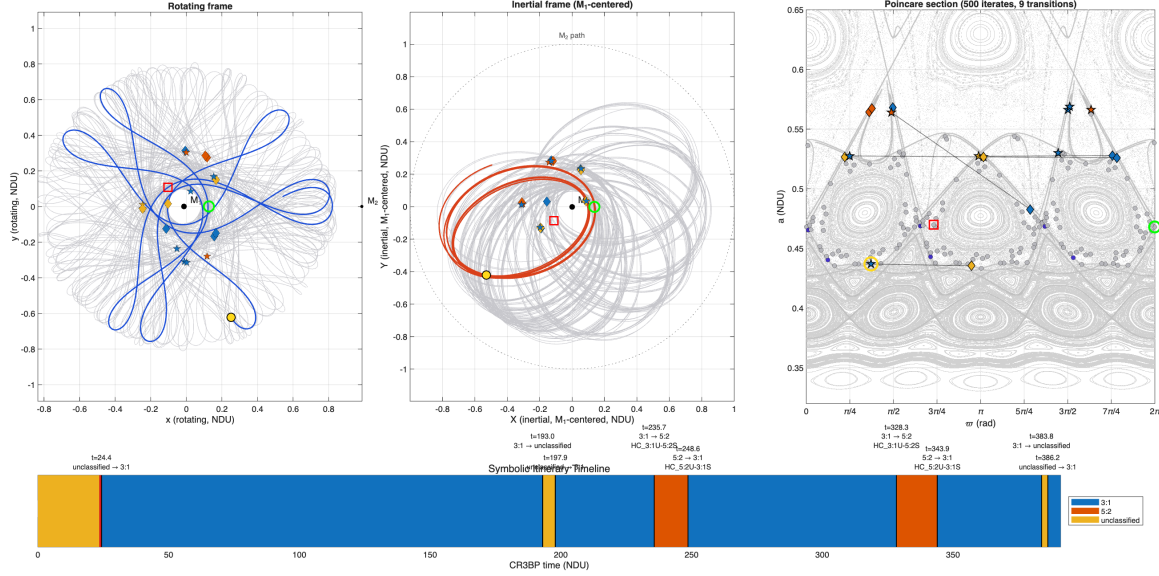


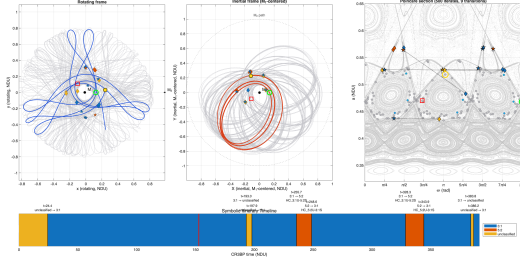
Figure 6: Application of the dynamical barcode methodology to the representative trajectory at $C = 3.10$: the trajectory history in rotating (left panel) and inertial (middle panel) frames, the partitioned Poincaré section with the trajectory’s zone visitation history (right panel), and the resulting barcode in color-mapped format (bottom panel). Markers highlighted in green and red represent the initial and final crossings, respectively. Trajectories that exit a zone are presented with a diamond marker, while the zone entrance trajectories are highlighted using a star marker.

Figure 7 shows the orbit progression of the initial state, demonstrating the consistency of the orbit behavior with MMR dynamics. Ultimately, this symbolic representation captures the full transport history of a trajectory. By compressing the complex, chaotic spatial dynamics into a discrete sequence, this tool enables the direct, large-scale comparison of trajectories across various initial states and facilitates the identification of dynamically driven trajectory classes.

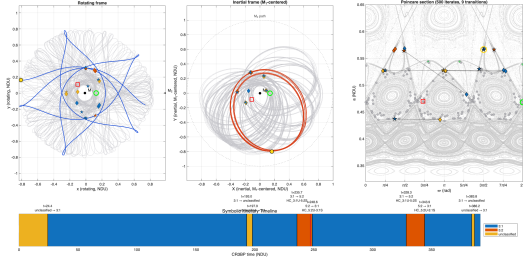
5 COMPARISON AT A HIGHER JACOBI CONSTANT

To assess the broader applicability of the framework, a simplified version of the analysis is repeated at $C = 3.20$ using the 3:1, 5:2, 7:2, and 4:1 MMRs, with the resulting partition shown in Figure 8. The increase in Jacobi constant corresponds to a lower-energy regime, and the resonance zone geometry changes accordingly: the 3:1 and 5:2 zones are wider, outlining a relatively larger stable region than their counterparts at $C = 3.10$, consistent with the energy-dependence of resonance widths reported by Rawat et al.²

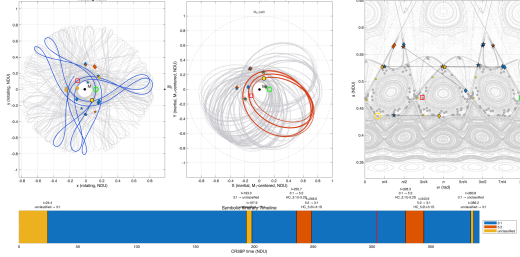
A representative trajectory is propagated for 500 perigee crossings on this energy surface, as shown in Figure 9. Because a state cannot lie on two energy surfaces simultaneously, the initial



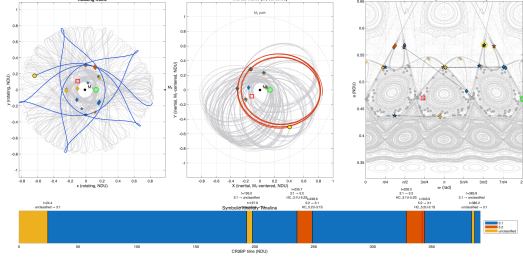
(a) Continuation in the 3:1 resonance region.



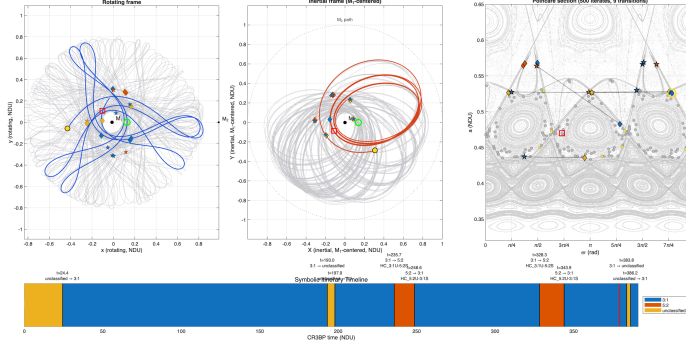
(b) Transition into the 5:2 resonance region.



(c) Return to 3:1 resonance region after the 5:2 resonance crossing.



(d) Return to 5:2 resonance region after the 3:1 resonance crossing.



(e) End in the 3:1 resonance region.

Figure 7: Perigee surface of section at $C = 3.10$ illustrating the evolution of the orbit within the 3:1 and 5:2 MMR zones. Panels (a)–(e) show representative sections progressing from the middle of the barcode in the 3:1 resonance region, through the transitions to the 5:2 resonance, and finally to the end of the barcode in the 3:1 resonance region.

condition is slightly perturbed ($X = [10^{-10}, 0, 0, 10^{-10}]$) from the 3:1 unstable resonance periodic point to exist in $C = 3.20$. Its transition matrix, Figure 10, and barcode, Figure 11, exhibit the same qualitative structure observed at $C = 3.10$: strongly diagonal transition probabilities reflecting resonance stickiness, punctuated by heteroclinic transitions between the 3:1 and 5:2 zones. The trajectory remains in the 3:1 resonance region for a majority of the orbit propagation, only remaining in the chaotic sea between the 3:1 and 5:2 regions for 81 crossings. The orbit propagation over time, Figure 11, show the progression represented by the symbolic barcode $N^{62}U_{3:1}^2N^{19}U_{3:1}^{417}$. Figure 12 shows the orbit continuation in the middle and end of the barcode, consistent with the expected 3:1

resonance behavior. The framework thus transfers across energy levels without modification; only the set of available resonance zones and their geometry change with the Jacobi constant.

The zone geometry includes a larger fraction of the domain with lower energies, accounting for more crossings that can be classified within the known dynamical regions. The set of resonances identifiable remains consistent across Jacobi constants studied, though individual resonances appear over different sub-ranges (e.g. 2:1 MMR).

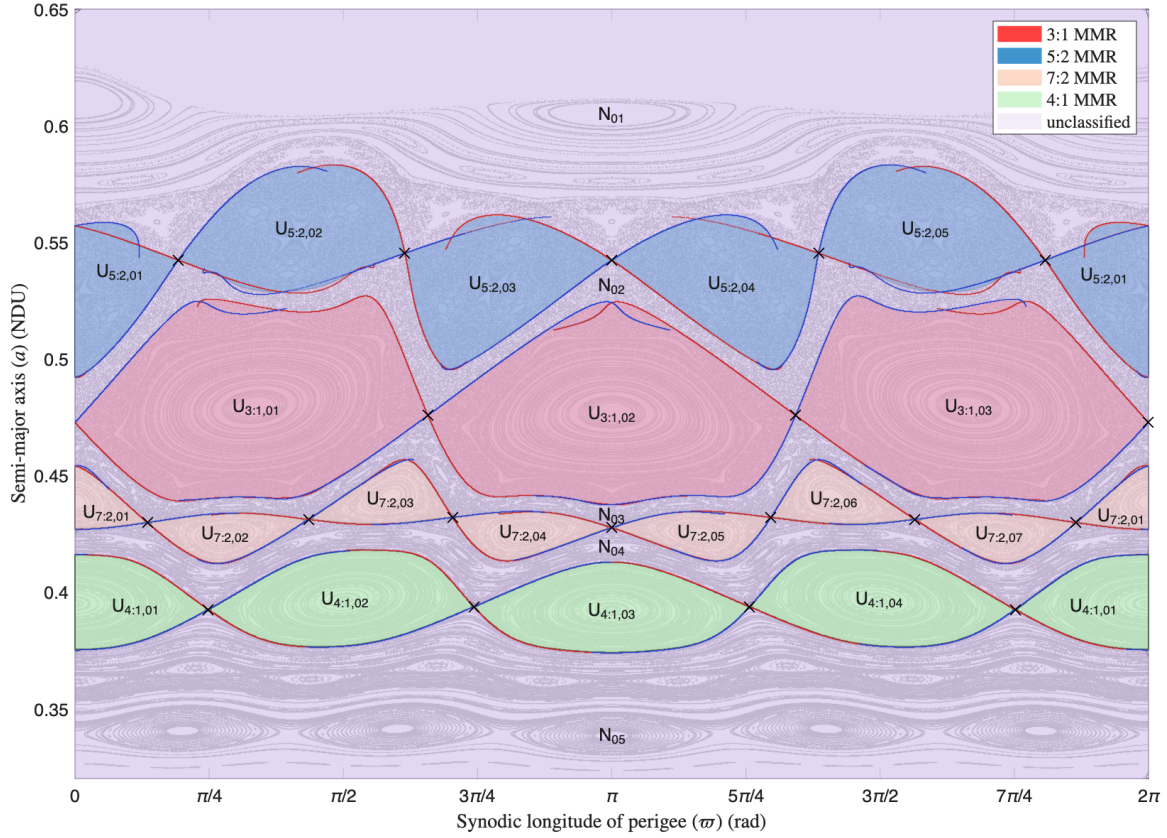


Figure 8: Perigee surface of section at $C = 3.20$ partitioned into unstable resonance zones governed by the 3:1, 5:2, 7:2, and 4:1 mean-motion resonances.

6 CONCLUSION

The highly chaotic nature of cislunar space presents a significant challenge for the astrodynamics community. While previous work highlights the underlying dynamics of this chaotic space using mean-motion resonances, this study leverages these structures to create a general, dynamically consistent classification framework for chaotic cislunar trajectories.

In this study, motion within the PCR3BP is parameterized by the Jacobi constant and reduced to a 2-dimensional discrete map using a Poincaré surface of section at perigee. Using $C = 3.10$, the unstable resonant periodic orbits are identified, and their 1-dimensional invariant manifolds are drawn to define local phase space boundaries. These boundaries establish the basis of the unstable regions of influence, allowing the phase space to be systematically partitioned into zones governed by the 3:1, 5:2, 4:1, and 7:2 MMRs. Furthermore, the transport behavior between these zones is

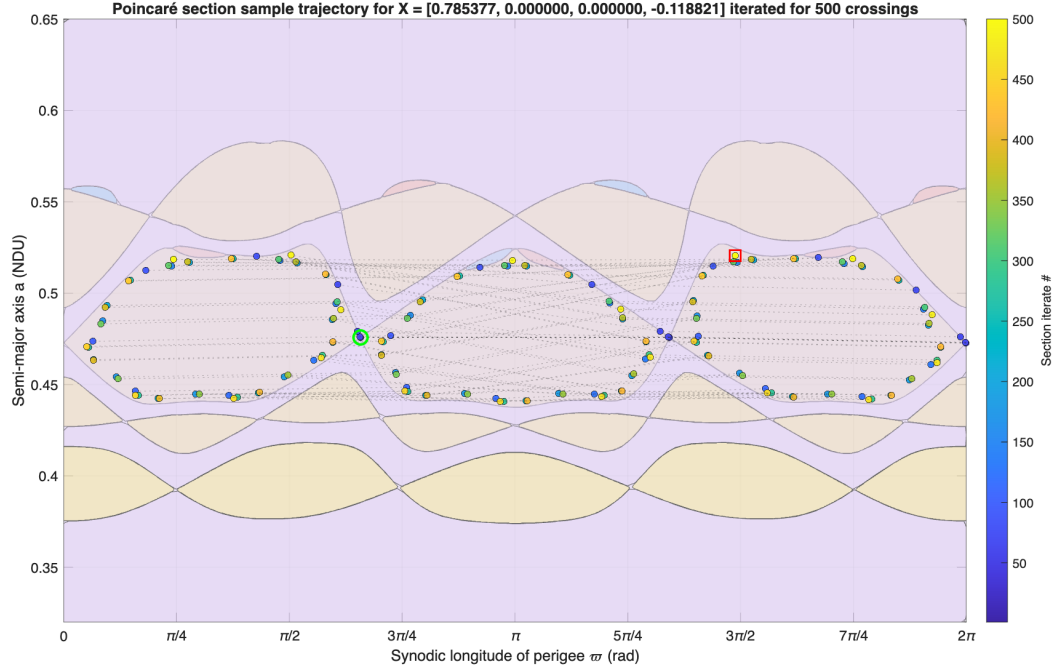


Figure 9: Sample trajectory with initial state $X = [0.785377, 0, 0, -0.118821]$ iterated for 500 perigee crossings, shown on a Perigee surface of section at $C = 3.20$.

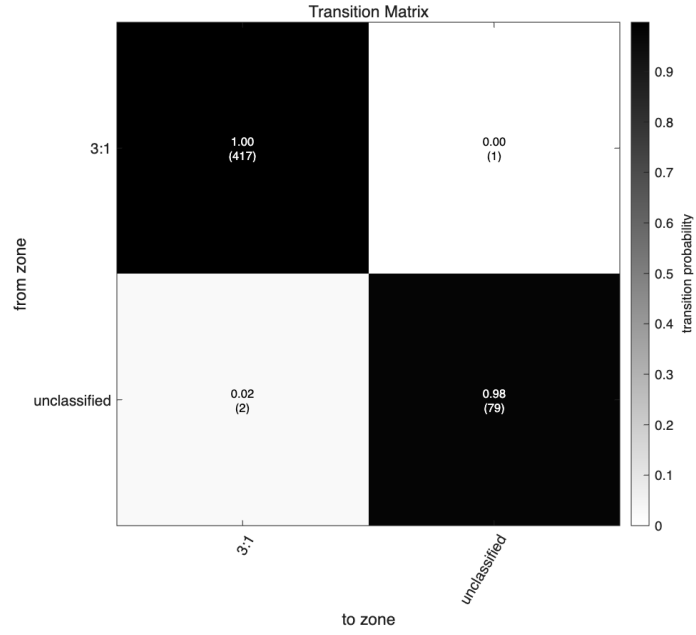


Figure 10: Transition matrix for sample trajectory with initial state $X = [0.785377, 0, 0, -0.118821]$ iterated for 500 perigee crossings at $C = 3.20$.

classified according to transition pathways and quantified via a transition matrix.

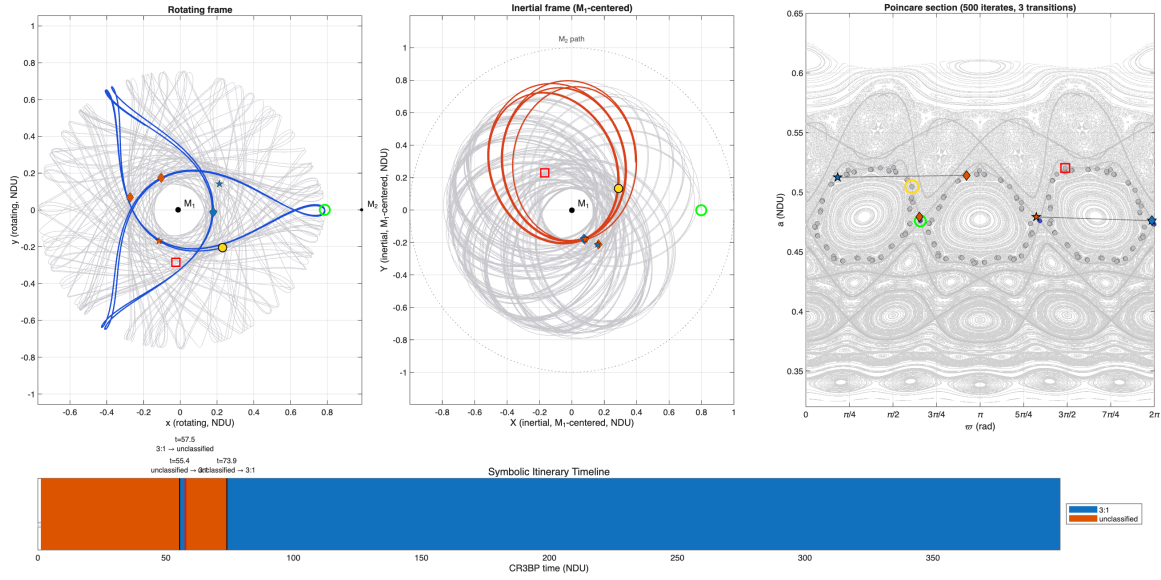


Figure 11: Application of the dynamical barcode methodology to the representative trajectory at $C = 3.20$: the trajectory history in rotating (left panel) and inertial (middle panel) frames, the partitioned Poincaré section with the trajectory's zone visitation history (right panel), and the resulting barcode in color-mapped format (bottom panel). Markers highlighted in green and red represent the initial and final crossings, respectively. Trajectories that exit a zone are presented with a diamond marker, while the zone entrance trajectories are highlighted using a star marker.

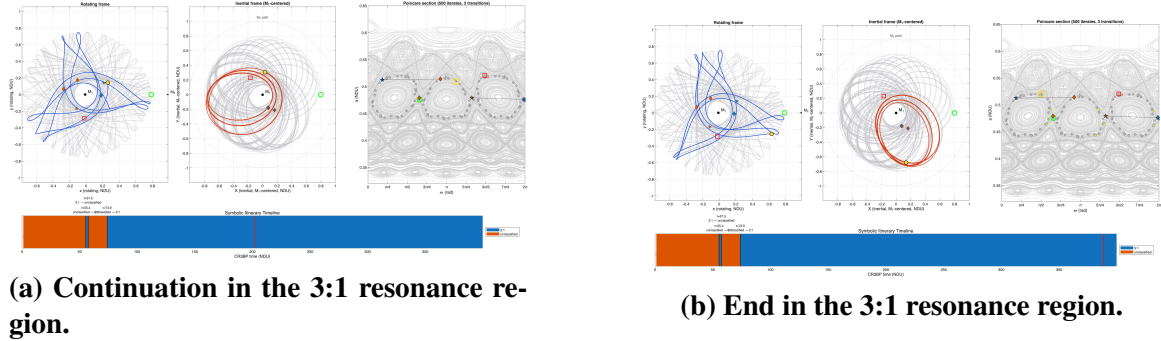


Figure 12: Perigee surface of section at $C = 3.20$ illustrating the evolution of the orbit within the 3:1 MMR zone. Panels (a)–(b) show representative sections progressing from the middle of the barcode in the 3:1 resonance region, through the end of the barcode, remaining in the 3:1 resonance region.

This spatial delineation of the Poincaré section facilitates the characterization of chaotic trajectories. Each trajectory is encoded as a symbolic sequence of resonance zone labels, forming a dynamical barcode that records the order and duration of zone visitations. When applied to a sample initial state, this tool successfully maps the zone visitation sequence, generating both a symbolic barcode and a transition matrix. The resulting orbital progression demonstrates the dynamical consistency of transport within sub-zones of a single MMR family. A comparative analysis conducted

at $C = 3.20$ using the 3:1, 5:2, 7:2, and 4:1 MMRs further demonstrates the consistency and broader applicability of this framework across varying energy levels.

The symbolic sequence provides a compact representation of a trajectory’s dynamical history, and is designed to enable the direct comparison of trajectories across large sets of initial conditions. This capability is particularly relevant for tracking uncooperative space objects subject to unexpected changes in orbital patterns.^{1,10} Because the resonance zones occupy known locations in semi-major axis, they correspond to identifiable families of high-eccentricity xGEO orbits, suggesting that cataloged objects could be assigned to zones—and their barcodes monitored—directly from orbit determination products. Moreover, this representation may support anomaly detection and long-term prediction of uncooperative objects. While a limitation of this methodology is the disappearance of specific low-order boundaries, this can be mitigated by incorporating higher order MMRs to comprehensively partition the remaining chaotic sea.

Overall, this investigation presents a structured, resonance-based classification framework that simplifies the description and analysis of chaotic trajectories in cislunar space. Several extensions follow naturally from the present work. Applying the barcode to a large ensemble of initial conditions would substantiate trajectory-class identification at scale and yield ensemble-based, rather than single-trajectory, transition probabilities. The barcode already contains the information needed to extract recurrence times—the intervals between successive visits to the same zone—which would add a temporal dimension to the classification. A systematic comparison against frequency-based classification approaches remains to be performed and would quantify the advantages of grounding the framework in resonance geometry. Finally, extending the analysis to a continuous spectrum of Jacobi constants, to the spatial problem, and to higher-order MMRs would more thoroughly classify the unresolved chaotic gaps remaining within the current phase-space partition.

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