

ANALYTICAL FRAMEWORK FOR COLLISION AVOIDANCE MANEUVER DESIGN USING REACTION CONTROL THRUSTERS WITH OPERATIONAL CONSTRAINTS

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The growing number of objects in space motivates the development of spacecraft autonomous operations. This paper presents an analytical model for the autonomous design of collision avoidance maneuvers using reaction control thrusters (RCTs). If the spacecraft is equipped with RCTs attitude changes are not required, but the maximum thrust is not uniform in all directions. This work introduces the analytical framework to model the zero-torque thrust authority polytope and the polar silhouette of the rotating polytope. Thruster failure constraints are also included. The resulting thrust authority is used to design the fuel-optimal collision avoidance maneuver. The model is then applied to a test case to showcase its capabilities. The results are verified against a numerical propagation, highlighting the accuracy of the model.

INTRODUCTION

The growth of the space sector and the advent of mega constellations are leading to an increase in Space Traffic Management related activities. The sudden rise in the number of objects in space has caused the overcrowding of specific orbital regimes, making conjunction events more frequent. Satellite operators must manage an increased number of conjunctions with many other operators, where reciprocal coordination is not always guaranteed.

The current protocol for Space Traffic Management relies on public and private entities that track objects in space through a network of ground-based radars and telescopes. The orbital information is then processed to spot conjunctions, that are characterized by the respective collision probability. When the collision probability exceeds a certain threshold, the operators of the satellites involved are alerted with a Conjunction Data Message (CDM) that contains the relevant information about the conjunction. Most notably, the state, covariance, and object size are provided at the Time of Closest Approach (TCA), i.e. the epoch at which the distance between the two objects is minimum. Conjunctions are generally spotted many revolutions prior to the TCA. This makes the preliminary conjunction assessment generally inaccurate because the state uncertainty is propagated for several revolutions. Further measurements are then performed at epochs closer to the TCA, and the new CDMs are sent to the operators.

The main challenges of this protocol are data availability¹ and the absence of an international

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regulation. However, the long and iterative processes currently required to manage a conjunction are not suitable to deal with the current pace at which the number of Resident Space Objects (RSOs) is rising. In this regard, there is a growing interest from both public and private sector towards autonomous and automatic operations. In the former, conjunction assessment, decision making and maneuver design are performed directly on-board of the spacecraft.² In the latter, they are performed on-ground and the data is later transmitted to the spacecraft. The key difference between the two approaches is that autonomous operations have access to limited computational resources but do not require communication with ground stations. Both modalities, on the other hand, require algorithms that are extremely reliable, to compensate the lack of human supervision.

Different tasks can be automated, such as conjunction assessment, decision making, maneuver design, and maneuver execution. This paper focuses on the autonomous design of Collision Avoidance Maneuvers (CAMs). Considering the limited computational resources available on-board of a spacecraft, maneuver design must be performed with reliable, lightweight algorithms, where accuracy become a secondary requirement. The guidance problem aims at minimizing fuel consumption, while reducing the collision risk below a predetermined safety threshold.

Guidance algorithms can be divided into impulsive and low-thrust. The latter are more complex but they can be used to model both impulsive and low-thrust maneuvers. Most low-thrust guidance algorithms available in literature are based on numerical optimization techniques, such as Convex Programming,³ Indirect Optimization,⁴ or Polynomial Programming.⁵ These methods can be very accurate, but they are not suitable for autonomous application because of their high computational effort. Additionally, the optimal solution is found via numerical gradient search of the state space, which is highly dependent on the initial guess. This limits their reliability and makes them unsuitable for unsupervised operations. This problem has been addressed by the authors⁶ by deriving the fuel-optimal solution through a combination of direct transcription and analytical primer vector theory.

Nonetheless, all methods cited above consider the maneuver to be performed by a single Orbit Control Thruster (OCT). However, there are spacecraft that are controlled with Reaction Control Thrusters (RCTs), for which these models are not applicable. The main difference between OCT and RCTs is that, in the first case, the spacecraft must re-orient itself to point the thruster along the desired thrust direction, while in the second case, the total thrust vector is defined by a linear combination of multiple, non-aligned thrusters. While for most spacecraft RCTs are used to control the attitude, maneuvers using RCTs are performed by spacecraft not equipped with OCTs, or in case the OCT has failed. Moreover, the capability of maneuvering with RCTs can be relevant when the spacecraft is unable to adjust its attitude, or when the OCT fuel is depleted.

The focus of this paper is to extend the semi-analytical low-thrust guidance algorithm developed in a previous work,⁶ to the design of maneuvers performed with RCTs.

The problem is tackled by first deriving the analytical framework to define the thrust authority of a spacecraft equipped with RCTs, which is still missing from the literature. The thrust authority is defined as the set of all possible thrust vectors that can be generated by the spacecraft. When the spacecraft can change its attitude, the thrust authority is a sphere of radius equal to the maximum thrust available. However, when the attitude is fixed and the thrust vector is defined by a linear combination of multiple thrusters, the thrust authority becomes a zonotope. A zonotope is a rotationally symmetric convex polytope that can be defined as the projection of a hypercube.

An unconstrained thrust authority can lead to the generation of an undesirable torque, that would change the attitude of the spacecraft. When the zero-torque constraint is imposed, the thrust authority becomes a constrained zonotope. The analytical characterization is then performed by converting

the constrained zonotope into the corresponding polytope.⁷

Finally, the case of thruster failure is also considered. When one or more RCTs are not operational, imposing the zero-torque constraint can lead to small thrust authorities that limit the feasibility of the maneuver. While the problem can be solved by allowing the spacecraft to adjust its attitude freely, this can affect spacecraft regular operations, that generally involve pointing a payload or antenna towards a specific direction. Because this can lead to scientific and economical losses, a new model is presented to characterize analytically the thrust authority of a spacecraft with RCTs, that can rotate only around a specific axis, corresponding to the pointing constraint.

Once the analytical characterization of the thrust authority is provided for all previous scenarios, the problem is converted into finding the thrusting time of every RCT. The overall optimal thrust direction can be derived from the analytical methods that consider only one OCT.^{6,8,9} Then, the thrust vector is converted into the respective RCT combination exploiting the Möller-Trumbore ray-triangle intersection algorithm.¹⁰

ANALYTICAL FRAMEWORK

This Section presents the analytical framework to derive the fuel-optimal maneuver performed with RCTs. At first, the unconstrained thrust authority is defined analytically as a zonotope. Then, the zero-torque constraint is introduced, and it is shown how the constraint converts the zonotope into a polytope. The case of thruster failure is also considered, and a rotational degree of freedom is introduced to restore three-dimensional authority. The resulting solid of rotation is defined analytically by parametrizing the rotation of every edge of the polytope. After all these scenarios are modelled, the Möller-Trumbore ray-triangle intersection algorithm is used to find efficiently the maximum thrust available along a specific direction. Finally, the analytical methods for fuel-optimal collision avoidance considering only one thruster are extended to the case of RCTs using the framework developed.

Unconstrained thrust authority

Let us assume that a spacecraft is equipped with N_T RCTs. Each thruster i is characterized by the thrust vector $\bar{\mathbf{g}}_i \in \mathbb{R}^{3 \times 1}$, such that $\bar{\mathbf{g}}_i$ is antiparallel to the thrust direction, and has magnitude equal to the maximum thrust. The vector $\bar{\mathbf{g}}_i$ is expressed in a generic body frame of the spacecraft that is centered at the spacecraft barycenter. The vectors $\bar{\mathbf{g}}_i$ are constant in the body frame as their orientation is fixed with respect to the spacecraft body. Let $\bar{\varepsilon}_i \in [0, 1] \forall i \in \{1, \dots, N_T\}$ be the thrust level, where 0 corresponds to no thrust, and 1 corresponds to maximum thrust. The resulting thrust $\mathbf{F}$ is given by the linear combination of all thrusters contributions as:

$$\mathbf{F} = \sum_{i=1}^{N_T} \bar{\mathbf{g}}_i \bar{\varepsilon}_i = \bar{\mathbf{G}} \bar{\varepsilon} \quad (1)$$

where $\bar{\mathbf{G}} \in \mathbb{R}^{3 \times N_T}$ is the matrix whose columns are the vectors $\bar{\mathbf{g}}_i$ and $\bar{\varepsilon} \in \mathbb{R}^{N_T}$ is the vector of thrust levels $\bar{\varepsilon}_i$.

The thrust authority defined by the configuration matrix $\bar{\mathbf{G}}$ corresponds to the mathematical set $\mathcal{Z}$ of achievable thrust vectors $\mathbf{F}$. As already mentioned, the unconstrained thrust authority $\mathcal{Z}$ is a zonotope, i.e. a polytope with centrally symmetric faces that can be defined as the Minkowski sum of line segments. For the unconstrained thrust authority of a spacecraft equipped with RCTs, the line segments correspond to the individual thruster contributions. In particular, the contribution

of a single thruster spans a line segment from $\mathbf{0}$ to $\mathbf{g}_i$, which can be modelled as a 1D zonotope. The linear combination, of the N_T 1D zonotopes results in a higher-dimensional zonotope whose dimension equals the rank of G . For the RCT problem, such dimension is generally 3, and the resulting 3D zonotope $\mathcal{Z}$ is defined as:

$$\mathcal{Z}(G, \mathbf{c}) = \{G\boldsymbol{\varepsilon} + \mathbf{c} : |\boldsymbol{\varepsilon}|_\infty \leq 1\} \quad (2)$$

where $G \in \mathbb{R}^{3 \times N_T}$ is the matrix whose columns are the line segments, $\boldsymbol{\varepsilon} \in \mathbb{R}^{N_T}$ is the vector of coefficients, $\mathbf{c} \in \mathbb{R}^3$ is the offset of $\mathcal{Z}$, and $|\cdot|_\infty$ is the infinity norm defined as the maximum absolute value of a vector components.

The definitions of the space of allowed thrust $\mathbf{F}$ from Eq. (1) and the zonotope $\mathcal{Z}$ from Eq. (2) are not exactly the same. The thrust vector problem requires the thrust level $\bar{\varepsilon}_i$ to be confined as $\bar{\varepsilon}_i \in [0, 1] \forall i \in \{1, \dots, N_T\}$, while the constraint on $|\boldsymbol{\varepsilon}|_\infty \leq 1$ assumes $\varepsilon_i \in [-1, 1] \forall i \in \{1, \dots, N_T\}$. Matching the two descriptions requires the change of variable:

$$\bar{\varepsilon} = \frac{\varepsilon + 1}{2} \quad (3)$$

Substituting Eq. (3) into Eq. (1) yields:

$$\mathbf{F} = \sum_{i=1}^{N_T} \bar{\mathbf{g}}_i \bar{\varepsilon}_i = \sum_{i=1}^{N_T} \bar{\mathbf{g}}_i \left(\frac{\varepsilon_i + 1}{2} \right) = \frac{1}{2} \sum_{i=1}^{N_T} \bar{\mathbf{g}}_i \varepsilon_i + \frac{1}{2} \sum_{i=1}^{N_T} \bar{\mathbf{g}}_i \quad (4)$$

Then, the zonotope $\mathcal{Z}$ defined in Eq. (2) that describes the space of allowed thrust directions $\mathbf{F}$ is characterized by:

$$G = \frac{1}{2} \bar{G} \quad \mathbf{c} = \frac{1}{2} \sum_{i=1}^{N_T} \bar{\mathbf{g}}_i \quad (5)$$

Note that if the RCT configuration is symmetrical, then $\mathbf{c} = \mathbf{0}$ and the zonotope is centered at the spacecraft barycenter.

The zonotope $\mathcal{Z}$ is defined in Eq. (2) using the half-space representation (H -rep) which describes $\mathcal{Z}$ as the intersection of half-spaces, i.e. oriented planes. An alternative representation, is the vertex-representation (V -rep) that defines $\mathcal{Z}$ through its vertices. The vertices can be pruned from the extreme values of ε_i (-1 and $+1$). This corresponds to the thrusters extreme settings where all thrusters are either off or at full throttle. The set of vertices $\mathcal{V}$ of a zonotope are the vertices of the convex hull generated from these extreme points:

$$\mathcal{V}_{\mathcal{Z}} = \text{conv} \left\{ \sum_{i=1}^{N_T} \bar{\varepsilon}_i \bar{\mathbf{g}}_i : \bar{\varepsilon}_i \in \{0, 1\} \right\} \quad (6)$$

Zero-torque thrust authority

An unconstrained thrust vector affects the attitude of the spacecraft by generating undesired torques. This is not suitable for common spacecraft operations because these torques can cause attitude changes during maneuver execution, or saturation of other attitude control sub-systems. To

avoid these complications, this Section presents the analytical definition of the zero-torque thrust authority.

Let $\bar{\mathbf{t}}_i \in \mathbb{R}^3$ be the position vector of the i^{th} thruster, expressed in the body frame centered at the spacecraft barycenter. The resulting torque $\boldsymbol{\tau}$ generated by the control action is given by the linear combination:

$$\boldsymbol{\tau} = \sum_{i=1}^{N_T} \bar{\mathbf{t}}_i \times \bar{\mathbf{g}}_i \bar{\varepsilon}_i = \sum_{i=1}^{N_T} \bar{\mathbf{a}}_i \bar{\varepsilon}_i = \bar{\mathbf{A}} \bar{\boldsymbol{\varepsilon}} \quad (7)$$

where $\bar{\mathbf{a}}_i \in \mathbb{R}^3$ is the maximum torque that the i^{th} thruster can generate, and $\bar{\mathbf{A}} \in \mathbb{R}^{3 \times N_T}$ is the matrix whose columns are the column vectors $\bar{\mathbf{a}}_i$.

Substituting Eq. (3) into Eq. (7) yields:

$$\boldsymbol{\tau}_{\text{RCT}} = \sum_{i=1}^{N_T} \bar{\mathbf{a}}_i \bar{\varepsilon}_i = \sum_{i=1}^{N_T} \bar{\mathbf{a}}_i \left(\frac{\varepsilon_i + 1}{2} \right) = \frac{1}{2} \sum_{i=1}^{N_T} \bar{\mathbf{a}}_i \varepsilon_i + \frac{1}{2} \sum_{i=1}^{N_T} \bar{\mathbf{a}}_i \quad (8)$$

Then, the zero-torque constraint can be enforced by imposing:

$$\mathbf{A} \boldsymbol{\varepsilon} = \mathbf{b} \quad (9)$$

where:

$$\mathbf{A} = \frac{1}{2} \bar{\mathbf{A}} \quad \mathbf{b} = -\frac{1}{2} \sum_{i=1}^{N_T} \bar{\mathbf{a}}_i \quad (10)$$

Similarly to Eq. (5), if both positions and thrusts vectors are symmetrical with respect to the barycenter, then $\mathbf{b} = \mathbf{0}$.

When the constraint from Eq. (9) is imposed on the zonotope $\mathcal{Z}$ defined in Eq. (2), the resulting set $\mathcal{Z}_C$ is a constrained zonotope defined as:

$$\mathcal{Z}_C = \{G\boldsymbol{\varepsilon} + \mathbf{c} : |\boldsymbol{\varepsilon}|_{\infty} \leq 1, \mathbf{A}\boldsymbol{\varepsilon} = \mathbf{b}\} \quad (11)$$

It can be shown⁷ that the constrained zonotope $\mathcal{Z}_C$ can be alternatively represented as a polytope $\mathcal{P}$, defined as:

$$\mathcal{P} = \{\mathbf{z} \in \mathbb{R}^3 : H\mathbf{z} \leq \mathbf{k}\} \quad (12)$$

where H and $\mathbf{k}$ are, respectively, the coefficient matrix and constant vector defining the polytope $\mathcal{P}$. The conversion from the constrained zonotope $\mathcal{Z}_C$ to the polytope $\mathcal{P}$ is based on imposing the torque constraint directly on the control vector $\boldsymbol{\varepsilon}$, such that the constraint $|\boldsymbol{\varepsilon}|_{\infty} \leq 1$ is converted into the polytope constraint $H\mathbf{z} \leq \mathbf{k}$. The set of solutions $\boldsymbol{\varepsilon}$ that satisfies Eq. (9) can be described in the $\mathbf{z}$ -space as the sum of a particular solution of the system and the null space, or kernel, of the coefficient matrix $\mathbf{A}$:

$$\varepsilon = A^+ \mathbf{b} + \mathbf{N}(A) \mathbf{z} \quad (13)$$

where A^+ is the Moore-Penrose pseudo-inverse of A , the null space $\mathbf{N}(A) \in \mathbb{R}^{N_T \times (N_T-3)}$ of A is the set of vectors in $\mathbb{R}^{N_T}$ that are mapped to $\mathbf{0}$ by A , and $\mathbf{z} \in \mathbb{R}^{N_T-3}$ is the new control vector.

Substituting Eq. (13) into the constraint $|\varepsilon|_\infty \leq 1$ yields:

$$|A^+ \mathbf{b} + \mathbf{N}(A) \mathbf{z}|_\infty \leq 1 \quad (14)$$

which can be rewritten as:

$$\begin{aligned} \mathbf{N}(A) \mathbf{z} &\leq \mathbf{1} - A^+ \mathbf{b} \\ -\mathbf{N}(A) \mathbf{z} &\leq \mathbf{1} + A^+ \mathbf{b} \end{aligned} \quad (15)$$

where $\mathbf{1} \in \mathbb{R}^{N_T}$ is a vector of all ones.

The linear inequality constraints $H\mathbf{z} \leq \mathbf{k}$ defining the polytope $\mathcal{P}$ in Eq. (12) can be obtained from the extended form of Eq. (15), by imposing:

$$H = \begin{bmatrix} \mathbf{N}(A) \\ -\mathbf{N}(A) \end{bmatrix} \in \mathbb{R}^{2N_T \times N_T-3} \quad \mathbf{k} = \begin{bmatrix} \mathbf{1} - A^+ \mathbf{b} \\ \mathbf{1} + A^+ \mathbf{b} \end{bmatrix} \in \mathbb{R}^{2N_T} \quad (16)$$

Note that the $H \in \mathbb{R}^{2N_T \times N_T-3}$ means that a minimum of 4 linearly independent thrusters is required to have 3D thrust authority.

Once the polytope $\mathcal{P}$ is defined, its V -rep can be obtained with any vertices enumeration algorithm. The V -rep is a set of vertices $\mathbf{z}_i$ expressed in the $(N_T - 3)$ -dimensional z -space. Substituting the values of $\mathbf{z}_i$ into Eq. (13), and then the resulting expression into Eq. (4), leads to the following definition of the set $\mathcal{V}_{\mathcal{P}}$, that is composed of the vertices $\mathbf{v}$ expressed in the the body frame of the thrust-space:

$$\mathcal{V}_{\mathcal{P}} = \text{conv} \{ \mathbf{v} \in \mathbb{R}^3 : \mathbf{v} = G(A^* \mathbf{b} + \mathbf{N}(A) \mathbf{z}) + \mathbf{c}, \mathbf{z} = \text{conv}(\mathcal{P}) \} \quad (17)$$

where G and $\mathbf{c}$ are defined according to Eq. (5).

Fig. 1 shows the zonotope $\mathcal{Z}$ (blue) for the unconstrained problem and the polytope $\mathcal{P}$ (red) for the constrained problem, expressed in the body frame. Both polyhedra are obtained from the 7-thruster configuration shown in the same figure, where the thruster length is proportional to the maximum thrust it can provide. The origin of the body-frame is the center of mass of the spacecraft, denoted by the black cross, which corresponds to the geometric center of the satellite body. The 3 thrusters on the right side corners are 80% less powerful than the 4 other thrusters on the left side corners. The asymmetry in the RCT configuration causes both $\mathcal{Z}$ and $\mathcal{P}$ to be more prominent on the right side. Moreover, $\mathcal{Z}$ and $\mathcal{P}$ have a common vertex on the far right. The corresponding thrust vector can be achieved with maximum throttle of the 4 left side thrusters. The zero-torque constraint is satisfied because all 4 thrusters involved are characterized by symmetric position, direction, and magnitude. The same does not apply to the far left vertex, because only 3 corners are equipped with a thruster.

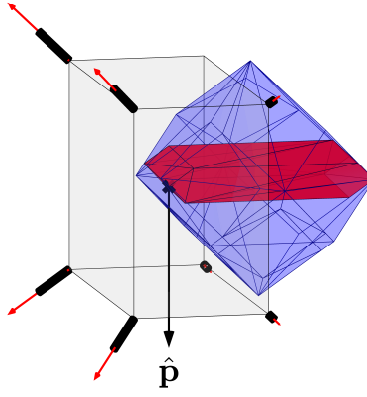


Figure 1. RCT configuration with respective zonotope $\mathcal{Z}$ (blue) and the polytope $\mathcal{P}$ (red) in body frame.

Rotating thrust authority

During the lifecycle of a spacecraft, it can happen that one or more RCTs fail. The reduced number of available thrusters negatively affects the thrust authority. This effect is enhanced when the zero-torque constraint is imposed, because asymmetries in the thrusters configuration increase the difficulty of compensating the torque. An example of this is shown in Fig. 1. There can be particular RCT configurations or thrusters failure events that lead to a limited, if not null, thrust availability in a sub-set of thrust directions. In such scenario, it can be useful to allow the spacecraft to freely adjust its attitude, hence restoring a spherical thrust authority. However, allowing full rotational freedom can lead to more complex maneuvers and to disruption of regular operations. For this reason, in this work, the rotation of the spacecraft is limited around a unique axis $\hat{p}$. This rotational degree of freedom transforms the space of allowed thrust into a solid of rotation generated by the rotation of either $\mathcal{Z}$ or $\mathcal{P}$ around the pointing direction $\hat{p}$. Both examples are shown in Fig. 2.

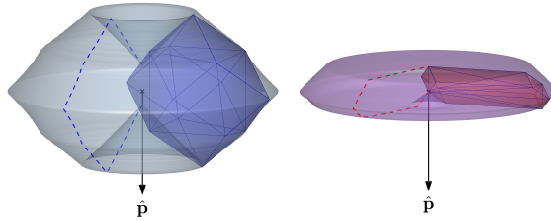


Figure 2. Unconstrained (left) and constrained (right) thrust authority of RCT configuration from Fig. 1, considering the rotational degree of freedom around $\hat{p}$.

When the spacecraft is allowed to rotate around $\hat{p}$, the maximum thrust $F_{\max}$ depends only on the angle α between the thrust direction $\mathbf{F}$ and the pointing direction $\hat{p}$. The set of thrust directions $\mathbf{F}$ that form an angle α with $\hat{p}$ defines the cone $\mathcal{C}_\alpha$ having half-angle equal to α and vertex at the spacecraft barycenter. The maximum thrust is constant on $\mathcal{C}_\alpha$ because both $\mathcal{C}_\alpha$ and the rotating thrust authority are solid of rotation generated around $\hat{p}$.

Let us consider the constrained case defined by the polytope $\mathcal{P}$, then the maximum thrust $F_{\max}|_\alpha$ available at an angle α is the maximum distance between the origin and the locus of points defined

by the intersection between $\mathcal{P}$, and the cone $\mathcal{C}_\alpha$:

$$\mathbf{F}_{\max}|_\alpha = \max_{\mathbf{x} \in \mathcal{C}_\alpha \cap \mathcal{P}} |\mathbf{x}| \quad (18)$$

Determining $\mathbf{F}_{\max}|_\alpha$ for all $\alpha \in [0\pi]$ defines the silhouette curve of the solid of rotation representing the thrust authority. The next paragraphs outline the methodology to compute this curve analytically for a generic polytope. The same exact approach can be applied to a zonotope.

Let $\mathcal{V}_\mathcal{P}$ be the set of vertices that define the convex hull of the thrust authority polytope $\mathcal{P}$, as defined in Eq. (17), and let $\mathbf{v} \in \mathbb{R}^{3 \times 1}$ be the vertices of $\mathcal{V}_\mathcal{P}$. Then, the silhouette of $\mathcal{V}_\mathcal{P}$ can be described by rotating every outer edge of $\mathcal{V}_\mathcal{P}$, i.e. the edges that lay on its surface. When an outer edge is considered to rotate around $\hat{\mathbf{p}}$, it generates a curve that can be parametrized based on the two vertices $\mathbf{v}_1$ and $\mathbf{v}_2$ that define the edge. The radial distance r from the pointing direction $\hat{\mathbf{p}}$ of the curve obtained from the rotating edge can be parametrized with the following quadratic function:

$$r^2 = (a^2 + b^2)z^2 + 2(ab + cd)z + (b^2 + d^2) \quad (19)$$

where:

$$\begin{aligned} a &= \frac{v_{x,2} - v_{x,1}}{v_{z,2} - v_{z,1}}, & c &= \frac{v_{y,2} - v_{y,1}}{v_{z,2} - v_{z,1}}, \\ b &= v_{x,1} - a v_{z,1}, & d &= v_{y,1} - c v_{z,1}. \end{aligned} \quad (20)$$

The quantities $v_{i,j}$ refers to the i^{th} component of the vertex $\mathbf{v}_j$.

It is important to highlight that the parametric description of the rotating edge in Eq (19) is valid only inside the interval:

$$z \in [\min(v_{z,1}, v_{z,2}), \max(v_{z,1}, v_{z,2})] \quad (21)$$

The silhouette of the rotating thrust authority is a 2D curve with radial component r perpendicular to $\hat{\mathbf{p}}$, and vertical component z parallel to $\hat{\mathbf{p}}$. Once the parametrization of the silhouette and definition of the domain is done for every edge, the maximum thrust direction $\mathbf{F}_{\max}|_\alpha$ corresponds to the intersection between the silhouette and the thrust direction that forms an angle α with $\hat{\mathbf{p}}$, projected on the same 2D space. The intersection point is computed with the silhouette of every edge, and the solutions that fall outside the domain defined in Eq. (21) are discarded. The projection $\mathbf{t}_{\text{ray}}$ of a generic thrust direction on the silhouette space can be parametrized as follows:

$$\mathbf{t}_{\text{ray}} = \begin{bmatrix} r \\ z \end{bmatrix} = \begin{bmatrix} t \sin \alpha \\ -t \cos \alpha \end{bmatrix} \quad (22)$$

where t is the parameter that controls the length of the thrust ray $\mathbf{t}_{\text{ray}}$.

Substituting Eq. (22) into Eq. (19) yields the following quadratic equation:

$$(a^2 + b^2 - \tan^2 \alpha)z^2 + 2(ab + cd)z + (b^2 + d^2) = 0 \quad (23)$$

Solving for z one obtains two intersection points between the ray t_{ray} and the silhouette of the rotating edges. All intersection points z are verified against the domain of every edge, and the solutions that do not satisfy this condition are discarded. The feasible solutions are used to compute the distance from the origin with Eq. (22), which is represented by the parameter t of the thrust ray t_{ray} . Finally, the maximum thrust $F_{\text{max}}|_{\alpha}$ for a particular angle α corresponds to the maximum distance from the origin t among all feasible solutions.

An example of the algorithm is shown in Fig. 3. The polytope has 16 vertices that result in a

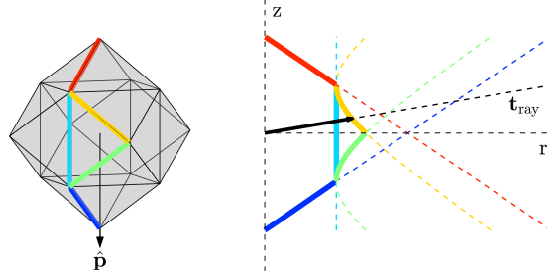


Figure 3. Thrust authority polytope with highlighted edges (left) and respective curves of rotation (right).

total of 120 edges. Computing the convex hull allows to consider only the outer edges, reducing the number to 36. Moreover, neglecting the horizontal edges that have same component along $\hat{p}$, allows to compute the coefficients from Eq. (20) only for 28 edges. Considering the symmetries of the polytope, the number of edges required is reduced to 5.

The right chart of Fig. 3 shows the curve obtained with Eq. (19). The curves have the same color of the respective edges on the left chart from which they originated. Furthermore, the curves are represented as solid lines only inside the domain defined in Eq. (21). The silhouette of the solid of rotation is defined by the outer most combination of the edges silhouettes. For a particular value of α , the intersection between the resulting thrust ray t_{ray} and the curves defined by the rotating edges can be found by solving Eq. (23). Considering the example in Fig. 3, the intersections between t_{ray} and the red, green, and blue curves are discarded because outside their respective domains. The feasible intersections are with the cyan and yellow curves. The latter is selected as $F_{\text{max}}|_{\alpha}$ because further from the center.

Maximum thrust along a specific direction

The maximum thrust F_{max} along a specific direction can be determined by formulating an optimization problem where the objective is to maximize the thrust along the desired direction. The optimization can be formulated as linear programming problem of the form:

$$\begin{aligned} & \text{maximize} && \mathbf{f}^T \mathbf{x} \\ & \text{subject to} && 0 \leq |\mathbf{x}|_{\infty} \leq 1 \\ & && \mathbf{C}\mathbf{x} = \mathbf{0} \end{aligned} \tag{24}$$

where $\mathbf{x}$ is the decision variable, $\mathbf{f}$ is the objective function, $\mathbf{C}$ the constraint matrix, and the infinity norm is defined in Eq. (2).

The quantities of the linear programming problem are defined as follows:

$$\mathbf{x} = \begin{bmatrix} \bar{\boldsymbol{\varepsilon}} \\ |\bar{\boldsymbol{\varepsilon}}| \end{bmatrix} \in \mathbb{R}^{N_T+1} \quad \mathbf{f} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \in \mathbb{R}^{N_T+1} \quad C = \begin{bmatrix} \bar{A} & \mathbf{0} \\ \bar{G} & \boldsymbol{\delta\hat{v}} \end{bmatrix} \in \mathbb{R}^{6 \times N_T+1} \quad (25)$$

where N_T is the number of thrusters, $\bar{\boldsymbol{\varepsilon}}$ and $\bar{G}$ are defined in Eq. (1), $\bar{A}$ is defined in Eq. (8), and $\boldsymbol{\delta\hat{v}}$ is the thrust direction.

While different methods exist to solve the problem in Eq. (24), the solution defines the maximum thrust only for a single direction $\boldsymbol{\delta\hat{v}}$, rather than characterizing the entire 3-dimensional behavior. For this reason, a more efficient method is proposed. From an operational perspective, the set of vertices $\mathcal{V}$ defining the thrust authority is constant in the body frame and can be pre-computed. In the context of autonomous operations, the computational effort to design the maneuver is limited to the more efficient Möller–Trumbore algorithm,¹⁰ which consists only of basic arithmetic operations, rather than a numerical optimization. Moreover, in case of thruster failure, updating the vertices of the set of allowed thrust directions can be easier than changing quantities and size of an on-board numerical optimization algorithm.

The Möller-Trumbore algorithm is a fast and efficient method to assess the ray-triangle intersection. Let the ray direction be defined by the thrust direction $\boldsymbol{\delta\hat{v}} \in \mathbb{R}^3$, centered at the origin, and let the triangle be defined by the three vertices $\boldsymbol{v}_i \in \mathbb{R}^3$, with $i = \{0, 1, 2\}$. The main idea of the algorithm is to parametrize a point P on both ray and triangle, and to equate those points. The parametrization is defined as:

$$P = F_{\max} \boldsymbol{\delta\hat{v}} \quad (26)$$

$$P = \boldsymbol{v}_0 + u \boldsymbol{e}_1 + v \boldsymbol{e}_2 \quad (27)$$

with:

$$u \geq 0 \quad v \geq 0 \quad u + v \leq 1 \quad (28)$$

and:

$$\begin{aligned} \boldsymbol{e}_1 &= \boldsymbol{v}_1 - \boldsymbol{v}_0 \\ \boldsymbol{e}_2 &= \boldsymbol{v}_2 - \boldsymbol{v}_0 \end{aligned} \quad (29)$$

Eq. (26) and Eq. (27) parametrize the point P on the ray and triangle, respectively. When the two expressions are set equal to each other, this leads to a linear system that is solved for the unknowns $F_{\max}$, u and v via Cramer's rule. First, the determinant D of the system matrix can be computed as:

$$D = \boldsymbol{e}_1 \cdot (\boldsymbol{\delta\hat{v}} \times \boldsymbol{e}_2) \quad (30)$$

If the determinant D is equal to zero, then the ray is parallel to the triangle plane and there is no intersection. Otherwise, the parameter u is given by:

$$u = -\frac{1}{D} \boldsymbol{v}_0 \cdot (\boldsymbol{\delta\hat{v}} \times \boldsymbol{e}_2) \quad (31)$$

If u is outside the range $[0, 1]$, then the intersection point lies outside the triangle. Otherwise, the parameter v is given by:

$$v = -\frac{1}{D} \delta \hat{v} \cdot (\mathbf{u}_0 \times \mathbf{e}_1) \quad (32)$$

Again, if v is outside the range $[0, 1]$, then the intersection point lies outside the triangle.

If all conditions are satisfied, then the maximum thrust $F_{\max}$ along $\delta \hat{v}$ is given by:

$$F_{\max} = -\frac{1}{D} \mathbf{e}_2 \cdot (\mathbf{u}_0 \times \mathbf{e}_1) \quad (33)$$

The Möller-Trumbore algorithm is applied to every face of the polytope until the solution that satisfies all conditions is found.

When the failure scenario is considered, the thrust authority is not defined by a polytope, but rather a solid of rotation. In this case, the maximum thrust is completely defined by the silhouette from Eq (23). The maximum thrust available along a specific direction $\delta \hat{v}$ can be determined by simply computing the angle α between $\delta \hat{v}$ and the rotation axis $\hat{p}$, and comparing it with the pre-computed values.

Maneuver design

The mathematical tools presented so far can characterize the thrust authority of a spacecraft equipped with RCTs, considering different scenarios. Moreover, it is also provided the methodology to determine the maximum thrust available along a specific direction. This Section utilizes these results to define the analytical procedure for CAM design. The main idea is to find the optimal maneuver assuming a single OCT, and then find the maximum thrust with RCT along that direction.

The state of the art method for analytical optimization of impulsive CAM relies on linearizing the dynamics using a State Transition Matrix (STM) and deriving the optimal thrust direction via spectral decomposition of its quadratic form. This method was first introduced by Conway¹¹ to maximize the miss distance in the context of asteroid deflection. It was later improved by Vasile and Colombo¹² to maximize the deflection on the b-plane. Finally, it was extended to collision avoidance by Bombardelli and Hernando-Ayuso⁸ to minimize the collision probability. The optimal control problem is reduced to an eigenvalue problem and the optimal thrust direction $\delta \hat{v}$ corresponds to the eigenvector conjugated to the largest eigenvalue. Then, the optimal thrust magnitude Δv is simply given by a quadratic equation.¹³

Once Δv is found, it must be verified against the thrust authority of the RCT equipped spacecraft. The results of the previous Section are used to obtain the maximum thrust $F_{\max}$ along the optimal thrust direction $\delta \hat{v}$. Let m be the mass of the spacecraft. Then, the thrusting time Δt_{thrust} can be obtained as:

$$\Delta t_{\text{thrust}} = \frac{m \Delta v}{F_{\max}} \quad (34)$$

where $F_{\max}$ is the magnitude of $\mathbf{F}_{\max}$.

When a larger deflection is required and the impulsive assumption does not hold anymore, the maneuver must be modelled as low-thrust. In this case, the same methodology is applied by modelling the low-thrust continuous profile as a series of consecutive impulsive maneuvers, using a

semi-analytical model developed in a previous work.⁶ The nominal trajectory is discretized into a series of equally spaced nodes, and the impulsive problem is solved analytically at each node, following the procedure described above. Fuel-optimality is achieved by pruning the nodes based on their optimality.

The advantage of this formulation is that CAMs generally introduce small changes to the nominal orbit. This allows to define the linearized dynamics analytically, without introducing any considerable error. There are three different analytical STMs available in literature, known as Yamanaka-Ankersen STM,¹⁴ D-matrix,¹⁵ and Keplerian STM.⁹

RESULTS

This Section applies the model outlined in the previous Section to a synthetic test case. First, the test case is defined, then the fuel-optimal collision avoidance maneuver using RCTs is computed at different epochs. The effects of the maneuver are analyzed onto the b-plane,¹⁶ and the accuracy of the model is analyzed considering both errors generated by the impulsive assumption and linearization of the dynamics.

Test case

The test case considered is inspired on the Sentinel mission led by the European Space Agency. The main objective of the constellation is to provide radar images of lands, oceans and support emergency services. The scenario analyzed involves the design of a CAM performed by the Sentinel-1A satellite to avoid an unknown debris. During maneuver, the Sentinel satellite is subjected to a pointing constraint that enforces the large radar antenna to be pointed towards Earth, along the anti-radial direction.

The *eoPortal* website* is used to obtain the physical parameters of the Sentinel-1A satellite. It has mass at launch of 2200 kg and it is composed of a main body modelled as a box with sizes 3.4 m, 1.3 m, and 1.3 m. The spacecraft is equipped with two solar panels that are 10 m long. The height is considered to be 2 m. Their support is assumed to be 2 m long. Finally, the antenna, located at the bottom of the spacecraft, is 12 m long and 1.3 m wide. The antenna height is assumed to be 20 cm. The satellite model is shown in Fig. 4.

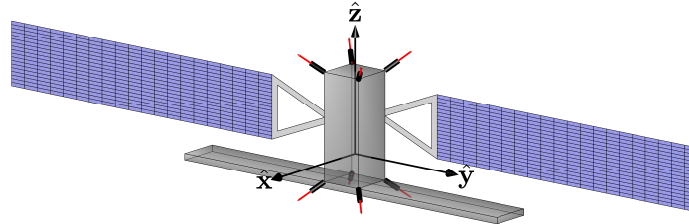


Figure 4. Modelling of Sentinel-1A satellite with body frame axes.

The Sentinel-1A is equipped with 8 RCTs. The position and thrust direction of the RCTs is not disclosed, therefore they are assumed to be placed in the corners of the main body. The thrust directions $\bar{g}$ is assumed to be along the outward direction such that it is equally inclined with respect to all three body frame axes. Every RCT can provide a thrust of 1 N and they are represented in Fig. 4 along with their respective thrust direction.

*<https://www.eoportal.org/satellite-missions/copernicus-sentinel-1#spacecraft>

The total mass at the maneuver epoch is assumed to be 2000 kg and the drag coefficient C_D is set to 2.2. Considering the presence of the large antenna, the center of mass of the spacecraft is assumed to be located 85 cm away from the geometric center of the main body, along the vertical direction. The spacecraft mission requires the antenna to be pointed along the anti-radial direction, and the long side of the antenna is assumed to stay perpendicular to the velocity vector. Considering that the spacecraft is on a quasi-circular orbit, the ram surface A_C of 14.68 m² is computed as the sum of the front surfaces of main body and antenna, and the average surface of the solar panels.

Let Sentinel-1A be the primary object that executes the maneuver, then its position vector $\mathbf{r}_1$ in ECI coordinates, velocity vector $\mathbf{v}_1$ in ECI coordinates, and position covariance Σ_1 in RTH coordinates at the TCA are:

$$\begin{aligned} \mathbf{r}_1 &= \begin{bmatrix} 1499.11 \\ 120.15 \\ 6920.59 \end{bmatrix} \text{ km} & \mathbf{v}_1 &= \begin{bmatrix} -5.0031 \\ -5.5510 \\ 1.1806 \end{bmatrix} \text{ km/s} \\ \Sigma_1 &= \begin{bmatrix} 0.0029 & -0.0271 & 0.0003 \\ -0.0271 & 2.7151 & 0.0124 \\ 0.0003 & 0.0124 & 0.0054 \end{bmatrix} \times 10^3 \text{ m}^2 \end{aligned} \quad (35)$$

Let the debris be the secondary object, then its position vector $\mathbf{r}_2$ in ECI coordinates, velocity vector $\mathbf{v}_2$ in ECI coordinates, and position covariance Σ_2 in RTH coordinates at the TCA are:

$$\begin{aligned} \mathbf{r}_2 &= \begin{bmatrix} 1500.10 \\ 120.46 \\ 6920.40 \end{bmatrix} \text{ km} & \mathbf{v}_2 &= \begin{bmatrix} -7.1373 \\ 1.5580 \\ 1.5193 \end{bmatrix} \text{ km/s} \\ \Sigma_2 &= \begin{bmatrix} 0.0124 & -0.0350 & 0.0057 \\ -0.0350 & 1.7943 & -0.0018 \\ 0.0057 & -0.0018 & 0.0062 \end{bmatrix} \times 10^6 \text{ m}^2 \end{aligned} \quad (36)$$

The respective Keplerian elements at the TCA are included in Table 1.

Table 1. Keplerian elements at TCA of the Sentinel test case.

Object	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]	θ [deg]
Sentinel-1A	7204.48	0.0170	98.28	46.65	260.75	0.18
Debris	7006.65	0.0108	86.53	348.41	77.76	180.46

Finally, the hard-body radius $r_{A,1}$ of the Sentinel-1A is computed geometrically from the minimum circumscribed sphere, while the hard-body radius $r_{A,2}$ of the Iridium 33 debris is, again, assumed from the minimum observable diameter of 10 cm:

$$r_{A,1} = 12.67 \text{ m} \quad r_{A,2} = 0.05 \text{ m} \quad (37)$$

The collision probability P_C of the conjunction event is equal to 4.32×10^{-4} , with respective nominal miss distance at TCA of 1044 m and squared Mahalanobis distance at TCA of 0.7948. The objective of the maneuver is to reduce the collision probability to the target value P_C^* of 10^{-6} , which corresponds to a target squared Mahalanobis distance w^* of 12.9234.

Maneuver analysis

The analysis of the test case presented in the previous Section is performed as follows. The available maneuver lead time Δt is discretized into 500 equally spaced epochs. At each epoch, the optimal thrust direction $\delta \hat{v}$ is computed with the spectral decomposition method. Then, it is derived the respective fuel-optimal thrust magnitude Δv required to achieve a target collision probability P_C^* of 10^{-6} . The pointing constraint direction $\hat{p}$ corresponds to the anti-radial direction at the maneuver epoch. The thrust authority is modelled enforcing the zero-torque constraint.

Two scenarios are considered. In the *Nominal* scenario, the propulsion system of Sentinel-1A is fully functioning and the spacecraft does not need to rotate around the pointing direction $\hat{p}$. In the *Failure* scenario, one faulty thruster restricts the thrust authority to approximately half of the available directions. Rotation around $\hat{p}$ is allowed to restore thrust capabilities along all directions. The thrust authorities of both scenarios are shown in Fig. 5.

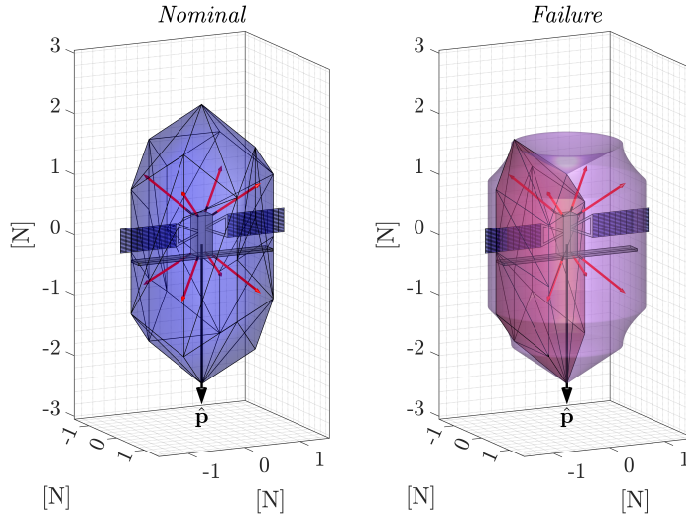


Figure 5. Thrust authority of *Nominal* (left) and *Failure* (right) scenarios.

The thrust authority of the *Nominal* scenario is a polytope with multiple planes of symmetry. The main observation is that it is narrower along the directions perpendicular to $\hat{p}$ because the low center of mass does not allow to use the top thrusters at full capacity for lateral movement. The faulty thruster in the *Failure* scenario is the bottom-left-back, as shown in Fig. 5. Without rotation, thrust is allowed only in the half-space defined by the plane containing the center of mass and the two bottom thrusters adjacent to the faulty one. When rotation around $\hat{p}$ is considered, the set of allowed thrust directions corresponds to the respective solid of rotation. This guarantees a thrust authority that is more uniform along all directions.

For both scenarios, Fig. 6 shows the total Δv of the maneuver, the total thrusting time Δt_{thrust} , the thrust direction $\delta \hat{v}$, the post-maneuver collision probability P_C computed with the linearized dynamics, and the time shift Δt_{TCA} of the TCA.

The top chart confirms the well-known result that, the sooner the maneuver is performed, the less fuel is consumed. Moreover, while having the conjunction close to the perigee would cause the optimal maneuver location to be close to the apogee, the relatively large eccentricity has the opposite effect because a change in velocity at the perigee is more effective. This makes the optimal maneuver location to move away from the usual true anomaly phase of 180 degrees, except for the

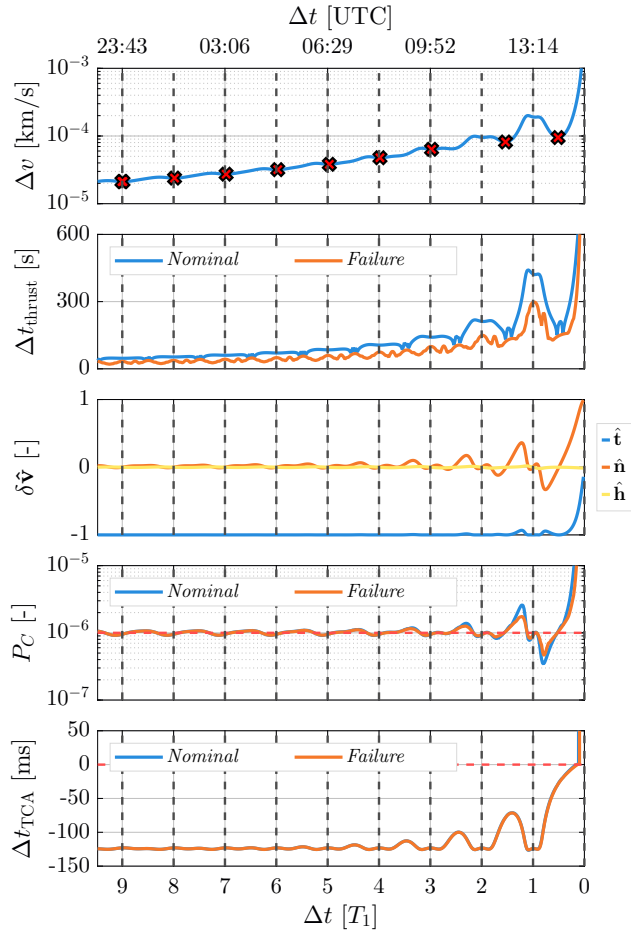


Figure 6. Results of the fuel-optimal CAM at different lead times Δt considering the Sentinel-1A test case.

last two revolutions where the geometric component of the maneuver becomes more prominent. The Δv required is the same for both *Nominal* and *Failure* scenarios. The main difference is the maximum acceleration available along the optimal thrust direction. From the thrusting time Δt_{thrust} represented in the second chart, it can be observed that the maneuver in the *Failure* scenario requires shorter thrusting times, highlighting the benefits of the rotational degree of freedom around $\hat{p}$.

The third chart represents the optimal thrust direction $\delta \hat{v}$ in the Tangential, Normal, Out-of-plane (TNH) reference frame at every node analyzed. The results confirm that tangential or anti-tangential thrusting is generally preferred until, approximately, half revolution before the TCA. The reason is that tangential thrusting in quasi-circular orbits is the most effective way of changing the semi-major axis. This, in turn, results in geometric effect that changes the Minimum Orbit Intersection Distance, also known as MOID, but also changes the orbital period, resulting in a phasing maneuver.

The post-maneuver collision probability P_C , computed using the analytical linear dynamics, are shown in the fourth chart. It can be noticed that the linearized collision probability is very close to the target value P_C^* of 10^{-6} , but regular oscillations can be identified. The error peaks grow after every revolution, similarly to the normal component $\hat{n}$ of the optimal thrust direction $\delta \hat{v}$,

represented in the third chart. When the lead time decreases, the normal component becomes more important because the effects of a phasing maneuver accumulate over every revolution. The fewer the revolutions of the primary between maneuver epoch and the TCA, the smaller the deviation along the time axis ζ . Contrary, the deviation along the geometry axis ξ does not depend on the lead time and its efficiency becomes comparable to the one of a phasing maneuver for small lead times Δt .

The accuracy of the model is analyzed for both *Nominal* and *Failure* scenarios. It can be noticed that the oscillations of the *Nominal* scenario are more prominent. The maneuver is exactly the same for both scenarios, the only difference rests in the thrusting time Δt_{thrust} . Because the thrust available in the *Nominal* scenario is smaller, the respective thrusting time Δt_{thrust} is longer, and the error related to the impulsive approximation is larger. Recalling that every horizontal grid line represents a logarithmic error $\log_{10} \Delta P_C$ of 0.1, it can be observed that maximum error is lower than 0.2 before the last revolution.

Finally, similar oscillations can be observed in the last chart, where the time shift Δt_{TCA} of the TCA is represented. When the maneuver is purely tangential, it can be observed that the effect on the TCA is the same. On the other hand, when the normal component of the thrust vector grows, the maneuver action along the time axis is reduced, and, accordingly, the time shift Δt_{TCA} from the nominal TCA.

The effects of the CAM for the *Nominal* scenario are shown on the b-plane in Fig. 7.

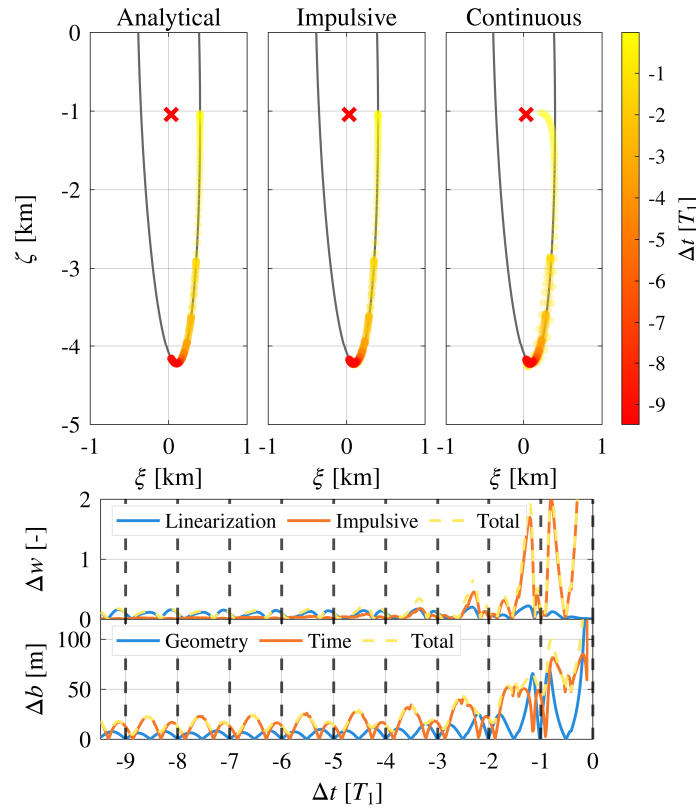


Figure 7. Representation on the b-plane of every maneuver designed for the *Nominal* scenario of the Sentinel test case.

The three top chart represent the b-plane with the nominal location, the target w^* - σ that corresponds

to the target collision probability P_C^* , and the miss distance onto the b-plane after every maneuver. From left to right, the b-plane representation is based on the analytical model, the numerical propagation with the impulsive assumption, and the numerical propagation with continuous thrust. The fourth chart shows the absolute error Δw between the square Mahalanobis distance achieved and the target one, divided per source. The final chart shows the error between the distance on the b-plane predicted with the analytical model, and the one achieved with numerical propagation and continuous thrust. The total error is divided into its geometric component ξ and time component ζ .

The b-plane representation shows that for large lead times Δt the maneuver acts solely along the time axis. This is expected because the thrust profile is mainly anti-tangential. Moreover, anti-tangential thrust is favoured over the tangential one because the nominal miss distance has negative ζ coordinate. When the maneuver epoch approaches the TCA, the deviation on the b-plane moves along the w^* ellipse until it becomes purely geometric. From both the b-plane representations and the squared Mahalanobis distance error Δw , it can be observed that the error is mainly related to the impulsive assumption. The main reason are the thrusting times Δt_{thrust} that are much longer than the one required for the Swarm test case.

CONCLUSIONS

This work addresses the absence from the literature of a model to design fuel-optimal CAMs performed with RCTs. The research is motivated by the need of autonomous Space Traffic Management procedures, and the presence in Low-Earth Orbit of spacecraft that are not equipped with an OCT, or it is malfunctioning.

The maneuver is computed using methods from literature based on single thruster optimizations, but the problem is then converted to find the control action of the RCTs. The computational efficiency is guaranteed by a complete analytical derivation of the thrust authority considering the unconstrained scenario, the zero-torque scenario, and the thruster failure scenario. The intersection between the optimal thrust direction and the thrust authority is then found using the Möller-Trumbore ray-triangle intersection algorithm.

The model is then applied to a synthetic test case, showcasing not only its capabilities, but most notably its accuracy. The accuracy is also guaranteed when the maneuver is performed unusually early, showing that the linearization assumption holds. The overall error is subdivided into its two main components. The linearization error depends on the analytical STM and it is shown to be always small. The impulsive error comes from the impulsive assumption but it can be avoided by using the low-thrust method instead. In the end, the results obtained prove the applicability of the model for autonomous operations.

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