

A COMPARATIVE STUDY OF OPTIMAL CONTROL AND NEURAL NETWORKS IN ASTEROID RENDEZVOUS MISSION ANALYSIS

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This paper presents a comparative study of the applicability and accuracy of optimal control methods and neural network-based estimators in the context of porkchop plots for preliminary asteroid rendezvous mission design. The scenario considered involves a deep-space CubeSat equipped with a low-thrust engine, departing from Earth and rendezvousing with a near-Earth asteroid within a three-year launch window. A low-thrust trajectory optimization model is formulated, incorporating variable specific impulse, maximum thrust, and path constraints. The optimal control problem is efficiently solved using Sequential Convex Programming (SCP) combined with a solution continuation strategy. The neural network framework consists of two models: one predicts the minimum fuel consumption (Δv), while the other estimates the minimum flight time (Δt) which is used to assess transfer feasibility. Case results demonstrate that, in simplified scenarios without path constraints, the neural network approach achieves low relative errors across most of the design space and successfully captures the main structural features of the porkchop plots. In cases where the SCP-based continuation method fails due to the presence of multiple local optima, the neural network still provides smooth and globally consistent predictions, significantly improving the efficiency of early-stage asteroid candidate screening. However, the deformation of the feasible region caused by path constraints leads to noticeable discrepancies in certain boundary regions, thereby limiting the applicability of the network in detailed mission design phases. Overall, the integration of neural networks with porkchop plot analysis offers a effective decision-making tool for mission designers and planetary scientists, with significant potential for engineering applications.

INTRODUCTION

Near-Earth asteroids (NEAs) offer unique scientific value for understanding the early evolution of the solar system, assessing extraterrestrial resources,¹ and enhancing planetary defense. They also possess engineering value due to their relative accessibility. It is estimated that the number of observable NEAs has exceeded ten thousand, and the number of detectable targets continues to increase rapidly.² In recent years, Japan's Hayabusa-2 mission successfully collected and returned samples from Ryugu, while NASA's OSIRIS-REx mission obtained samples from Bennu. The DART mission further demonstrated, for the first time, the feasibility of kinetic impactor technology for asteroid deflection.³⁻⁵ With the maturation of deep-space CubeSats and small satellite platforms, opportunities for low-cost and low-thrust missions to NEAs are increasing.^{6,7} However, such missions must perform complex trajectory design under strict mass and power constraints, highlighting the need for more efficient preliminary mission assessment tools.

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The “porkchop” plot provides an intuitive cost landscape of fuel consumption and transfer time for interplanetary missions by sweeping departure epochs and flight durations. However, for low-thrust spacecraft, each grid point requires solving an optimal control problem that may involve multiple constraints, variable specific impulse, and other nonlinearities, leading to a computational burden significantly higher than that of chemical propulsion.^{8,9} Existing optimal control methods include direct methods such as pseudospectral methods, sequential convex programming (SCP), and dynamic programming,^{10–12} as well as indirect methods based on Pontryagin’s maximum principle.^{13,14} Even with advanced optimal control methods and continuation techniques,¹⁵ generating a porkchop plot for a single transfer target through dense sampling can require tens to hundreds of CPU hours, and is sensitive to initial guesses and local minima. This remains computationally prohibitive when applied to a large number of NEA targets.

In recent years, deep learning has been increasingly applied in astrodynamics, using multilayer perceptrons to directly regress fuel consumption or assess reachability.^{16–19} Machine learning models have also been employed for real-time trajectory guidance.^{20,21} Neural networks, with their universal approximation capabilities²² and microsecond-level inference efficiency, are well-suited to large-sample scenarios such as porkchop plot generation. However, existing studies have mostly focused on specific transfer scenarios, and are often validated under simplified assumptions such as constant thrust and the absence of path constraints. Moreover, quantitative comparisons with advanced optimal control methods are often lacking, especially when addressing realistic engineering constraints such as variable specific impulse, thermal limits, and power restrictions.^{23,24}

To this end, this study aims to evaluate the accuracy of neural networks (NNs) in realistic, constrained low-thrust mission scenarios, and to assess whether they can serve as a fast surrogate tool for asteroid preselection during early mission design, while also clarifying their limitations. A representative scenario, porkchop plot generation for rendezvous missions to near-Earth asteroids, is selected for analysis. The applicability and accuracy of optimal control methods and neural networks are compared in this context.

The main contributions of this paper are summarized as follows: 1. Under realistic scenarios involving variable specific impulse, maximum thrust, and path constraints, this paper conducts a quantitative comparison between neural networks and optimal control methods at the porkchop plot level. 2. Under simple transfer conditions, the neural network maintains a relative error of less than 10% across most of the feasible domain. 3. The study demonstrates that the porkchop plot is a suitable scenario for applying neural networks. It not only provides fast and direct support for engineering design, but also allows for visual validation of neural network performance across a wide range of transfer scenarios.

The remainder of this paper is organized as follows: Section 2 describes the mission model and algorithms; Section 3 presents the experimental results; Section 4 discusses the sources of error and potential improvements; and Section 5 concludes the paper.

PROBLEM DESCRIPTION AND METHODS

This paper investigates the porkchop plot in the context of a practical asteroid rendezvous mission. The scenario involves a CubeSat launched from Earth to rendezvous with an asteroid. The rocket allows for a maximum departure $v_{\infty}^{\max}$ of 4 km/s, with no constraint on direction. The departure date ranges from January 1, 2029, to December 31, 2031, with a flight time between 200 and 1200 days and a time step of 10 days. Each point in the porkchop plot represents a computation aimed at minimizing the total velocity increment required for the rendezvous. The initial mass of the satellite is 26.5 kg, with 2 kg of propellant allocated to the thruster. The maximum thrust and specific impulse of the low-thrust engine vary with the spacecraft’s distance from the Sun. Path constraints are also imposed in this scenario.

This section introduces the optimal control problem corresponding to the porkchop plot analysis of the

asteroid rendezvous mission studied in this paper. It covers the definition of the low-thrust trajectory optimization problem, the optimal control method used for its solution, and the neural network model employed for prediction. First, the mathematical formulation of the low-thrust trajectory optimization problem is presented, including the dynamical equations, the modeling of variable specific impulse and maximum thrust, and the path constraints. Then, the optimal control method used to solve the problem, namely the Sequential Convex Programming (SCP) algorithm, is introduced. Finally, the structure and training process of the neural network model are described, along with the indirect method used for data generation, and the approach to use the trained model for prediction.

Problem Definition

The problem of interest is the optimization of low-thrust trajectories for asteroid rendezvous missions. The objective is to minimize the total velocity increment required for a CubeSat to rendezvous with an asteroid.

The equations of motion governing the spacecraft include gravitational forces and thrust forces

$$\dot{\mathbf{r}} = \mathbf{v} \quad (1)$$

$$\dot{\mathbf{v}} = -\frac{\mu}{r^3}\hat{\mathbf{r}} + \frac{\mathbf{\Gamma}}{m} \quad (2)$$

where $\mathbf{r}$ is the position vector, $\mathbf{v}$ is the velocity vector, μ is the gravitational parameter of the Sun, $r = \|\mathbf{r}\|_2$. Fuel consumption obeys

$$\dot{m}(t) = -\frac{\|\mathbf{\Gamma}(t)\|}{I_{sp}(t) g_0} \quad (3)$$

where $\mathbf{\Gamma}(t)$ is the thrust force and $I_{sp}(t)$ is the specific impulse.

A realistic thruster model is implemented by mapping the maximum thrust $T_{\max}$ and specific impulse I_{sp} variation over the instantaneous input power. The available input power P is powered by solar arrays and depends on the spacecraft distance r from the Sun. The coefficients of the fourth-order polynomials describing $P(r)$, $T_{\max}(P)$ and $I_{sp}(P)$ are detailed in Table 1,

Table 1. Electric propulsion thruster coefficients.

Input power limits	$T_{\max}$ [$\mu\text{N}/\text{W}^i$] coeff.	I_{sp} [s/W^i] coeff.	P [W/AU^i] coeff.
$P_{\text{in},\text{min}} = 80 \text{ W}$ $P_{\text{in},\text{max}} = 130 \text{ W}$	$a_0 = -1234.3$	$b_0 = -5519.5$	$c_0 = 1281.4$
	$a_1 = 26.498$	$b_1 = 225.44$	$c_1 = -2518.8$
	$a_2 = 0$	$b_2 = -1.8554$	$c_2 = 1828.5$
	$a_3 = 0$	$b_3 = 5.0836 \cdot 10^{-3}$	$c_3 = -475.80$
	$a_4 = 0$	$b_4 = 0$	$c_4 = 0$

Maximum thrust and specific impulse can also be expressed directly as a function of the distance from the Sun, as shown in Figure 1.

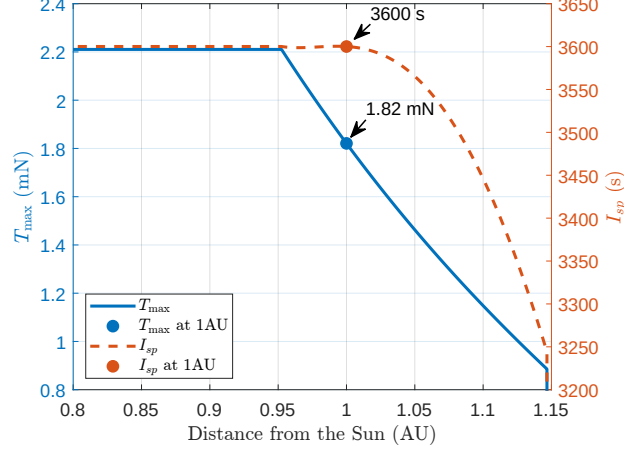


Figure 1. Variable $T_{\max}$ and I_{sp} curves with respect to the distance from the Sun.

Variations of $T_{\max}$ and I_{sp} with heliocentric distance (in astronomical units, AU) are modeled by fourth order polynomials between 0.9525 AU to 1.1467 AU

$$T(x) = \begin{cases} 2.2 \times 10^{-3} \text{ N}, & x \leq 0.9525, \\ \sum_{i=0}^4 c_{T,i} x^i, & 0.9525 < x < 1.1467, \\ 0, & x \geq 1.1467, \end{cases} \quad I_{\text{sp}}(x) = \begin{cases} 3600 \text{ s}, & x \leq 0.9525, \\ \sum_{i=0}^4 c_{I,i} x^i, & 0.9525 < x < 1.1467, \\ 0, & x \geq 1.1467, \end{cases} \quad (4)$$

where the thrust coefficients and the I_{sp} coefficients are summarized in Table 2.

Table 2. Fourth-order polynomial coefficients for thrust $T(x)$ and specific impulse $I_{\text{sp}}(x)$

i	$c_{T,i}$ [N/AU ^{i}]	$c_{I,i}$ [s/AU ^{i}]
0	0.0327202372	3.1951149396×10^5
1	-0.0667431624	$-1.1891014516 \times 10^6$
2	0.0484515930	1.6645687780×10^6
3	-0.0126077484	$-1.0253620793 \times 10^6$
4	0	2.3398323687×10^5

Due to thermal and power constraints, the CubeSat trajectory is required to stay above 0.8 AU, and the engine must be shut down beyond 1.15 AU throughout the mission, representing a path constraint.

Optimal Control Method

To efficiently analyze the reachability of multiple asteroids, a fast and reliable method is needed to compute thousands of fuel-optimal low-thrust trajectories per target. This work employs a direct Sequential Convex Programming (SCP) algorithm, solving a convex subproblem for each combination of departure date and time of flight. The solution is then used as an initial guess for neighbouring problems across the time grid, ensuring robust convergence and smooth porkchop plots.

The nonconvex low-thrust trajectory optimization problem is solved by considering a sequence of convex subproblems whose solution converges, under defined hypotheses, to the solution of the original one.¹²

Exploiting a correct change of variables and convexification techniques, at each iteration of the SCP, the following problem is solved

$$\min_{\mathbf{u}(t)} -w(t_f) + \lambda_1 \int_{t_i}^{t_f} \max(0, \eta(t)) dt + \lambda_2 \int_{t_i}^{t_f} \|\mathbf{v}(t)\|_1 dt \quad (5a)$$

$$\text{subject to: } \dot{\mathbf{x}}(t) = \mathbf{f}(\tilde{\mathbf{x}}(t), \tilde{\mathbf{u}}(t)) + \mathbf{A}(\tilde{\mathbf{x}}(t))(\mathbf{x}(t) - \tilde{\mathbf{x}}(t)) + \mathbf{B}(\mathbf{u}(t) - \tilde{\mathbf{u}}(t)) + \mathbf{v}(t) \quad (5b)$$

$$\Gamma(t) \leq T_{\max} e^{-k\theta} (1 - w(t) + \tilde{w}(t)) + \eta(t) \quad (5c)$$

$$\|\mathbf{v}(t)\|_2 \leq \Gamma(t) \quad (5d)$$

$$\|\mathbf{x}(t) - \tilde{\mathbf{x}}(t)\|_1 \leq R \quad (5e)$$

$$\mathbf{r}(t_i) = \mathbf{r}_i, \quad \mathbf{v}(t_i) = \mathbf{v}_i, \quad w(t_i) = w_i \quad (5f)$$

$$\mathbf{r}(t_f) = \mathbf{r}_f, \quad \mathbf{v}(t_f) = \mathbf{v}_f \quad (5g)$$

$$\mathbf{x}_l \leq \mathbf{x}(t) \leq \mathbf{x}_u, \quad \mathbf{u}_l \leq \mathbf{u}(t) \leq \mathbf{u}_u \quad (5h)$$

where $\mathbf{x} = [\mathbf{r}, \mathbf{v}, w]$ and $\mathbf{u} = [\tau_x, \tau_x, \tau_x, \Gamma] = [\boldsymbol{\tau}, \Gamma]$ are the state and control variables, respectively. The quantities $\eta(t)$ and $\mathbf{v}(t)$ are slack variables introduced to avoid artificial infeasibility, and λ is a large weight parameter. The initial time t_i is assumed to be any epoch in the three-year departure window uniformly discretized, and $T_{\max}$ is the maximum available thrust. The variable R is the radius of the trust region where the convexification of the problem is valid. In Eq. (5b), the vector of dynamics $\mathbf{f}$ and matrices $\mathbf{A}$ and $\mathbf{B}$ are defined as

$$\mathbf{f}(t) = \begin{bmatrix} \mathbf{v}(t) \\ -\frac{\mu}{r^3} \mathbf{r} + \boldsymbol{\tau}(t) \\ -\frac{\Gamma(t)}{I_{sp} g_0} \end{bmatrix} \quad (6)$$

$$\mathbf{A}(\tilde{\mathbf{x}}(t)) := \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\tilde{\mathbf{x}}(t)}, \quad \mathbf{B} := \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\tilde{\mathbf{u}}(t)} \quad (7)$$

where $r = \|\mathbf{r}(t)\|_2$, I_{sp} is the specific impulse and g_0 is the gravitational acceleration at sea level. In Eq. (5f), $\mathbf{v}_i$ is given by the sum between the heliocentric velocity of the Earth at t_i and the initial velocity provided by the launcher $\mathbf{v}_\infty$, which translates as

$$\begin{cases} \mathbf{v}_i = \mathbf{v}_E(t_i) + \mathbf{v}_\infty \\ \|\mathbf{v}_\infty\|_2 \leq v_\infty^{\max} \end{cases} \quad (8)$$

where $v_\infty^{\max}$ is equal to 4 km/s. Note that the ecliptic angles of the vector $\mathbf{v}_\infty^{\max}$, namely the right ascension and declination of the launcher velocity, are not constrained.

The problem in Eqs. (5) is a Second-Order Cone Program (SOCP) and can be solved by efficient convex solvers. In this work, the Embedded Conic Solver (ECOS)²⁵ has been used. Additionally, an arbitrary-order Legendre-Gauss-Lobatto method based on Hermite interpolation is used.²⁶

Despite the robustness of the SCP algorithm, the solution of a trajectory optimization problem depends on the initial guess provided. Therefore, to obtain a homogenous porkchop plot a *continuation method* has been used. It consists of exploiting the solutions of already-solved problems and using them as initial guesses for their neighbours. The continuation scheme adopted follows the one implemented in Morelli et al. (2024).²⁷

In particular, the solution at earliest departure date and maximum time of flight (which corresponds to the top-left corner of the porkchop plot) is solved by trying two different initial guesses: a simple shape-based method approximated by a cubic polynomial function²⁸ and a more sophisticated sub-optimal trajectory resulting from an adapted version of the Q-law guidance algorithm.²⁹ Then, this solution is used as the initial guess of lower time of flights and later departure dates. Each time a new problem is solved, its solution becomes the initial guess for neighbour ones. The idea behind continuation is that, if the search space of a porkchop plot is discretized with a fine enough time grid, the nearby problems are characterized by fairly similar initial and final conditions, and therefore the solution of one represents a good initial guess for the other.

Still, artificial discontinuities in the solutions space can be introduced by mission assumptions, such as the spacecraft thermal limits or thruster model constraints. In particular, the spacecraft thermal limits introduce a hard-constraint on the minimum distance from the Sun of 0.8 AU, below which the spacecraft cannot go. On the other hand, the cut-off threshold of the input power for the thruster forces the shutdown of the engine at distances above approximately 1.15 AU. These discontinuities can become relevant, increasing the complexity of the problem and leading to local optimal convergence. To tackle these problems when encountered, the gradual introduction of the constraints is exploited through a two-step relaxation strategy. Namely, the porkchop plot is computed first without some constraints to ensure a smooth solution space. Afterward, each solution from the relaxed problem is used as the initial guess of the refined porkchop plot, which considers the original constrained problem. Although this approach is undoubtedly time-consuming, it improves the convergence and robustness when computing porkchop plots.

However, in a limited number of instances, some discontinuities could remain visible in the porkchop plots. This is not necessarily due to a convergence problem of the solver; rather, it could be related to the actual unfeasibility of solutions in certain combinations of time of flight and departure date. Such combinations indeed determine the relative configuration between the departure and arrival points, which may lead to unfeasible trajectories once the thruster model and distance constraints are injected into the problem.

Neural Network Approximator

The neural network is trained on data generated by an indirect method solver with no path constraints and with constant specific impulse I_{sp} and maximum thrust T_{max} . The neural networks predict the optimal transfer time and the minimum fuel consumption. In this subsection, we first describe how the indirect method is used to generate the training data, then we present the architecture of the neural network, and finally we explain how the trained network is used to produce pork-chop diagrams.

Indirect Method for Data Generation: Define the normalized thrust control $\mathbf{u}(t)$, with $\|\mathbf{u}\| \leq 1$, and thrust vector defined as

$$\mathbf{\Gamma} = T_{max}(r) \mathbf{u}.$$

Instead of using Cartesian coordinates $(\mathbf{r}, \mathbf{v})$ or classical orbital elements $(a, e, i, \Omega, \omega, f)$, the state vector $\mathbf{S}$ is represented using modified equinoctial elements (MEE),^{30,31} which are non-singular and well-suited for solving the majority of low-thrust trajectory optimization problems.

$$\begin{aligned} p &= a(1 - e^2), & f &= e \cos(\omega + \Omega), & g &= e \sin(\omega + \Omega), \\ h &= \tan(i/2) \cos \Omega, & k &= \tan(i/2) \sin \Omega, & L &= \omega + \Omega + v, \end{aligned} \quad (9)$$

where a , e , i , Ω , ω , and v represent the semi-major axis, eccentricity, inclination, right ascension of the ascending node, argument of perigee, and true anomaly, respectively.

Defining the spatial state vector as $\mathbf{s} = [p, f, g, h, k, L]^T$, the dynamics are then expressed as

$$\dot{\mathbf{s}} = \mathbf{D}(\mathbf{s}) + \frac{T_{\max}(r)}{m} \mathbf{M}(\mathbf{s}) \boldsymbol{\alpha} u. \quad (10)$$

Detailed expressions of $\mathbf{M}$ and $\mathbf{D}$ are provided in.^{21,31,32}

Two optimization objectives are considered:

- Fuel-Optimal: Minimize

$$J_{\text{fuel}} = \lambda_0 \int_{t_0}^{t_f} \frac{T_{\max}}{I_{\text{sp}} g_0} \{u - \varepsilon \ln[u(1-u)]\} dt \quad (11)$$

subject to the dynamics $\dot{\mathbf{s}}, \dot{m}$ and boundary conditions $\mathbf{s}(t_0) = \mathbf{s}_0, \mathbf{s}(t_f) = \mathbf{s}_f, m(t_0) = m_0$.

- Time-Optimal: Minimize

$$J_{\text{time}} = \lambda_0 \int_{t_0}^{t_f} 1 dt \quad (12)$$

subject to the same dynamics and endpoint constraints, with free final time t_f .

The Hamiltonian for the fuel problem is given by

$$H_{\text{fuel}} = \boldsymbol{\lambda}_s^T \dot{\mathbf{s}} + \lambda_m \dot{m} + \lambda_0 L_{\text{fuel}}(\mathbf{s}, \mathbf{u}) = \frac{T_{\max}}{m} \boldsymbol{\lambda}_s^T \mathbf{M} \boldsymbol{\alpha} u + \boldsymbol{\lambda}_s^T \mathbf{D} - \lambda_m \frac{T_{\max}}{I_{\text{sp}} g_0} \boldsymbol{\alpha} u + \lambda_0 \frac{T_{\max}}{I_{\text{sp}} g_0} [u - \varepsilon \ln(u(1-u))]. \quad (13)$$

Costate dynamics are given by

$$-\dot{\boldsymbol{\lambda}} = \frac{\partial H}{\partial(\mathbf{s}, m)}.$$

Pontryagin's minimum principle yields

$$\mathbf{u}^* = -\frac{\boldsymbol{\Phi}}{\|\boldsymbol{\Phi}\|}, \quad \boldsymbol{\Phi} \equiv \frac{T_{\max}(r)}{m} \boldsymbol{\lambda}_v - \lambda_m \frac{T_{\max}(r)}{I_{\text{sp}}(r) g_0} \frac{\boldsymbol{\lambda}_v}{\|\boldsymbol{\lambda}_v\|}.$$

The final-time transversality condition for free $m(t_f)$ gives $\lambda_m(t_f) = 0$. Normalization of the costate norm $\|\boldsymbol{\lambda}(t_0)\| = 1$ fixes the scaling.

For time-optimal control, the running cost reduces to λ_0 , the control is always full throttle ($\|\mathbf{u}\| = 1$), and the free-final-time condition $H(t_f) = 0$ determines t_f . The time-optimal Hamiltonian becomes

$$H_{\text{time}} = \frac{T_{\max}}{m} \boldsymbol{\lambda}_s^T \mathbf{M} \boldsymbol{\alpha} u + \boldsymbol{\lambda}_s^T \mathbf{D} - \lambda_m \frac{T_{\max}}{I_{\text{sp}} g_0} \boldsymbol{\alpha} u + \lambda_0. \quad (14)$$

The resulting two-point boundary value problem is solved using a single shooting method, yielding the optimal state trajectories $\mathbf{r}(t), \mathbf{v}(t), m(t)$, and thrust history $\boldsymbol{\Gamma}(t)$.

- Fuel-Optimal Shooting Function: Let the state be $\mathbf{x} = [\mathbf{s}; m]$ with costate $\boldsymbol{\lambda} = [\lambda_r; \lambda_v; \lambda_m]$ and multiplier λ_0 . The unknowns are the initial costates $\boldsymbol{\lambda}(t_0)$. The shooting function is

$$\Phi_{\text{fuel}}[\boldsymbol{\lambda}(t_0)] = \begin{bmatrix} \mathbf{s}(t_f) - \mathbf{s}_f \\ \lambda_m(t_f) \\ \|\boldsymbol{\lambda}(t_0)\| - 1 \end{bmatrix} = \mathbf{0}. \quad (15)$$

- Time-Optimal Shooting Function: Let t_f be free and the terminal state depend on t_f . The unknowns are $\lambda(t_0)$ and t_f . The boundary value function is

$$\Phi_{\text{time}}[\lambda(t_0); t_f] = \begin{bmatrix} s(t_f) - s_f(t_f) \\ \lambda_m(t_f) \\ H_{\text{time}}(t_f) - \lambda_L(t_f) \frac{\sqrt{\mu p}}{r^2} \\ \|\lambda(t_0)\| - 1 \end{bmatrix} = \mathbf{0}. \quad (16)$$

Neural Network Model: The inputs to interface of neural network model include the departure and arrival positions and velocities, transfer time, maximum thrust, specific impulse, and initial mass. Two separate models are used for the outputs: one predicts the solution to the Δv fuel-optimal problem, and the other predicts the Δt solution to the time-optimal problem. The time-optimal Δt model is used to assess reachability—if the given transfer time is shorter than the predicted minimum transfer time, the transfer is considered infeasible.

The final deep neural network employed for both the Δv and Δt models consists of $n_{\text{layer}} = 9$ fully connected hidden layers, each containing $n_{\text{neuron}} = 128$ neurons. The ReLU activation function is used throughout all hidden layers to introduce nonlinearity, while the output layer applies a linear activation to directly predict the continuous transfer cost. Training was performed on a 100-million-sample dataset, which requires approximately three days to produce each 1-million-sample block, culminating in a total generation time of three days for the full dataset. Model training, executed on the same computational platform, also completes in about three days.

On the test sets, the Δv model achieves a mean absolute error (MAE) of 3.38 m/s and a mean relative error (MRE) of 0.78%, significantly outperforming prior methods¹⁷¹⁸.¹⁶ For the Δt model, the MAE on minimum transfer time is 2.56 days, with an MRE of 0.63%. Additionally, the neural network model leverages dynamic transformations, and the generated dataset spans a maximum acceleration range of 2.5×10^{-6} to 1.2×10^{-2} m/s² and a specific impulse range of 700 to 9000 s, enabling the network’s strong generalization across arbitrary orbital parameters (a, e, i) and propulsion configurations (a_s, I_{sp}) to support almost all current low-thrust missions without requiring dataset retraining.

Multi-revolution Trajectory Optimization: Note that the neural network model used in this study is applicable only to single-revolution cases and cannot be directly applied to multi-revolution transfers. To address this limitation, a simple optimization model was developed. The multi-revolution trajectory is divided uniformly into several segments based on time, with each segment being shorter than one full revolution. The optimization variables include the orbital states of the intermediate connection points between segments. In addition, three more optimization variables are the components of the launch velocity provided by the rocket. The optimization process consists of two stages: a global search using particle swarm optimization, followed by local refinement with a local optimization algorithm. The objective function is the total velocity increment. In addition, each segment incorporates a feasibility constraint using another neural model, which is implemented as a penalty function.

Note that the neural network model developed in this study is applicable only to single-revolution transfers and cannot be directly applied to trajectories involving multiple revolutions. To overcome this limitation, we propose a hybrid optimization framework combining global search, local refinement, and feasibility checking via the Δt model.

First, the full multi-revolution transfer is partitioned into N fixed time segments $\{[t_k, t_{k+1}]\}_{k=0}^{N-1}$, each spanning less than one orbital period. The optimization variables are:

$$s_k = [r(t_k), v(t_k)]_{k=1}^{N-1}, \quad \Delta v_{\text{launch}} = [\Delta v_x, \Delta v_y, \Delta v_z]$$

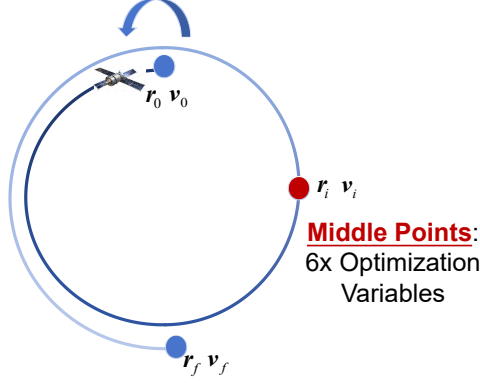


Figure 2. Connecting trajectories for computing multi-revolution transfers.

where $\mathbf{r}(t_k)$, $\mathbf{v}(t_k)$ denotes the spatial state at the k -th intermediate waypoint, and $\Delta \mathbf{v}_{\text{launch}}$ represents the three components of the initial injection velocity supplied by the launch vehicle. The specific impulse I_{sp} and maximum thrust T_{max} can either be fixed or vary at each intermediate waypoint according to Eq. 4, the latter case being referred to as "piece-wise variable".

The total velocity increment objective is

$$J = \sum_{k=0}^{N-1} \Delta v_k^*$$

where Δv_k^* is the optimal velocity increment for segment k , as predicted by our single-revolution neural Δv model. To ensure dynamic consistency and mission feasibility, each segment's end state must satisfy both boundary matching and feasibility constraints. The latter are imposed via a penalty function

$$P_{\text{feas}}(\mathbf{s}_k, \mathbf{s}_{k+1}) = \rho \max\{0, (t_{k+1} - t_k) - \mathcal{N}_{\Delta t}(\mathbf{s}_k, \mathbf{s}_{k+1})\}^2,$$

where $\mathcal{N}_{\Delta t}$ is the output of the time-optimal Δt model, with ρ a large penalty weight.

The overall optimization proceeds in two stages:

1. Global Search: A particle swarm optimization (PSO) algorithm explores the high-dimensional optimization space $\{\mathbf{s}_k, \Delta \mathbf{v}_{\text{launch}}\}$, minimizing $J + \sum_k P_{\text{feas}}$.
2. Local Refinement: The best candidate from PSO is passed to a gradient-based local solver³³ to fine-tune the intermediate states and launch velocity, further reducing the objective while strictly enforcing segment continuity and feasibility.

RESULTS

Among the numerous near-Earth asteroid analyzed, three representative porkchop plots, corresponding to asteroids 2012 LA, 2008 ST, and 2022 OC3, are presented for demonstration. Their orbital elements are shown in Table 3 and Figure 3.

The porkchop plots for these three asteroids are shown in Figures 4, 5, and 6. In each plot, the horizontal axis represents the launch date, the vertical axis represents the time of flight, and the color scale indicates fuel consumption. For comparison, the direct method results are compared with those from the neural network model. The direct method (numerical results) accounts for variable specific impulse, maximum

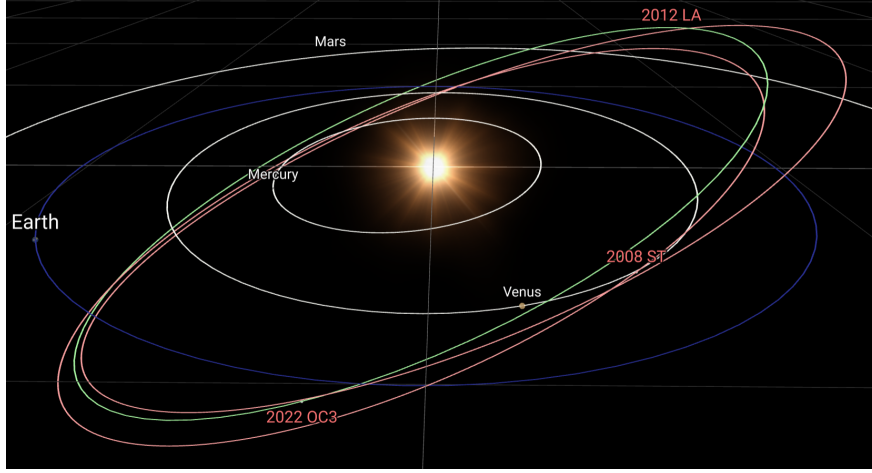


Figure 3. Orbits of selected asteroids. Credits: ESA NEO Toolkit

Table 3. Classical orbital elements of selected asteroids

Epoch	Asteroid	a (km)	e	i (°)	Ω (°)	ω (°)	ν (°)
2029 Jan 01 12:00:00 UTC	2012 LA	1.55604×10^8	0.0216877	21.7262	353.0774	320.1778	257.0437
	2008 ST	1.43986×10^8	0.1265623	21.5865	359.3513	120.5766	157.4352
	2022 OC3	1.52899×10^8	0.1286078	22.9596	0.5957	49.6797	113.2483

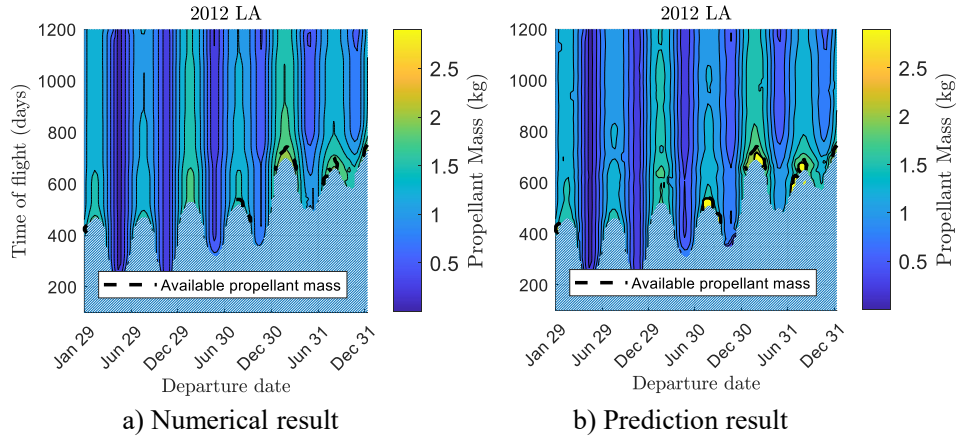


Figure 4. Porkchop plot for the 2012 LA Asteroid.

thrust, and includes path constraints, whereas the neural network model (prediction results) is based on an indirect method using piece-wise variable specific impulse and maximum thrust.

For asteroid 2012 LA, large regions of low fuel consumption appear in the porkchop plot, and the neural network model results closely match the direct method. The porkchop plot for asteroid 2008 ST exhibits a more complex structure, with multiple high fuel consumption regions, and significant discrepancies between the neural network model results and the direct method in some areas. The porkchop plot for asteroid 2022 OC3 exhibits even more pronounced discrepancies, as the optimal control direct method results display numerous discontinuities and gaps. The characteristics of these distinct porkchop plots will be discussed in

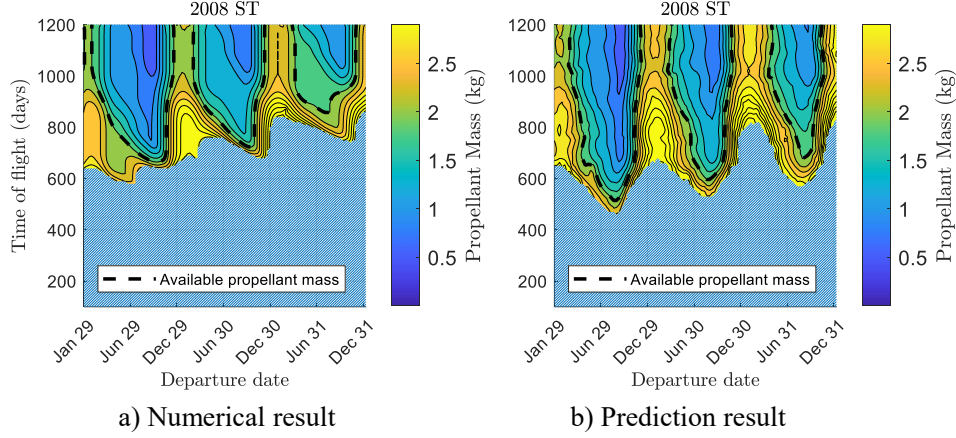


Figure 5. Porkchop plot for the 2008 ST Asteroid.

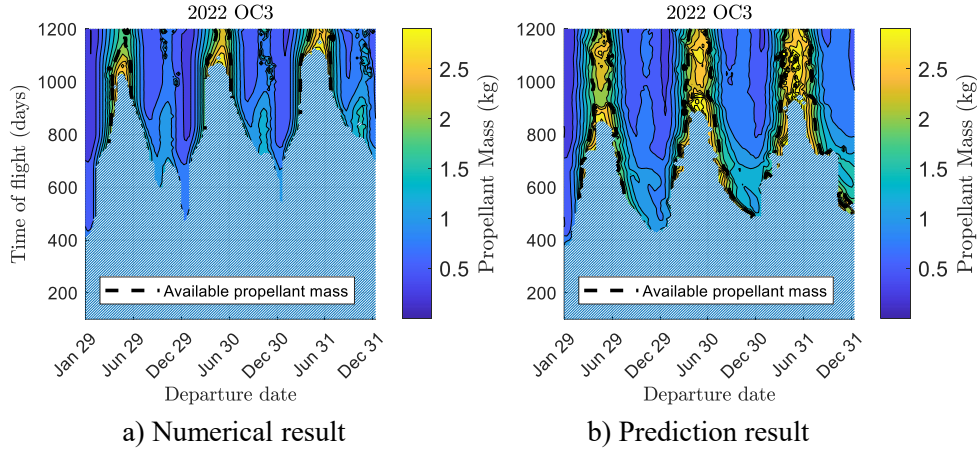


Figure 6. Porkchop plot for the 2022 OC3 Asteroid.

detail in the following section.

ANALYSIS AND DISCUSSION

This section further investigates the causes of the non-negligible errors observed in the previous section. Intuitively, these errors may arise from the approximation inherent in the neural network model or from the complexity of the orbital dynamics and constraint. To isolate the effect of the neural network, variable specific impulse, thrust, and path constraints are removed in this experiment, leaving only the core dynamical model. The resulting relative error plots are shown in Figures 7, 8, Fig. 9. As illustrated in the figures, the neural network consistently captures the overall trends and salient features of the trajectory across all asteroid scenarios. This demonstrates the model's capability to represent the underlying orbital dynamics and validates the effectiveness of the proposed learning-based approach.

The discrepancies between the porkchop plots and the neural network predictions for asteroid 2008 ST are further analyzed to investigate the underlying reasons for the observed differences. As shown in Figure 8, removing the constraints leads to significantly different porkchop plot structures compared to the original ones in Figure 5. This indicates that the new structures arise from the inclusion of variable specific impulse

I_{sp} , maximum thrust T_{max} , and path constraints specific to this scenario.

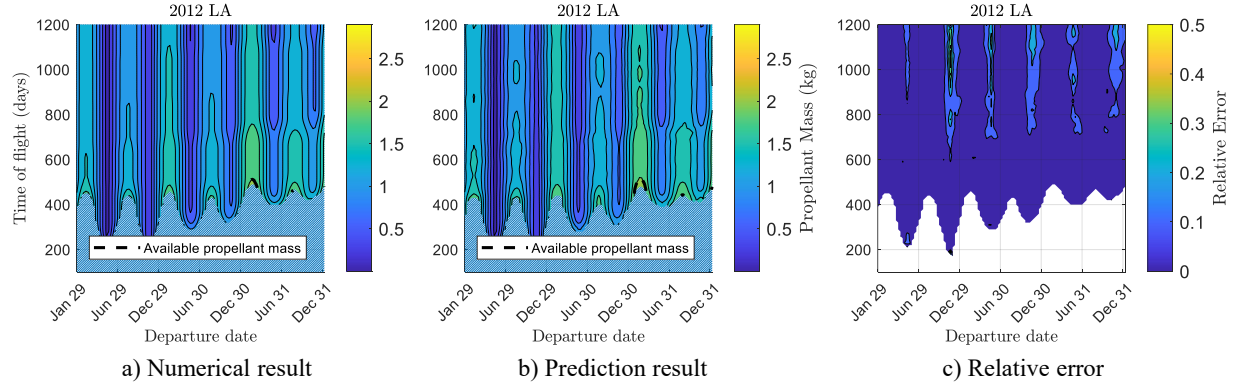


Figure 7. Porkchop plot for the Asteroid 2012 LA without variable I_{sp} , T_{max} , and path constraints.

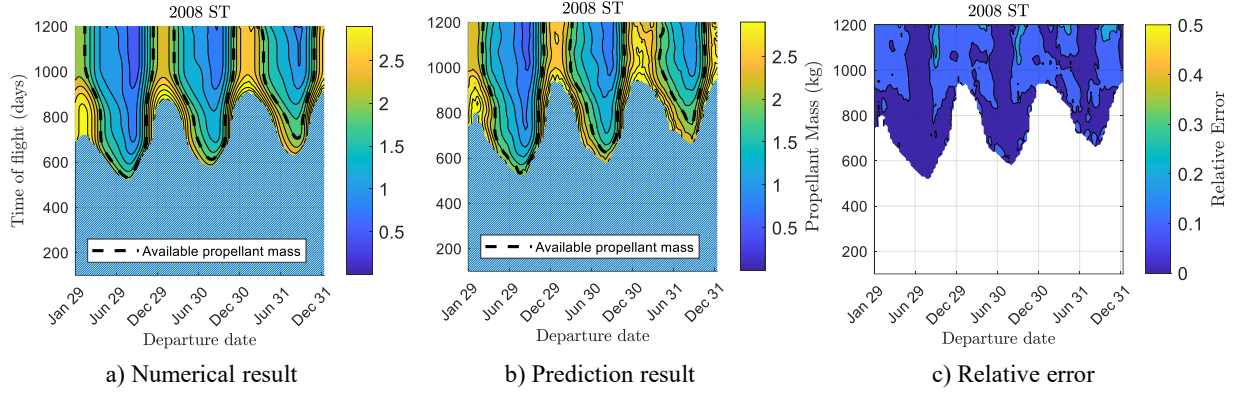


Figure 8. Porkchop plot for the Asteroid 2008 ST without variable I_{sp} , T_{max} , and path constraints.

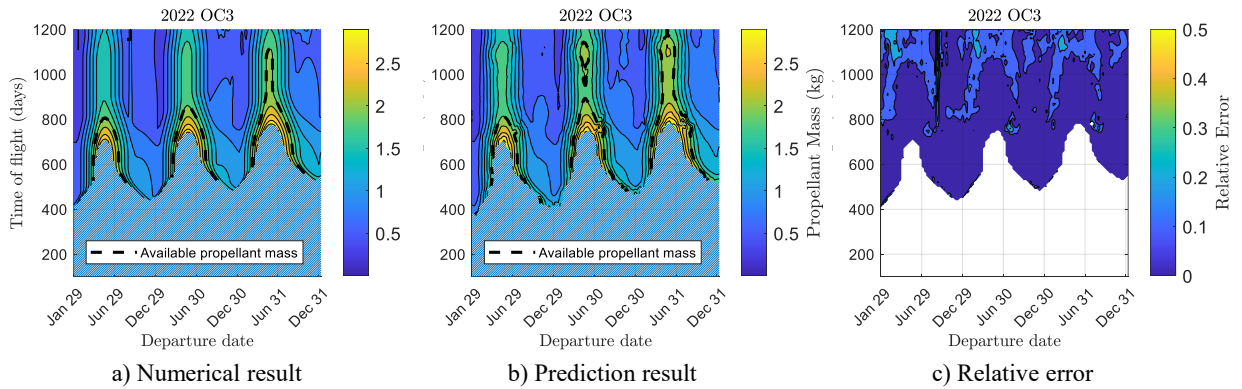


Figure 9. Porkchop plot for the Asteroid 2022 OC3 without variable I_{sp} , T_{max} , and path constraints.

Therefore, the individual effects of variable specific impulse, maximum thrust, and path constraints on

the porkchop plots are further investigated. To this end, porkchop plots with and without variable specific impulse, maximum thrust, and path constraints are generated using a direct method of optimal control, as shown in Fig. 10.

The variations in specific impulse and maximum thrust have a relatively minor impact on the orbital transfer, resulting in only slight structural changes in the porkchop plots. In contrast, the introduction of path constraints significantly affects the porkchop plot, particularly near its boundaries, and substantially alters the overall plot structure. This suggests that, in the later stages of mission planning, the influence of path constraints must be carefully accounted for to optimize launch window selection.

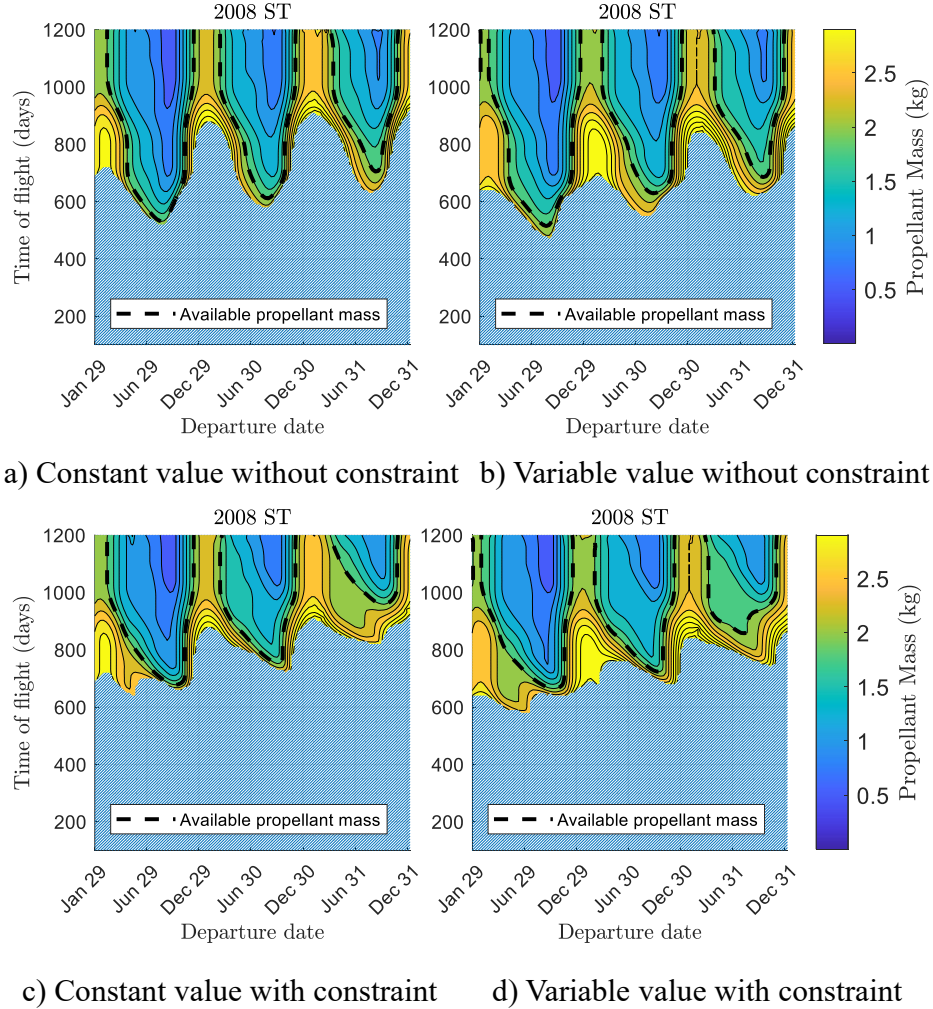


Figure 10. Comparison of porkchop plots with variable I_{sp} , T_{max} and path constraints by optimal control method.

Further analysis of the reverse-computed trajectories, shown in Fig. 11, reveals that the optimal control method, constrained by path boundaries, produces trajectories that closely follow the 0.8 AU limit. In contrast, the neural network approach, which does not impose path constraints, results in infeasible trajectories in certain regions. This shows that at the point with the largest prediction error, there is a significant difference between subplots (d) and (e), which correspond to the path-constrained scenarios. This indicates that the current neural network model may produce large estimation errors when physical path constraints are present. This also highlights the need for future research to incorporate such constraints into the network

model.

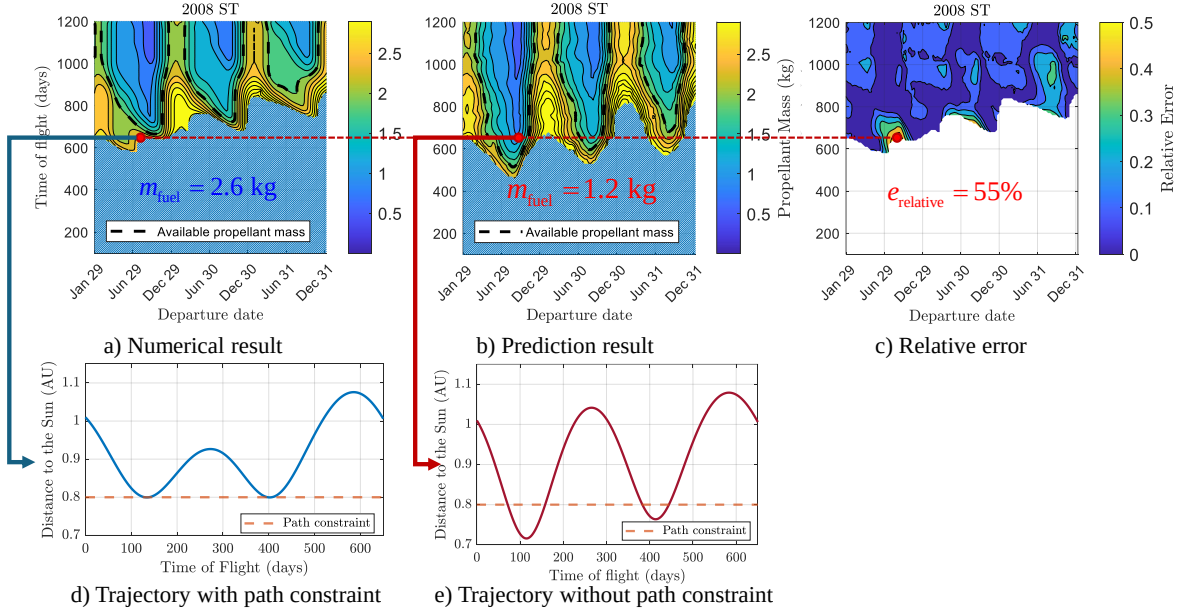


Figure 11. Porkchop plot for the 2008 ST Asteroid with path constraint.

In some cases, the presence of multiple local optima causes the optimal control method based on the solution-continuation technique to fail in capturing the complete global cost landscape, as illustrated in Fig. 12. Although this issue can be mitigated through extensive trial runs, such an approach would inevitably lead to significantly increased computation time. This challenge becomes particularly pronounced when screening a large number of candidate asteroids for exploration, where the global computational burden becomes increasingly prohibitive.

This scenario presents a compelling case for the use of neural network models. When trained on a sufficient amount of high-quality data, neural networks can leverage their global approximation capability to provide smooth and reasonable fuel consumption estimates. Moreover, due to their high computational efficiency, neural networks enable the use of global optimization algorithms within a reasonable time frame, even in scenarios requiring further optimization like tuning the launcher’s velocity in this paper’s study, making them a promising alternative.

CONCLUSION

This work presents a comparative study of the performance of optimal control methods and neural network-based estimation approaches on porkchop plots, a widely adopted tool in mission analysis. In simple cases without path constraints, the neural network approach is able to reproduce the structural characteristics of the original porkchop plot while maintaining relative prediction errors below 10% in the majority of regions. In scenarios involving multiple local optima where continuation-based optimal control methods may fail, the neural network approach remains stable due to its independence from initial guesses. This facilitates efficient identification of asteroid targets with scientific or engineering value by mission designers.

Currently, practical engineering constraints, such as path constraints, have not yet been incorporated

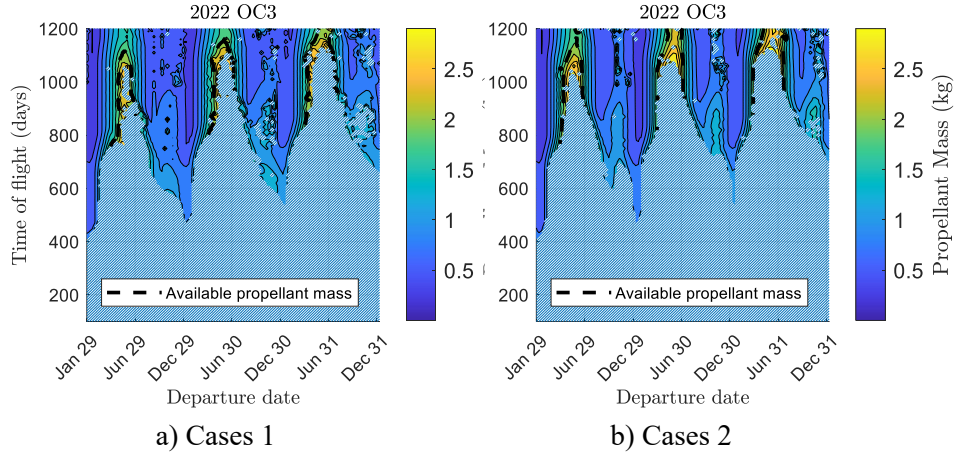


Figure 12. Example of failed cases of solution continuation with direct method, due to discontinuities in I_{sp} , T_{max} .

into the neural network model, which limits its applicability in some cases. Future research could focus on integrating these constraints into the learning framework, or exploring approaches guided by physical constraints.

This paper also presents a new perspective on the application of neural networks in astrodynamics. The generation of porkchop plots is inherently intuitive and easy to visualize, while enabling the validation and verification of a large number of orbital transfer scenarios, thus allowing for a straightforward assessment of prediction quality. More importantly, this tool is widely used in mission preliminary design and is particularly valuable in early-stage planning, where a higher tolerance for solution inaccuracy is often acceptable. The introduction of neural networks can significantly enhance the efficiency of mission designers. Furthermore, due to their independence from domain-specific expertise, planetary science teams can quickly perform preliminary filtering during the initial mission phase, thereby reducing communication and iteration costs, which offers practical value for engineering applications.

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