TWO NOVEL APPROACHES TO ANTENNA-PATTERN SYNTHESIS

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PRESENTATION DESCRIBES AN ITERATIVE APPROACH TO PATTERN SYNTHESIS USING A MATRIX BASED ON SPECIFIED LOBE MAXIMA . . .

• THE BASIC IDEA

• SEVERAL EXAMPLES

. . . AND PATTERN SYNTHESIS USING SPATIAL POLES

• SOME BACKGROUND

• PRONY’S METHOD AS A WAY TO DETERMINE SOURCE LOCATIONS AND STRENGTHS FOR SPECIFIED PATTERNS

• THE SINUSOIDAL CURRENT FILAMENT

• SEVERAL EXAMPLES OF PRONY SYNTHESIS

• SYNTHESIZING EXPONENTIATED PATTERNS
ANTENNA PATTERN SYNTHESIS REMAINS A TOPIC OF INTEREST . . .


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VARIOUS SOURCE DISTRIBUTIONS AND/OR PATTERNS FROM THE FOLLOWING SOURCES WERE USED


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• SYNTHESIZING EXPONENTIATED PATTERNS
THE APPROACH IS STRAIGHTFORWARD:

1) A LINEAR-ARRAY GEOMETRY IS CHOSEN
   --TYPICALLY UNIFORM SPACING IS USED, BUT THIS IS NOT MANDATORY

2) AN INITIAL SET OF ELEMENT CURRENTS IS SPECIFIED
   --IT’S CONVENIENT TO USE UNIT-AMPLITUDE CURRENTS WITH A UNIFORM PHASE OF ZERO OR A SMALL POSITIVE ANGLE

3) THE FAR-FIELD PATTERN IS COMPUTED

4) THE ANGLES AT WHICH THE PATTERN MAXIMA OCCUR ARE LOCATED AND A NEW SET OF ELEMENT CURRENTS ARE OBTAINED USING THESE ANGLES AND THE DESIRED VALUES OF THE LOBE MAXIMA

5) RETURNING TO 2) THESE NEW CURRENTS ARE USED TO COMPUTE A NEW PATTERN & THE PROCESS CONTINUES UNTIL THE PATTERN CONVERGES
EVEN AND ODD NUMBERS OF ELEMENTS WERE USED FOR SYMMETRIC ARRAYS

• FOR SYMMETRIC ARRAYS THE PATTERN CAN BE WRITTEN AS . . .

\[
P(\theta) = \sum_{n=1}^{N} S_n \cos[(2n - 1)u]
\]

OR

\[
P(\theta) = \sum_{n=0}^{N} S_n \cos(2nu)
\]

FOR AN EVEN OR ODD NUMBER OF ELEMENTS RESPECTIVELY, WHERE

\[
u = \left[\left(\frac{\pi d}{\lambda}\right)\cos\theta\right]
\]
LOBE MAXIMA GENERATE A MATRIX . . .

1) The initial pattern $P_1(\theta)$ is sampled finely enough in $\theta$ to accurately locate its positive and negative maxima at the angles $\theta_{1,n}, n = 1, \ldots, N$ with the corresponding pattern maxima denoted by $P_1(\theta_{1,n})$.

2) A matrix is then developed from the cosines of the angles where the maxima are found, since these multiply the source currents in Equation (1), to determine the lobe maxima from

$$
\begin{bmatrix}
\cos(u_{11}) & \cos(3u_{11}) & \cdots & \cos[(2N-1)u_{11}] \\
\cos(u_{12}) & \cos(3u_{12}) & \cdots & \cos[(2N-1)u_{12}] \\
\vdots & \vdots & \ddots & \vdots \\
\cos(u_{1N}) & \cos(3u_{1N}) & \cdots & \cos[(2N-1)u_{1N}]
\end{bmatrix}
$$
WHICH IS THEN INVERTED TO SOLVE FOR A NEW SET OF CURRENTS $S_{1,n}$ FROM

$$
S_{l,1} = \begin{bmatrix}
\cos(u_{11}) & \cos(3u_{11}) & \cdots & \cos[(2N-1)u_{11}]
\end{bmatrix}^{-1} L_1
$$

$$
S_{l,2} = \begin{bmatrix}
\cos(u_{12}) & \cos(3u_{12}) & \cdots & \cos[(2N-1)u_{12}]
\end{bmatrix} L_2
$$

$$
\vdots
$$

$$
S_{l,N} = \begin{bmatrix}
\cos(u_{1N}) & \cos(3u_{1N}) & \cdots & \cos[(2N-1)u_{1N}]
\end{bmatrix} L_N
$$

WHERE THE $L_n$ ARE THE MAXIMUM VALUES DESIRED FOR THE LOBES OF THE SYNTHESIZED PATTERN
A SECOND SET OF PATTERN MAXIMA

\( P_2(\theta_{2,n}) \) AND MATRIX \([M_{2,N}]\) ARE COMPUTED TO OBTAIN AN UPDATED SET OF CURRENTS . . .

\[
\begin{bmatrix}
S_{2,1} \\
S_{2,2} \\
\vdots \\
S_{2,N}
\end{bmatrix} =
\begin{bmatrix}
\cos(u_{21}) & \cos(3u_{21}) & \cdots & \cos((2N-1)u_{21}) \\
\cos(u_{22}) & \cos(3u_{22}) & \cdots & \cos((2N-1)u_{22}) \\
\vdots & \vdots & \ddots & \vdots \\
\cos(u_{2N}) & \cos(3u_{2N}) & \cdots & \cos((2N-1)u_{2N})
\end{bmatrix}^{-1}
\begin{bmatrix}
L_1 \\
L_2 \\
\vdots \\
L_N
\end{bmatrix}
\]
WHICH RESULTS IN A THIRD SET OF PATTERN MAXIMA $P_3(\theta_3, n)$, etc., UNTIL THE PATTERN CONVERGES ACCEPTABLY

--ITERATION IS NECESSARY BECAUSE THE ANGLES AT WHICH MAXIMA OCCUR DEPEND SLIGHTLY ON THE CURRENT
For the more general case of a non-symmetric array the pattern can be written . . .

\[ P(\theta) = \sum_{n=1}^{N} S_n \exp \left( i(kx_n \cos \theta + \beta_n) \right) \]

. . . which leads to a current computation of the form . . .

\[
S_{i,1} = \begin{bmatrix} 
\exp(ikx_1 \cos \theta_{i1}) & \exp(ikx_2 \cos \theta_{i1}) & \cdots & \exp(ikx_N \cos \theta_{i1}) \\
\exp(ikx_1 \cos \theta_{i2}) & \exp(ikx_2 \cos \theta_{i2}) & \cdots & \exp(ikx_N \cos \theta_{i2}) \\
\vdots & \vdots & \ddots & \vdots \\
\exp(ikx_1 \cos \theta_{iN}) & \exp(ikx_2 \cos \theta_{iN}) & \cdots & \exp(ikx_N \cos \theta_{iN}) 
\end{bmatrix}^{-1} \begin{bmatrix} 
L_1 \\
L_2 \\
\vdots \\
L_N 
\end{bmatrix}
\]

. . . for the \( i \)'th iteration
SOME ADJUSTMENT MAY BE NEEDED DURING THE ITERATION PROCESS

• IF THE NUMBER OF LOBES CHANGES
  -- INCREASE OR DECREASE THE NUMBER OF ARRAY ELEMENTS
  -- INCREASE OR DECREASE THE ARRAY LENGTH
  -- ADJUST THE PATTERN SPECIFICATION

• IF THE NEAR END-FIRE LOBES BECOME ILL FORMED
  -- AS ABOVE
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. . . AND PATTERN SYNTHESIS USING SPATIAL POLES

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• SYNTHESIZING EXPONENTIATED PATTERNS
A SEQUENCE OF PATTERNS THAT CONVERGES TO ONE HAVING -20 dB & -40 dB SIDELOBES ON THE LEFT AND RIGHT ILLUSTRATES THE APPROACH

• 15 ELEMENTS, 0.5 WAVELENGTHS APART
A SEQUENCE OF PATTERNS . . .

• ITERATION #1
A SEQUENCE OF PATTERNS . . .

ITERATION #2
A SEQUENCE OF PATTERNS . . .

ITERATION #3
A SEQUENCE OF PATTERNS . . .

ITERATION #4
A SEQUENCE OF PATTERNS . . .

ITERATION #5
A SEQUENCE OF PATTERNS . . .

ITERATION #6
A SEQUENCE OF PATTERNS . . .

ITERATION #7 AND THE FINAL PATTERN
THE PATTERN DETERIORATES FOR LOWER FREQUENCIES . . .

Normalized pattern (dB) vs. angle from array axis (degrees). The pattern is shown for a separation of 0.1 wavelengths. The graph includes a note for D-Left20Right40D0.1to0.8 and G-N15L20R40Sep0.1to0.2.
THE PATTERN DETERIORATES FOR LOWER FREQUENCIES . . .
WITH SIDELOBES MAINTAINED OVER A NEARLY 2:1 BANDWIDTH . . .
... WITH SIDELOBES MAINTAINED OVER A NEARLY 2:1 BANDWIDTH ...
\[ \ldots \text{WITH SIDELOBES MAINTAINED OVER A NEARLY 2:1 BANDWIDTH} \ldots \]
... AND DEVELOPS GRATING LOBES FOR HIGHER FREQUENCIES ...
AND DEVELOPS GRATING LOBES FOR HIGHER FREQUENCIES . . .
AND DEVELOPS GRATING LOBES FOR HIGHER FREQUENCIES . . .
A STANDARD DOLPH-CHEBYSHEV PATTERN IS READILY GENERATED . . .

-9-ELEMENT ARRAY 4 WAVELENGTHS LONG
VARIATIONS ON THE DOLPH-CHEBYSHEV DESIGN ARE EASY TO DEVELOP . . .

• 15-ELEMENT ARRAY, 7 WAVELENGTHS LONG
VARIATIONS ON THE DOLPH-CHEBYSHEV DESIGN ARE EASY TO DEVELOP . . .

• 15-ELEMENT ARRAY, 7 WAVELENGTHS LONG
A pattern designed with 15 lobe maxima increasing in steps of 5 dB.
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

ITERATION #1
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

ITERATION #2
A pattern designed with 15 lobe maxima increasing in steps of 5 dB.
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

ITERTATION #4
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

NORMALIZED PATTERN (dB)

ANGLE FROM ARRAY AXIS (degrees)

ITERATION #5
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

NORMALIZED PATTERN (dB)

ANGLE FROM ARRAY AXIS (degrees)

ITERATION #6
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

ITERATION #8
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

NORMALIZED PATTERN (dB)

ANGLE FROM ARRAY AXIS (degrees)

ITERATION #9
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB
A PATTERN DESIGNED WITH 15 LOBE MAXIMA INCREASING IN STEPS OF 5 dB

• A 15-ELEMENT ARRAY, 7 WAVELENGTHS LONG

ITERATION #11
THE DOLPH-CHEBYSHEV PATTERN DOES NOT REQUIRE UNIFORM SPACING

• VARIABLE SPACINGS OF 0.4 AND 0.6 WAVELENGTHS
THE DOLPH-CHEBYSHEV PATTERN DOES NOT REQUIRE UNIFORM SPACING

- VARIABLE SPACINGS OF 0.4 TO 0.7 WAVELENGTHS
THE DOLPH-CHEBYSHEV PATTERN DOES NOT REQUIRE UNIFORM SPACING

• ARRAY LENGTHS OF 4 (BLACK) AND 5 (BLUE) WAVELENGTHS RESPECTIVELY
THE DOLPH-CHEBYSHEV PATTERN DOES NOT REQUIRE UNIFORM SPACING

- VARIABLE SPACING (BLACK) AND UNIFORM SPACING (RED) RESPECTIVELY
The pattern for the -20 dB and -40 dB array when the initial element currents are all zero except for unit-amplitude currents on elements 1 and 15, and for the first two iterations.
SOME EXTENSIONS OF THE BASIC IDEA MIGHT INVOLVE SUCH THINGS AS CONTROLLING:

- NULLS
- SIDE-LOBE ANGLES
- MAIN LOBE ANGLE
- THE NUMBER OF SIDE LOBES
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PRONY’S METHOD OR ITS EQUIVALENT PROVIDES THE ARRAY PARAMETERS FROM PATTERN SAMPLES

• GIVEN A DESIRED PATTERN \( P_{\text{desired}}(\theta) \) . . .

\[
P_{\text{desired}}(\theta) \approx P_{\text{DSA}}(\theta) = \sum_{\alpha=1}^{N} S_{\alpha} e^{kz_{\alpha} \cos(\theta)}
\]

• . . . THE \( N \) SOURCE STRENGTHS \( S_{\alpha} \) AND \( N \) LOCATIONS \( z_{\alpha} \) CAN BE OBTAINED

• FOR THE ARRAY TO BE REALIZABLE USING ISOTROPIC SOURCES \( z_{\alpha} \) MUST BE PURE IMAGINARY

• OTHERWISE A SOURCE DIRECTIVITY WOULD BE REQUIRED AS GIVEN BY

\[
D_{\alpha} = e^{kz_{\alpha,\text{real}} \cos(\theta)}
\]
IMPLEMENTING PRONY’S METHOD FOR PATTERN SYNTHESIS INVOLVES CHOOSING 3 PARAMETERS . . .

• THE ANGLE SAMPLING INTERVAL $\Delta \cos \theta$
  -- MUST BE SMALL ENOUGH TO AVOID ALIASING

• THE TOTAL ANGLE OBSERVATION WINDOW $W$
  MEASURED IN UNITS OF $\cos \theta$
  -- MUST BE WIDE ENOUGH TO AVOID ILL CONDITIONING OF THE DATA MATRIX
THE LOBES OF A LINEAR ARRAY ARE SPACED UNIFORMLY IN\( \cos(\theta) \).

- This shows that sampling as a function of \( \cos(\theta) \) rather than \( \theta \) is more appropriate.
- Besides which Prony’s method requires that sampling use equal steps in \( \cos(\theta) \).
IMPLEMENTING PRONY’S METHOD FOR PATTERN SYNTHESIS INVOLVES CHOOSING 3 PARAMETERS . . .

• THE NUMBER OF POLES OR EXPONENTIALS $N$
  -- FOR WHICH THE NUMBER OF PATTERN SAMPLES REQUIRED IS $2N = (W/\Delta \cos \theta) + 1$

. . . WHICH RESULTS IN REQUIRING THAT $N$ BE THE LARGER OF

$$N \geq WL + 1$$

AND

$$N \geq R$$

WITH $L$ THE SOURCE SIZE IN WAVELENGTHS, $R$ THE PATTERN RANK AND $W$ THE WINDOW WIDTH
THE RESULTS THAT FOLLOW WERE GENERALLY OBTAINED USING THE FOLLOWING STEPS:

• BEGINNING THE FITTING-MODEL COMPUTATION USING A SLIGHTLY SMALLER VALUE FOR $N$ THAN GIVEN ABOVE

• SUCCESSIVELY INCREASING $N$ UNTIL THE FITTING MODEL CONVERGES TO WITHIN 0.1 dB (UNLESS OTHERWISE NOTED) OF THE GENERATING-MODEL PATTERN

• SOMETIMES VARYING THE WIDTH OF THE OBSERVATION WINDOW

• ROUTINELY COMPUTING THE SVD SPECTRUM OF THE DESIRED PATTERN

• USING A COMPUTE PRECISION OF 24 DIGITS
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A USEFUL INITIAL TEST IS A MODIFIED PATTERN OF A SINUSOIDAL CURRENT FILAMENT

• ITS FAR-FIELD PATTERN IS GIVEN BY

\[ P_{MSCF}(\theta) = \sin \theta \times P_{SCF}(\theta) = \sin \theta \left[ \frac{e^{(ikL/2)\cos \theta} + e^{-(ikL/2)\cos \theta} - 2\cos(kL/2)}{\sin \theta} \right] \]

• \( P_{MSCF}(\theta) \) IS SEEN TO BE THE SUM OF THREE POINT SOURCES

• THE FIRST TWO TERMS ARE DUE TO THE ENDS OF THE FILAMENT

• THE LAST IS A LENGTH-DEPENDENT CONTRIBUTION DUE TO A CURRENT-SLOPE DISCONTINUITY AT THE CENTER
TWO DIFFERENT WINDOW WIDTHS PRODUCE ESSENTIALLY IDENTICAL PATTERN MATCHES

- LENGTH OF SCF IS 5 WAVELENGTHS
- WINDOWS OF -0.999 TO + 0.999 AND -0.05 TO + 0.05 IN $\cos \theta$ WERE USED
- TWO ARROWS INDICATE THE EXTENT OF THE LATTER
SINGULAR-VALUE SPECTRA FOR SEVERAL WINDOW WIDTHS EXHIBIT A PATTERN RANK OF 3 FOR $P_{MSCF}$ . . .

$N$ WAS INCREASED FOR EACH WINDOW UNTIL THE PATTERN CONVERGED

RESULT IS CONSISTENT WITH A 3 POINT SOURCES
AS IS REVEALED BY A PLOT OF THE PRONY-DERIVED SOURCES

- Source strengths are plotted as arrows on a 3-decade logarithmic scale.
- Phase is shown on a polar plot.
- The X’s denote the physical SCF extent.
THE NUMBER OF FITTING MODELS NEEDED FOR A CONVERGED PATTERN INCREASES SYSTEMATICALLY WITH WINDOW WIDTH

• TO AVOID ALIASING
THE CENTER SOURCE DISAPPEARS FOR A SCF 5.5 WAVELENGTHS LONG

• SAMPLED OVER A -0.05 TO +0.05 COS θ WINDOW
AN 11-TERM FITTING MODEL MATCHES THE ACTUAL PATTERN OF A 5-WAVELENGTH SCF DOWN TO -60 dB . . .

• THE BLACK DOTS DENOTE THE GENERATING-MODEL SAMPLES USED TO COMPUTE THE 11-POLE FITTING MODEL
. . . BUT THE DERIVED SOURCE DISTRIBUTION IS NOT PHYSICALLY REALIZABLE . . .

• . . . BECAUSE SOME OF SCF 9 SOURCES HAVE REAL COMPONENTS IN THE COMPLEX SPACE PLANE
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THE PATTERN OF A ±1 SQUARE-WAVE APERTURE IS GRAPHICALLY INDISTINGUISHABLE FROM AN 11-TERM FM . . .

\[ P_\pm = L \left( 1 - \cos \left( \frac{\pi L}{\lambda} \cos \theta \right) \right) \]
... WHOSE SYNTHESIZED SOURCES ARE NOT UNIFORMLY SPACED

• FOR A 5-WAVELENGTH APERTURE

• AND AN 11-POLE FITTING MODEL
THE PATTERN OF AN APERTURE VARYING AS $\cos^2(\pi/L)$ IS ALSO GRAPHICALLY IDENTICAL TO ITS PRONY FM . . .

- ITS PATTERN FACTOR IS GIVEN BY

$$P_{\cos^2} = \frac{\sin(u)}{u} \left[ \frac{\pi^2}{\pi^2 - u^2} \right]$$

WHERE $u = \left(\frac{\pi L}{\lambda}\right) \sin \theta$
... WHOSE SOURCE DISTRIBUTION IS ALSO NONUNIFORM

- FOR A 5-WAVELENGTH APERTURE
- USING AN 11-TERM FITTING MODEL
PATTERN OF UNIFORM CURRENT OF LENGTH \( L \) TIMES \((\sin \theta)^P\) HAS TAPERED SIDELOBES WITH INCREASING \( P \)

- ITS PATTERN FACTOR IS

\[
P_{UCF} = (\sin \theta)^P \frac{\sin(kL \cos \theta)}{kL \cos \theta}
\]
ITS SOURCES ARE ALSO NON-UNIFORMLY SPACED

... USING 11 EXPONENTIALS IN THE FITTING MODEL

• AND FOR A 5-WAVELENGTH APERTURE
A DOLPH-CHEBYSHEV ARRAY IS READILY SYNTHESIZED

- 5-WAVELENGTHS LONG WITH -26 dB SIDELOBES AND 10 ELEMENTS
A MODIFIED DOLPH-CHEBYSHEV ARRAY IS ALSO SYNTHESIZED

-20 AND -40 dB SIDELOBES

15 ELEMENTS UNIFORMLY SPACED

7 WAVELENGTHS LONG
THIS ARRAY STEPS UP IN 5 dB INCREMENTS FROM LEFT TO RIGHT

- 15 ELEMENTS UNIFORMLY SPACED
- 7 WAVELENGTHS LONG
SVD SPECTRA FOR SEVERAL ARRAYS ILLUSTRATE THEIR DIFFERENCES

- THE DOLPH-CHEBYSHEV ARRAY CLEARLY SHOWS THE NUMBER OF ELEMENTS IT CONTAINS
- THE SPECTRA OF THE CONTINUOUS DISTRIBUTIONS FALL OFF MORE SMOOTHLY
THIS PATTERN FROM ELLIOTT WAS REPLICAED USING PRONYS' METHOD


- 12 ELEMENTS IN 5.5 WAVELENGTHS.
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CONSIDER EXPONENTIATING A PATTERN:
  • FOR EXAMPLE THE PATTERN OF A 10-ELEMENT DOLPH-CHEBYSHEV ARRAY AS GIVEN BY . . .

\[ P_{10}(\theta) = 2.798 \cos(D) + 2.496 \cos(3D) + 1.974 \cos(5D) + 1.357 \cos(7D) + \cos(9D) \]

WITH \( D = \left[ (\pi d / \lambda) \cos \theta \right] \) AND \( d \) THE ELEMENT SPACING

. . . TO YIELD SUCCESSIVELY LOWER SIDELOBES

• INITIAL ARRAY LENGTH IS 4.5 WAVELENGTHS
REFINING THE D-C -26 dB PATTERN USING MATRIX-SYNTHESIS* APPROACH YIELDS A WIDENING MAIN LOBE FOR FIXED $L$

SYNTHESIZING THE D-C -26 dB D-C PATTERN USING PRONY’S METHOD PROVIDES A 0.1 Db OR BETTER MATCH

- AT LEVELS $\geq -110$ dB
- USING 10, 19, 28, & 37 POLES 4.5, 8.5, 13.5 AND 18.5 WAVELENGTHS LONG
ARRAYS FOR SUCCESSIVELY LOWER SIDE LOBES EXPAND PROPORTIONATELY IN SIZE

\[ M = 2 \quad M = 3 \quad M = 4 \]

... WHILE RETAINING UNIFORM SPACING AND THE SAME NUMBER OF SIDE LOBES
PATTERNS WITH SAME SIDELOBE LEVELS WERE GENERATED WITH THE MATRIX APPROACH

... Using 10, 18, 28 and 38 elements and array lengths of 4.5, 8.5, 13.5 and 18.5 wavelengths with 0.5 WL spacing
SIMILAR RESULTS ARE OBTAINED WHEN THE D-C ARRAY IS 7.5 WAVELENGTHS LONG . . .

![Normalized Pattern Graph](image)
... FOR WHICH THE SINGULAR-VALUE SPECTRA INDICATE THE NUMBER OF ARRAY ELEMENTS

• FOR EXPONENT $M = 1$ TO 4 ARE 10, 19, 28, 37 RESPECTIVELY
THE NUMBER OF SINGULAR VALUES INCREASES LINEARLY WITH THE EXPONENT $M$

**FOR A 7.5-WAVELENGTH, 10-ELEMENT DOLPH-CHEBYSHEV ARRAY**
THE MAIN BEAMWIDTH DECREASES FROM ABOUT 7.4 TO 3.6 DEGREES FOR AN EXPONENT PARAMETER VALUE OF 4 . . .

• FOR THE 7.5 WAVELENGTH ARRAY
... AT THE -3 dB LEVEL
THE PRONY-DERIVED ARRAYS CAN HAVE WIDELY VARYING SOURCE STRENGTHS:

- The number of sources varies from 10, 19, 28 to 35 for $M$ varying 1 to 4.
- For a 5-wavelength D-C array normalized to end elements.
- With implications for noise sensitivity.
WITH EACH ARRAY SIZE VARYING LINEARLY WITH INCREASING EXPONENT

- AS $M \times$ INITIAL ARRAY WIDTH

• FOR A 10-ELEMENT PATTERN GIVEN BY
  
  \[0.4463 \times \cos(U) + 0.4306 \times \cos(3. \times U) + 0.4003 \times \cos(5. \times U) + 0.3576 \times \cos(7. \times U) + \cos(9. \times U)\]

THE EXponentiated PATTERN main beamWIDTH successively decreases

The “standard” -20 dB pattern (red) is given by

\[ 1.5585 \cos(U) + 1.4360 \cos(3U) + 1.2125 \cos(5U) + 0.9264 \cos(7U) + \cos(9U) \]

The 19-element prony pattern (black) comes from

\[ (0.4463 \cos(U) + 0.4306 \cos(3U) + 0.4003 \cos(5U) + 0.3576 \cos(7U) + \cos(9U))^2 \]
THE EXPONENTIATED PATTERN MAIN BEAMWIDTH SUCCESSIVELY DECREASES

- THE “STANDARD” -30 dB PATTERN (RED) IS GIVEN BY
  \[ 3.8830 \cos(U) + 3.4095 \cos(3.0U) + 2.5986 \cos(5.0U) + 1.6695 \cos(7.0U) + \cos(9.0U) \]

- THE 28-ELEMENT PRONY PATTERN (BLACK) COMES FROM
  \[ (0.4463 \cos(U) + 0.4306 \cos(3.0U) + 0.4003 \cos(5.0U) + 0.3576 \cos(7.0U) + \cos(9.0U))^3 \]
THE EXPONENTIATED PATTERN MAIN BEAMWIDTH SUCCESSIVELY DECREASES

• THE “STANDARD” -40 dB PATTERN (RED) IS GIVEN BY
  \[ 7.9837 \cos(U) + 6.6982 \cos(3. \times U) + 4.6319 \cos(5. \times U) + 2.5182 \cos(7. \times U) + \cos(9. \times U) \]

• THE 35-ELEMENT PRONY PATTERN (BLACK) COMES FROM
  \[(0.4463 \cos(U) + 0.4306 \cos(3. \times U) + 0.4003 \cos(5. \times U) + 0.3576 \cos(7. \times U) + \cos(9. \times U))^4\]
ABOVE PRONY-SYNTHESIZED ARRAYS ARE UNIFORMLY SPACED FOR $M \leq 3$ BUT EXHIBIT A TAPERED SPACING FOR $M \geq 4$

• THE RESPECTIVE NUMBER OF ARRAY ELEMENTS ARE 10, 19, 28, 35, AND 42 FOR AN INITIAL ARRAY 5-WAVLENGTHS LONG
THE DOLPH-CHEBYSHEV SVD SPECTRUM ROLLS OFF SLOWER WITH INCREASING WINDOW WIDTH

• FOR A 10-WAVELENGTH, 10-ELEMENT ARRAY
ANALYTIC EXPRESSIONS FOR THE EXPONENTIATED PATTERNS CAN BE DERIVED* 

• CONSIDER THE 4-ELEMENT D-C ARRAY WHOSE PATTERN IS \( P_4 = A_1 \cos(u) + A_2 \cos(3u) \) 
WHERE \( A_1 = 0.8794 \) and \( A_2 = 1 \) 

• ITS EXPONENTIATED PATTERN IS THEN 

\[
P_4^M = \left[ A_1 \cos(u) + A_2 \cos(3u) \right]^M.
\]

• FOR \( M = 2 \) THIS BECOMES 

\[
P_4^2 = \frac{A_1^2 + A_2^2}{2} + \left( \frac{A_1^2}{2} + A_1 A_2 \right) \cos(2u) + A_1 A_2 \cos(4u) + \frac{A_2^2}{2} \cos(6u).
\]

*G. J. BURKE, PRIVATE COMMUNICATION, 2013 VIA MATHEMATICA
ITS PATTERNS FOR $M = 3$ AND $M = 4$ ARE GIVEN BY

$$P_4^3 = \frac{3}{4} [A_1^3 + A_1^2 A_2 + 2A_1 A_2^2] \cos(u) + \frac{1}{4} [A_1^3 + 6A_1^2 A_2 + 3A_2^3] \cos(3u)$$

$$+ \frac{3}{4} [A_1^2 A_2 + A_1 A_2^2] \cos(5u) + \frac{3}{4} A_1 A_2^2 \cos(7u) + \frac{1}{4} A_2^3 \cos(9u)$$

AND

$$P_4^4 = \frac{3}{2} \left[ \frac{1}{4} A_1^4 + \frac{1}{3} A_1^3 A_2 + A_1^2 A_2^2 + \frac{1}{4} A_2^4 \right] \cos(2u)$$

$$+ \frac{3}{2} \left[ \frac{1}{12} A_1^4 + A_1^3 A_2 + \frac{1}{2} A_1^2 A_2^2 + A_1 A_2^3 \right] \cos(4u) + \frac{3}{2} \left[ \frac{1}{3} A_1^3 A_2 + A_1^2 A_2^2 + \frac{1}{3} A_2^4 \right] \cos(6u)$$

$$+ \frac{3}{2} \left[ \frac{1}{2} A_1^2 A_2^2 + \frac{1}{3} A_1 A_2^3 \right] \cos(8u) + \frac{1}{2} A_1 A_2^3 \cos(10u) + \frac{1}{8} A_2^4 \cos(12u).$$

RESPECTIVELY
PRONY-SYNTHESIZED AND ANALYTIC PATTERNS FROM THE PREVIOUS FORMULAS AGREE TO WITHIN 0.1 dB . . .

. . . FOR A 2-WAVELENGTH ARRAY . . .
... WHOSE ELEMENT STRENGTHS ARE FOUND TO BE...

<table>
<thead>
<tr>
<th>Element Number</th>
<th>Basic Array</th>
<th>M = 2</th>
<th>M = 3</th>
<th>M = 4</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>4 Elements</td>
<td>7 Elements</td>
<td>10 Elements</td>
<td>13 Elements</td>
</tr>
<tr>
<td>1</td>
<td>0.8794</td>
<td>1.773</td>
<td>9.640</td>
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<tr>
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</tbody>
</table>

WITH THEIR DYNAMIC RANGE INCREASING FROM 1.14:1 TO 30.4:1
EXPONENTIATED PATTERNS OF A UNIFORM CURRENT FILAMENT ARE NOT SYNTHESIZED AS WELL

• FOR A 5-WAVELENGTH FILAMENT

• DIFFERENCES BETWEEN SYNTHESIZED AND ACTUAL PATTERNS BECOME SIGNIFICANT AT LEVELS ≤ -50 TO -60 dB
SYNTHESIZED EXPONENTIATED PATTERNS FOR A TRIANGLE CURRENT FILAMENT ARE IMPROVED OVER THE UCF

• FOR A 5-WAVELENGTH CURRENT FILAMENT
WIDE DYNAMIC RANGE OF SOURCE STRENGTHS CAN MAKE PATTERNS NOISE SENSITIVE . . .

• FOR $M = 2$, $L = 7.5$ WAVELENGTHS
• WITH A MAXIMUM OF 10% RANDOM VARIATION IN THE ELEMENT STRENGTHS
WIDE DYNAMIC RANGE OF SOURCE STRENGTHS CAN MAKE PATTERNS NOISE SENSITIVE . . .

• FOR $M = 3$, $L = 5$ WAVELENGTHS
• WITH A MAXIMUM OF 1% RANDOM VARIATION IN THE ELEMENT STRENGTHS
WIDE DYNAMIC RANGE OF SOURCE STRENGTHS CAN MAKE PATTERNS NOISE SENSITIVE . . .

\[ M = 3, \ L = 7.5 \text{ WAVELENGTHS} \]

\[ \text{WITH A MAXIMUM OF 1}\% \text{ RANDOM VARIATION IN THE ELEMENT STRENGTHS} \]
PRESENTATION HAS DESCRIBED AN ITERATIVE APPROACH TO PATTERN SYNTHESIS USING A MATRIX BASED ON SPECIFIED LOBE MAXIMA . . .

• THE BASIC IDEA

• SEVERAL EXAMPLES

. . . AND PATTERN SYNTHESIS USING SPATIAL POLES

• PRONY’S METHOD AS A WAY TO DETERMINE SOURCE LOCATIONS AND STRENGTHS FOR SPECIFIED PATTERNS

• THE SINUSOIDAL CURRENT FILAMENT

• SEVERAL EXAMPLES OF PRONY SYNTHESIS

• SYNTHESIZING EXPONENTIATED PATTERNS