Strategic asset allocation for pension plans

Camillo Biasca
University of Neuchâtel
Master of Science in Finance
camillo.biasca@unine.ch

December 2009

In collaboration with
DE Planification
Institutional Consulting

Responsible:
Nicolas Marmagne

Supervisor:
Prof. Michel Dubois

The views expressed in this paper are those of the author and do not necessarily reflect those of the University, the employer and the supervisor.
Strategic asset allocation for pension plans

Abstract

This study explores the strategic asset allocation for pension plans. I develop an Asset-Liability Management (ALM) model in a multiperiod framework. I discover that pension plans must heavily invest in bonds, real estate and hedge funds. This finding is strongly in contrast to current investment decisions in practice that allocate high percentage in stocks and very little in real estate and hedge funds. To demonstrate the appropriateness of the ALM model, I compare this approach to three alternative models (two Markowitz and a naïve model) by applying a funding ratio return analysis and a Sharpe ratio comparison over a three years out-of-sample period. I find that ALM is the most adequate solution because it yields constant moderate risk-adjusted return and it significantly reduces the funding ratio volatility. These characteristics allow pension plans to guarantee the payment of current and future obligations on a continuing basis avoiding drastic variations of the funding ratio. Conversely, I find that alternative models and current investment decisions in practice highly expose pension plans to mismatch risk.

Keywords: Pension funds, Strategic asset allocation, Portfolio choice, Financial Forecasting, Asset-liability management, Performance measurement

JEL classification: G11, G23, C32, C53
1. Introduction

During the last decade most pension funds have experienced a strong decrease in the level of funding ratio and their survival is nowadays shaking. These entities exclusively justify their bad results by the underperformance of financial markets. Numerous authors have rather claimed that the main reason of such failure must be searched in the inappropriateness of the investment strategies designed by pension managers. Among others, Plantinga (2005) demonstrates that the bad shape of pension fund system is directly relatable to the reckless exposure to stocks. Particularly important is the switch from conservative asset allocations towards more aggressive exposures to Equity that took place at the end of the 90’s. UBS reports that at the end of 2001 Equities made up around 71 percent of total long term asset holdings. A European survey carried out by JP Morgan (2006) reveals that in 2005 Stocks still accounted for 40 percent of the entire asset allocation (see appendix).

Although different authors have proposed tailored alternative models to solve the portfolio choice problem for pension plans, few practical implementations have followed. Von Ah (2008) claims that pension managers tend to adopt wrong models and meaningless market benchmarks when they design the strategic asset allocation. Specifically, they seem to neglect that the ultimate purpose of pension plans is to provide enough assets to pay for present and future obligations. Such indifference with respect to liability have driven the pension fund system into what has been called the “perfect storm” by Ritchie and Dumbrek. Two effects basically eroded the funding ratio of pension plans. One is the strong decrease in the value of assets, while the second is the sharp drop in interest rates that caused a consequent increase in liability. Wilshire Associates reports that 89 percent of the US S&P 500 Defined Benefits (DB) pension plans found themselves underfunded at the end of 2002. At the end of the subsequent year, total underfunding in the US private pension funds exceeded USD 350 million. At the end of 2008, the US 100 largest corporate pension plans were underfunded by $217 billion, holding only 79 percent of the assets needed to cover estimated long-term liabilities.

---

The goal of this paper is to determine the optimal strategic asset allocation for a typical US pension plan by implementing an Asset-Liability Management (ALM) approach. The innovation of this model consists in the separation of the asset allocation in two funds: a Liability-Hedging Portfolio (LHP) and a Return Optimizing Portfolio (ROP). The LHP aims to construct a portfolio that reduces the volatility of the funding ratio (i.e. mismatch risk), while the ROP seeks to yield high adjusted returns (as in a classic Markowitz context). In contrast with the majority of previous studies, I work in a multiperiod framework where assets and liability returns are modeled by a first order Vector Autoregressive Return model (VAR). To be as close as possible to the industry practices, I extend the common asset-menu composed by T-bills, Stocks and Bonds to alternative assets. I incorporate Hedge Funds, Public and Private Real Estate, Commodities and Gold. I solve the portfolio choice problem for different investment horizons (3, 5, 10, 20 and 25 years) and I analyze in details the evolution of variances and correlations across time. I also present three alternative strategies: a conditional asset-only (AO), an unconditional Jorion-Stein (JS) and a naïve 1/N (1/N) model⁶. The addition of these strategies leads to test over a three years out-of-sample period if and which strategy outperforms the others. For that, I use a funding ratio return analysis and a Sharpe ratio comparison (using Jobson-Korkie statistics).

I find that ALM tends to outperform JS and AO strategies. This result indicates that taking into consideration the behavior of liabilities in the construction of the strategic asset allocation is crucial. Bonds and Public Real Estate are fundamental asset classes, regardless of the investment horizon and the level of risk aversion. The high exposure to these asset classes is mainly explained by their great liability-hedging potentials. Hedge Funds and Private Real Estate are also appealing classes because of their interesting risk diversification and Sharpe ratio properties. Commodities are marginal but still interesting investments for low risk-averse agents having short-term horizons. Conversely, T-bills, Stocks and Gold look unattractive and they are completely ignored in the strategic asset allocation.

The reminder of this paper is organized as follows. The next section describes the different forms of pension plans present in the industry. Section 3 provides an overview of the relevant literature regarding strategic asset allocation, portfolio choice problem, asset-liability management, asset allocation, asset-liability management, and portfolio choice problem.

---

⁶ AO strategy: corresponds to a classic Markowitz optimization in a multiperiods framework where only assets are considered.

JS strategy: corresponds to a classic Markowitz optimization in a single period where expected returns are shrunk thanks to the use of a Jorion-Stein technique.

1/N strategy: corresponds to a naïve allocation where weights are assigned in a 1/N manner, where N refers to the number of asset classes.
alternative assets and financial forecasts. Section 4 introduces the methodology for estimating dynamics of returns for conditional and unconditional approaches. Section 5 describes the portfolio choice problem of these previous models. Section 6 reports the performance measurement technique adopted to test out-of-sample the four strategies. Section 7 describes the data set composed by assets, liability and state variables. Section 8 reports the estimation results from the models used to forecast asset and liability returns. The same section also offers a detailed analysis of the risk diversification and liability-hedging properties across different investment horizons. Section 9 illustrates the strategic asset allocations for different levels of risk aversion and investment horizons. Section 10 reports the results inherent to the out-of-sample test for the four strategies.

Schema 1: an overview of the study
2. Pension system: DB and DC pension plans

Defined Benefits (DB) pension plan system has been strongly criticized for its inability to offer tailored solutions for diverse risk averse participants. Although Morin and Suarez (1983) and Brown (2007) show that the risk aversion and therefore the investment choice change with age, race, ethnicity and social class, pension plans continue to extensively use single utility functions to solve the portfolio choice problem\(^7\). This way of proceeding cannot fairly represent the investment preferences of all persons belonging to the pension schemes (pensioners, dynamic young managers and employees) and implies that DB pension plans often offer sub-optimal allocations to most of their participants. Foley, Serjantov and Smith (2006) suggest that an alternative solution would be to identify a unique utility function that would describe the average pension plan stakeholder’s risk preference. This proposition has arisen strong criticisms about its concrete practical implementation, although this approach can be theoretically justified.

The changeover to a Defined Contributions (DC) scheme was assumed to offer potential benefits in satisfying the willingness of participants to select more appropriate investments according to their own preferences. In the extreme United States version, DC pension plans are in charge of executing only restricted administrative task, offering a wide range of investment funds and paying pensions on a lifetime contribution schema. Employees are allowed to choose among various risk-return profiles to invest their total contributions\(^8\). This way of proceeding enables DC pension plans to transfer all the risk in the hands of participants.

However, Rooij, Kool and Prast (2004) report that the vast majority of respondents in their survey remains in favor of a DB pension compulsory saving retirement system. The average respondent considers himself too financially unsophisticated to decide for their pension fund investments. Benartzi and Thaler (2002) highlight that the autonomy with regard to the investment choice has little value because agents typically have difficulties in selecting their portfolio in a consistent manner. Thus, the shift from DB to DC system has generally advantaged pension plans and disadvantaged employees.

Because of this evident unsophistication problem among employees, some countries have introduced a hybrid form of DC system that does not totally transfer the risk and the task of the portfolio choice to participants. Sunden (2000) informs that the Swedish government was one of the first movers to adopt in 1998 a hybrid DC scheme primarily financed on a pay-as-you-go basis with a funded component. Under this system, participants must pay a contribution of 18.5 percent, where

---

\(^7\) Generally, the most employed utility functions are Constant Relative Risk Aversion (CRRA) and Hyperbolic Absolute Risk Aversion (HARA)

\(^8\) Examples of funds offered by Defined Contribution plans in the United States are Individual Retirement Account (IRAs) and 401(k) plans.
16 percent is credited to a “notional” account, and the 2.5 percent is contributed to an “individual” account. Palme, Sunden and Söderlin (2004) report that Swedes can choose their risk profile for the individual account among a broad range of domestic and international funds. In 2000, this menu was composed of 650 mutual funds. This particular DC system allows the government to create a clear link between contributions and benefits. Different from a DB plan, benefits are determined by a lifetime contribution and participants can individually decide how to invest part of their pension wealth.

Under both an hybrid DC and a DB pension form, the pension plan remains in charge of designing the strategic asset allocation. The ALM model proposed in this study is assumed to help these entities to find an optimal solution.

3. Literature Review

3.1 Strategic asset allocation

The primordial goal of a pension plan is to design a strategic asset allocation that enables the pension plan to meet its current and future obligations. The value of liability mainly depends on interest rates, demography, mortality rates, inflation rates and contribution rates of participants. Today financial markets do not offer specific products that fully hedge liability risk. For that reason, Hoevenaars et al. (2008) affirm that markets are incomplete. Brennan and Xia (2000), Campbell and Viceira (2002,2005) and Van Binsbergen and Brandt (2006) suggest that pension funds should exclusively invest in TIPS (Treasury Inflation Protected Securities) because they are the safest way to guarantee liability-hedge and inflation-indexed benefits. De Jong, Schotman and Werker (2008) rather highlight that this option is usually rejected by pension funds because of their high cost. More advanced Asset-Liability Management models are therefore preferred by pension managers.

Von Ah (2008) describes two traditional ALM approaches. One consists of an immunization strategy (also called duration-matching) whose goal is to match the sensitivity of assets and liabilities to interest rate variations. This solution has however several drawbacks. First, duration-matching would eliminate the exposure of pension funds to parallel shifts of the term structure, but not to more complicated changes. Second, interest rate risk is only one of the numerous factors affecting liabilities. As a result, pension funds would still remain partly exposed to other risks. Third, Leibowitz, Kogelman and Bader (1994) emphasize that duration-matched solution is generally quite unattractive in terms of portfolio performance, because the volatility of long-duration investments approaches the volatility of equities. The second approach consists of a cash flow matching. Pension managers use dynamic optimization to determine a portfolio of assets
(generally bonds) that generates cash flows matching with the liability side. This strategy engenders a significant level of trading and consequent high transaction costs.

The incompleteness of financial markets and the high cost of traditional ALM approaches induce pension funds (and implicitly their members) to be willing to accept funding risk. Thus, pension managers need to find an adequate portfolio choice model to solve the asset allocation problem.

3.2 Portfolio choice

Despite its long existence and its extensive use in practice, the modern portfolio theory formulated by Markowitz (1952) still represents the basic concept used by pension plans to select their optimal portfolios. Wilkie (1985) and Sherris (1992) emphasize that the classic Markowitz optimization is however not suitable for pension plans and insurance companies. A new model that takes into account the current and future obligations would be required. These authors propose a portfolio choice model that maximizes the pension scheme absolute level of surplus.\(^9\)

Sharpe and Tint (1990) introduce the concept of “surplus return” which permits the conversion of the absolute level of surplus to a more familiar notion of return. The “surplus return” basically consists of the difference between assets and liability returns weighted by the inverse of the funding ratio.\(^10\) By including liability returns in the portfolio choice model, pension managers must now achieve a twofold objective: combining assets that have low correlations with each other (as in a Markowitz model) and that have high correlation with changes in pension liabilities. Keel und Muller (1995) investigate the characteristics of the set of efficient portfolios in this surplus return framework. They find that efficient portfolios can be decomposed into three elements: 1) minimum risk component, 2) liability component and 3) return generating component.

Leibowitz, Kogelman and Bader (1994) develop the concept of “funding ratio return” (FRR)\(^11\). Pension schemes having the same asset allocation and the same characteristics of liabilities will also have the same distribution of funding ratio return, regardless of the initial funding ratio. Based on

---

\(^9\) The absolute level of pension scheme is: \( S = A - L \) where \( S \) stands for surplus, \( A \) for assets and \( L \) for liabilities.

\(^10\) Sharpe and Tint (1990) propose the concept of surplus return: \( SR = \bar{R}_a - k \frac{L_o}{A_o} \bar{R}_l \) where \( SR \) is the surplus return, \( \bar{R}_a \) is the return on assets, \( k \) is the factor weighting the importance of the liabilities, \( \frac{L_o}{A_o} \) is the inverse of the initial funding ratio, and \( \bar{R}_l \) is the return on liabilities.

\(^11\) The Funding ratio return (FRR) is \( \frac{A_1}{L_1} - \frac{A_0}{L_0} = F_1 - F_0 \) where \( F_1 \) is the funding ratio at date 1 and \( F_0 \) is the funding ratio at date 0.
this evidence, the FRR becomes a valuable universal measure for pension plans. De Groot and Swinkels (2007) show that when pension fund assumes full surplus optimization (i.e. the value of $k = 1$ in Sharpe and Tint’s formula), “surplus return” and “funding ratio return” concepts coincide.

Based on the FRR perspective, Hoevenaars et al. (2008) develop a new “two funds separation” theory consisting of maximizing the logarithm funding ratio return. The peculiarity of such approach is the decomposing of the strategic asset allocation into a Liability Hedge Portfolio (LHP) and a Return Optimizing Portfolio (ROP). On the one hand, LHP is a portfolio with a low risk vis-à-vis liabilities whose goal is to control mismatch risk by approximating as close as possible the inexistent risk free asset for pension funds. On the other hand, ROP is an overlay portfolio whose purpose is to yield the highest risk adjusted return. This way of solving the portfolio choice problem seems to be most appropriate, assuming that pension funds agree to accept the mismatch risk.

### 3.3 Financial forecasts

There exists ample evidence of predictability in returns. Solnik (1993) among others shows that excess Stock returns can be partly predicted with a number of lagged variables in linear models. Using a statistical model selection criterion, Bossaerts and Hillion (1999) investigate whether Stock index returns in excess of risk-free rate are indeed predictable or of the predictability is rather caused by the model overfitting. They discover an effective in-sample predictability and they also find that manifestly unit root non-stationary variables as Dividend yield have good forecasting ability. Keim and Stambaugh (1986), Fama and French (1988), Chen (1991) show that not only Dividend yield, but also Default spread and Term spread have significant predictive power in explaining future Stock returns. Cochrane and Piazzesi (2005) and Campbell, Chan and Viceira (2003) discover the ability of Forward rates and Yield spread in predicting Bond returns.

Bossaerts and Hillion (1999) provide also evidence that the inclusion of lagged Stock and lagged Bond returns effectively generate the moving-average prediction that have become widely used among professionals in AR models lately. This result supports previous findings of Fama (1965), Hamao, Masulis and Ng (1990) and Harvey (1991) indicating that returns do not display only partial predictability, but also important autocorrelation in most of the markets all around the world. Barberis (2000) uses a VAR model to forecast returns for a dynamic asset allocation strategy. Campbell and Viceira (2002) also employ it to highlight the difference between short-term and long-term investments. In the ALM framework, recent studies of Binsbergen and Brandt (2006), Inkmann and Blake (2007) and Hovenaars et al. (2008) use VAR model for defining the strategic asset allocation for pension plans where investment opportunities are time-varying.
The widespread use in recent strategic asset allocation studies and the evidence that predictability in returns actually exists lead me to choose a VAR model to forecast asset and liability returns.

3.4 Alternative asset classes

Pension managers have to choose within a broad asset menu in order to determine the optimal strategic asset allocation. A restriction to common T-bills, Stocks and Bonds would not allow to exploit important features as 1) low correlation with other asset classes, 2) high Sharpe ratios and 3) high correlation with liabilities, generally characterizing alternative assets.


Gorton and Rouwenhorst (2006) study the behavior of Commodity futures over different horizon. They conclude that Commodity futures are particularly effective in providing a diversification to both Stocks and Bonds due to their negative correlation with these asset classes. Froot (1995) provides evidence that risk averse investor should be willing to bear an amount of Commodity risk. He demonstrates that long positions in high Energy component as Oil futures and Commodity futures provide strong hedging properties that enable asset managers to reduce the risk of the portfolio. He also proves that investments in Gold and Commodity-linked equities tend to increase the portfolio variance strongly.


Although the role of alternative asset classes has been deeply analyzed in a stand-alone manner, few researches have been devoted to address this issue in an ALM context. Brennan and Xia (2002), Barberis (2000), Watcher (2002), Campbell and Viceira (2002), Martellini and Ziemann (2005), Binsbergen and Brandt (2006) and Inkmann and Blake (2007) are examples of studies that investigate the long-term investor perspective in different settings and preferences frameworks,
where asset allocation is however restricted to Cash, Equities and Bonds. Hoevenaars et al. (2008) come to fill this gap by explicitly investigating the behavior of different asset classes (Cash, Stocks, Bonds, Real Estate, Commodities and Hedge Funds) in presence of liability and across different horizons. In a similar way, I also include a wide range of alternative assets in order to explore their role in an ALM framework.

4. Estimation methodologies

4.1 ALM and AO strategy: Vector Autoregressive model

This section describes the first order Vector Autoregressive Return model (VAR) used to model return dynamics for ALM and AO strategies. Following Hoevenaars and al. (2008), I extend the VAR model proposed by Campbell and Viceira (2005) by increasing the number of assets and by including liability. I represent the VAR model as:

\[ z_t = \begin{pmatrix} x_t \\ s_t \end{pmatrix} \] (1)

where \( x_t \) is a vector of assets and liability returns and \( s_t \) is a vector of state variables having predicting power on \( x_t \).

The use of a unique unrestricted VAR would have caused serious problems in reliably estimating the coefficients of the model basically due to the large number and the different historical length of asset classes. For that reason, I decompose the vector \( x_t \) and the VAR model in two parts: a core VAR (\( x_1 \)) and a additional OLS model (\( x_2 \)).

\[ x_t = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \] (2)
contains traditional asset classes (real T-bills interest rate, Stocks and Bonds), while \( x_2 \) includes liability and alternative assets (Hedge Funds, Public and Private Real Estate, Commodities and Gold).

The core VAR for the first subset of variables \( x_1 \) can be represented as

\[
y_t = \begin{pmatrix} x_1 \\ s_t \end{pmatrix}
\]  

(3)

It can be further specified as:

\[
y_{t+1} = a + By_t + \varepsilon_{t+1}
\]  

(4)

where \( B \) is a \((7 \times 7)\) matrix containing coefficients displaying relationships between the three asset classes and the four state variables with their corresponding lags. \( a \) is a \((7 \times 1)\) vector of constant terms. The shocks \( \varepsilon_{t+1} \) are normally distributed with zero mean and covariance matrix \( \Sigma_{\varepsilon\varepsilon} \).

The OLS model used for the second subset of variables \( x_2 \) can be represented as

\[
x_{2,t+1} = c + D_0 y_{t+1} + D_1 y_t + H x_{2,t} + \eta_{t+1}
\]  

(5)

where \( D_0 \) is a \((6 \times 6)\) matrix that captures contemporaneous relationships between \( x_{2,t+1} \) asset classes and the \( y_{t+1} \) variables. \( D_1 \) is a \((6 \times 6)\) matrix containing coefficients displaying relationships between the lagged \( y_t \) variables and \( x_{2,t+1} \) asset classes. The \( H \) matrix has a diagonal form, because I impose that additional assets only affect their own returns. \( c \) is a vector of constant terms and \( \eta_{t+1} \) are normally distributed shocks with zero mean and covariance matrix \( \Omega \).

The presence of contemporaneous variables among the regressors helps to estimate the covariances between \( y_t \) shocks in and \( x_{2,t} \), because it facilitates the task when the number of observations of \( x_{2,t} \) is shorter than \( y_t \). This way of proceeding, initially proposed by Stambaugh (1997), leads to make an optimal use of all information even when time series have different size.

Considering the core VAR and the OLS model, the complete VAR can be represented as

\[
y_{t+1} = \Phi_0 + \Phi_1 y_t + \nu_{t+1}
\]  

(6)

where

\[
\Phi_0 = \begin{pmatrix} a \\ c + D_0 a \end{pmatrix}, \quad \Phi_1 = \begin{pmatrix} B \\ D_1 + D_0 B \end{pmatrix}
\]

(7)
and $\nu_{t+1}$ is normally distributed with zero mean and covariance matrix

$$
\Sigma = \begin{pmatrix}
    \Sigma_{xx} & D_0 \Sigma_{ex} \\
    D_0 \Sigma_{xe} & \Omega + D_0 \Sigma_{xx} D_0'
\end{pmatrix}
$$

To have a stable VAR model, I test whether the eigenvalues of the matrix $\Phi_1$ have modulus less than one. This condition ensures that, in absence of shocks, the variables entering in the first order VAR converge to their long-run means in a finite number of periods. This excludes explosive (non stationary) behavior in these variables. I find that the highest root is equivalent to 0.9726. The process results stationary although this value is very close to the unit root. Errors are assumed to be homoskedastic.

### 4.2 JS strategy: Jorion-Stein estimator

Schema 3: Jorion-stein estimator

It has been widely documented that estimations over historical series contain remarkable estimation errors. To enhance the quality of the optimization, more accurate estimations of mean, variance and covariance are required. Chopra and Ziemba (1993) show that estimation errors in mean are the most acute compared to the ones encountered in variance and covariance. I adjust the estimated mean using a James-Stein estimator, letting in contrast the variance unchanged. This method basically consists of shrinking the estimated sample mean towards any point $Y_0$ to reduce the estimation risk. Jorion (1986) tests different propositions for the value of $Y_0$, and he finds that Global Minimum Variance Portfolio (GMVP) obtains good performance as prior portfolio. The Jorion-Stein estimator model is represented as follows:

$$
E_{JS}(R) = \hat{\mu}_{JS} = (1-x)\hat{Y} + xY_0
$$

where $\hat{Y}$ is the sample mean, $Y_0$ is the GMVP mean, and $x$ is the shrinking factor.\(^{12}\)

\(^{12}\) The mean of the GMVP is obtained by: $Y_0 = \frac{\hat{\Sigma}^{-1}}{\hat{\Sigma}^{-1}1} \hat{Y}$ where $\hat{\Sigma}$ is the sample variance-covariance matrix.
5. Portfolio choice problem

### 5.1 ALM strategy

Following Hovenaars et al. (2008), I approach the portfolio choice problem from a log funding ratio return perspective\(^{13}\). I define the log funding ratio return \( r_F \) as

\[
r_F = r_{A,t} - r_{L,t}
\]

where \( r_{A,t} \) is the logarithm asset return and \( r_{L,t} \) is the logarithm liability return. Both log asset and log liability returns are in excess of 3 months real T-bills rate (i.e. 3 months nominal T-bills minus the realized inflation). I denote excess log asset returns as \( x_{A,t} = r_{A,t} - r_{b,t} \) and excess log liability returns as \( x_{L,t} = r_{L,t} - r_{b,t} \).

In a multiperiod framework, logarithm returns are suitable for their nice time-aggregation properties. However, aggregating returns across assets becomes a more complex task and I need to refer to the log-linear approximation presented by Campbell and Viceira (2002, 2005):

\[
r_{A,t+1} = r_{b,t+1} + \alpha_i(x_{A,t+1} + \frac{1}{2} \sigma_A^2) - \frac{1}{2} \alpha_i \Sigma_{AA} \alpha_i
\]

where \( \alpha_i \) is the vector of weights invested in each asset class, \( r_{b,t+1} \) is the 3 months real T-bills rate, \( x_{A,t+1} \) is the vector of asset returns, \( \sigma_A^2 \) is the vector of asset variances and \( \Sigma_{AA} \) is the asset variance-

---

**LogFRR** = \( \ln \left( \frac{F_t}{F_0} \right) = \ln \left( \frac{A_t}{L_t} \frac{A_0}{L_0} \right) = \ln \left( \frac{A_t}{A_0} \frac{L_0}{L_t} \right) = \bar{r}_a - \bar{r}_l \)

---

\(^{13}\) LogFRR
covariance matrix. Once I have defined \( r_{A,t+1} \), I replace this value in (10) and I obtain the following funding ratio return equation:

\[
r_{F,t} = r_{b,t+1} + \alpha_t' \left( x_{A,t+1} + \frac{1}{2} \sigma_A^2 \right) - \frac{1}{2} \alpha_t' \Sigma_{A,t} \alpha_t - r_{L,t+1}
\]

(12)

Thanks to the use of log returns, computing the cumulative funding ratio return consists of summing the values from date \( j \) to the horizon date \( T \).

\[
r_{F,j,T} = \sum_{j=1}^{T} r_{F,j} = \alpha_t^{(T)} \left( x_{A,t+1} + \frac{T}{2} \sigma_A^2 \right) - \frac{T}{2} \alpha_t^{(T)} \Sigma_{A,t} \alpha_t - x_{L,T}
\]

(13)

I describe expected funding ratio return as

\[
E_t \left[ r_{F,j,T} \right] = T \left( \alpha_t^{(T)} \left( \mu_{A,t+T} + \frac{T}{2} \sigma_A^2 \right) - \frac{T}{2} \alpha_t^{(T)} \Sigma_{A,t} \alpha_t - \mu_{L,t+T} \right)
\]

(14)

where \( T \) is the forecasting horizon, \( \alpha_t^{(T)} \) is a vector of weights, \( \mu_{A,t+T}^{(T)} \) is a vector of unconditional expected asset returns, \( \mu_{L,t+T}^{(T)} \) corresponds to the unconditional expectation of liability return and \( \Sigma_{A,t} \) is the asset variance-covariance matrix.

I define the variance of the funding ratio return as

\[
Var_t \left[ r_{F,j,T} \right] = T \left( \sigma_L^{(T)2} - 2 \alpha_t^{(T)} \sigma_{AL}^{(T)} + \alpha_t^{(T)} \Sigma_{AL,t} \alpha_t^{(T)} \right)
\]

(15)

where \( \sigma_L^{(T)2} \) is the variance of liability returns and \( \sigma_{AL}^{(T)} \) is a vector of covariances between assets and liability returns. I refer to the volatility of the funding ratio as mismatch risk.

I define the annual expected return for all asset classes in \( x_t \) as

\[
\mu_t^{(T)} = \frac{1}{T} E_t \left( x_t^{T} \right) = \begin{pmatrix} \mu_{A,t}^{(T)} \\ \mu_{L,t}^{(T)} \end{pmatrix}
\]

(16)

and the annual variance of returns as

\[
\Sigma_t^{(T)} = \frac{1}{T} Var_t \left( x_t^{T} \right) = \begin{pmatrix} \Sigma_{AA}^{T} & \sigma_{AL}^{T} \\ \sigma_{LA}^{T} & \sigma_L^{(T)2} \end{pmatrix}
\]

(17)

where \( \mu_{A,t}^{(T)} \) and \( \mu_{L,t}^{(T)} \) are cumulative returns of assets and liabilities, \( \Sigma_{AA}^{T} \) is the variance-covariance matrix of assets, \( \sigma_{AL}^{T} \) is the covariance matrix between assets and liability and \( \sigma_L^{(T)2} \) corresponds to the variance of liability returns.
The objective of a pension fund is to maximize its funding ratio return depending on its level of risk aversion. As in Van Binsbergen and Brandt (2006) I assume CRRA utility function\(^{14}\). I can write the objective function as:

\[
U^{(T)}_t = \max_{\{\alpha_t, \ldots, \alpha_{T}\}} \left[ \frac{F^{1-\gamma}_t}{1-\gamma} \right]
\]

Assuming that returns are normally distributed, the optimization problem reduces to

\[
\max_{\alpha_t^{(T)}} E_t \left[ r_{F,t+T}^{(T)} \right] + \frac{1}{2} (1-\gamma) Var_t \left[ r_{F,t+T}^{(T)} \right] \\
\text{s.t.} \\
\alpha_t^{(T)} 1 = 1
\]

I use mean-variance analysis to expressively investigate the risk horizon effect of the funding ratio return. The strong underlying assumption that investors follow a buy-and-hold strategy (i.e. they design a one-time asset allocation at date \(t\) for an investment horizon of \(T\) periods) do not completely reflect the exact behavior of pension funds. To be as close as possible to the industry practices, I stabilize the allocation weights through the entire investing period. In other words, I assume that pension funds rebalance their portfolio back to the original strategic asset allocation designed at time \(t\).

Differentiating, the optimal weights vector becomes

\[
\alpha_t^{(T)} = \frac{1}{\gamma} \left( \left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(T)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \left( \mu_t^{(T)} + \frac{1}{2} \sigma_A^2 - (1-\gamma) \sigma_{AL}^{(T)} \right)
\]

The optimal strategic asset allocation can be decomposed in two parts: a Return Optimizing Portfolio (ROP) and a Liability Hedge Portfolio (LHP).

The ROP corresponds to

\[
\alpha_{ROP,t}^{(T)} = \frac{1}{\gamma} \left( \left(1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(T)} + \frac{1}{\gamma} \Sigma_{AA} \right)^{-1} \left( \mu_t^{(T)} + \frac{1}{2} \sigma_A^2 \right)
\]

\(^{14}\) In this study I aim to solve the strategic asset allocation of a typical US pension plan that have to partly or entirely (hybrid DC or DB pension plan) manage the pension plan’s wealth. At the moment, there are no valuable alternative solutions to the use of single utility functions (see section 3). As a result, I employ a unique CRRA utility function for solving the portfolio choice problem.
while the LHP correspond to
\[
\alpha_{LHP}^{(T)} = \frac{1}{\gamma} \left( \left( 1 - \frac{1}{\gamma} \right) \Sigma_{AA}^{(T)} + \frac{1}{\gamma} \Sigma_{AA}^{(T)} \right)^{-1} \left( (1-\gamma)\alpha_{AL}^{(T)} \right)
\] (22)

The decomposition of the portfolio into ROP and LHP is called “two funds separation” theorem. The ROP represents a portfolio that aims to yield the highest risk-adjusted return, while the goal of LHP is to reduce as much as possible the mismatch risk. If I let the risk aversion tend to infinity, the optimal portfolio converges to the Global Minimum Variance Portfolio (GMVP), which corresponds to invest all wealth in LHP.
\[
\alpha_{LHP}^{(T)} = \left( \Sigma_{AA}^{(T)} \right)^{-1} \sigma_{AL}^{(T)}
\] (23)

Besides imposing the restriction that all weights must sum up to one, pension funds must often comply with further constraints imposed by governmental regulatory commissions. Although most of these restrictions are country specific, short-selling is surely one of the most common. Adding this last constrain the optimization program becomes
\[
\max E_t \left[ r_{F,t+T}^{(T)} \right] + \frac{1}{2} (1-\gamma) \text{Var}_t \left[ r_{F,t+T}^{(T)} \right]
\]
\[\text{s.t.} \]
\[
\alpha_{t}^{(T)} 1 = 1
\]
\[
\alpha_{t,i}^{(T)} \geq 0
\]

5.2 AO strategy

Most of the literature regarding long term investors has approached the portfolio choice problem from an asset-only (AO) perspective. Some examples are Campbell and Viceira (2002, 2005) and Chan, Campbell and Viceira (2003). I solve the optimization portfolio choice problem from an asset-only perspective in a multiperiod framework where variance and covariance are time-varying (based on VAR estimations\textsuperscript{15}).

The optimization problem can be written:
\[
\max E_t \left[ r_{A,t+T}^{(T)} \right] + \frac{1}{2} (1-\gamma) \text{Var}_t \left[ r_{A,t+T}^{(T)} \right]
\]
\[\text{s.t.} \]
\[
\alpha_{t}^{(T)} 1 = 1
\]
\[
\alpha_{t,i}^{(T)} \geq 0
\]

\textsuperscript{15} Further details about mean and variance of asset returns can be found in the Appendix.
This optimizing program represents a classic Markowitz optimization in a multiperiod framework with the imposition that all wealth must be invested and no short sales are allowed.

5.3 JS strategy

The portfolio choice problem of the Jorion-Stein (JS) strategy is also based on a typical Markowitz optimization. Mean and variance of funding ratio return are estimated through the Jorion-Stein estimator and on historical data, respectively. The portfolio choice problem is solved in a single period\(^{16}\). The optimization problem can be written as

\[
\max_a E[r_a] + \frac{1}{2}(1 - \gamma)\operatorname{Var}[r_a]
\]

\[s.t\]

\[
a'1 = 1
\]

\[
\alpha \geq 0
\]

Different from ALM and AO strategies, mean and variances of asset returns do not depend on time.

I also impose the restrictions that all wealth must be invested and no short sales are allowed.

5.4 Naïve strategy

The use of a naïve strategy is inspired by the results of De Miguel, Garlappi and Uppal (2007). These authors provide evidence that more advanced portfolio choice techniques do not consistently outperform a naïve 1/N portfolio. Precisely, they suggest to employ such heuristic portfolio as benchmark for assessing the performance of other asset allocations. The construction of a naïve portfolio consists of holding a weight of \( w_i = 1/N \) in each of the N asset classes. This method does obviously not require neither estimations of mean and variance nor portfolio choice model for deriving optimal weights. Thus it is not exposed to either model misspecification or estimation errors.

6. Performance measurement

To determine the superiority of one strategy with respect to others, I need a measure of performance that allows to rank funds. I use the popular risk-adjusted measure of Sharpe ratio because of its simplicity, its intuitive understanding and its exemption of the Dybvig and Roll’s (1985) criticisms. Strong assumptions requiring that returns are normally distributed, that risk-free rate is not correlated with other assets and that agent must invest all his wealth in this portfolio characterize the use of Sharpe ratio.

\(^{16}\) Further details about mean and variance of returns can be found in the Appendix.
The simple comparison of Sharpe ratios does not lead to determine the superiority of one strategy with respect to another because this measure is a proportion of two random variables (mean and standard deviation). To reliably compare two Sharpe ratios, I have to test the null hypothesis that the difference between the two is not statistically different from zero. For that purpose, I use the Jobson-Korkie $t$-test (which follows a standard normal distribution):

$$ z_{i,j} \sim N(0,1) = \frac{\bar{r}_i s_j - \bar{r}_j s_i}{\sqrt{s_i^2 s_j^2 / T + 2 - 2 \rho_{i,j} + \frac{1}{2} \left( \frac{\bar{r}_i}{s_i} \right)^2 + \frac{\bar{r}_j}{s_j}^2 - \frac{\bar{r}_i^2}{s_i s_j} \left( \rho_{i,j}^2 + 1 \right) \right]} $$

(27)

where $\bar{r}_i$ and $s_i$ are the mean and the standard deviation of the realized excess returns for the strategy $i$ (in excess of the 3 months T-bills), $T$ is the sample size and $\rho_{i,j}$ is the correlation between strategy $i$ and $j$.

7. Data

I use US quarterly data. All returns are logarithmic, expressed in USD currency and in excess of the 3 months real T-bills rate. The time series of the core VAR begin in June 1956 and finish in December 2005. The time series of the OLS model have different starting dates but all have December 2005 as ending date. The JS strategy’s estimations are based on relatively short period, starting in March 1994 and ending in December 2005. This is due to the short existing life of Hedge Funds.

7.1 Data set

The data set is basically composed of three elements: assets, liability and state variables. Assets are split in two sub-categories: traditional and alternative asset classes. The common T-bills, Stocks and Bonds compose the group of traditional assets. Most of the papers in asset allocation literature consider these three assets to be the core investment classes. Conversely, the use of alternative assets in the literature does not encounter the same convergence. For a more careful selection of these asset classes, I use a Principal Components Analysis (PCA) to reduce their number without losing much of information. This restriction basically relies on the elimination of those assets that possess low significance level. I analyze eight alternative asset classes: Hedge Funds (HEDGE), Private Real Estate (NCREIF), Public Real Estate (NAREIT), Global Commodity Index (GSCI), Gold Index (GOLD), Agricultural Index (AGRI), Oil Crude Index (OIL) and Energy Index (ENERGY).

---

17 In the appendix I propose the entire process for carrying out the Principal Components Analysis.
Table 1 reports the eigenvalues and the significance percentage of alternative assets ranked in descending order. Global Commodity (GSCI) and Public Real Estate (NAREIT) exhibit evident superiority in term of significance. Private Real Estate (NCREIF), Hedge Funds (HEDGE) and Gold index (GOLD) have modest but still important significance with respect to Oil index (OIL), Agriculture index (AGRI) and Energy index (ENERGY). These last three indices do not seem to display interesting characteristics in the data set. As a result, I restrict the original height alternative asset classes to Global Commodity (GSCI), Public Real Estate (NAREIT), Private Real Estate (NCREIF), Hedge Funds (HEDGE) and Gold index (GOLD).

Liability is represented by the Citigroup Pension Liability Index, which well reflects the behavior of a typical US pension fund. It also allows to deal with real data and to provide implementable solutions without requiring actuarial calculations (which would be out of the scope of this paper).

The state variables are represented by four factors having predicting power in returns: 3 months real T-bills, Dividend Yield, Credit Spread and Term Spread.

7.2 Asset classes

The asset menu available to the pension manager is composed of eight asset classes: T-bills, Stocks, Bonds (traditional assets), and Hedge Funds, Public and Private Real Estate, Commodity, Gold indices (alternative assets).

I use 3 months Treasury bills as a proxy for Cash. The Equity investment type is represented by the S&P500 composite index (dividends are reinvested), which is considered the most representative index for the US market.

I construct the Bond return series from the 20 years constant maturity yields on US Bonds using the method of Campbell, Lo and MacKinlay (1997):
where $n$ is the Bond maturity, $Y_{n,t}$ is the Bond yield, the log Bond yield is $y_{n,t} = \ln(1 + Y_{n,t})$ and $D_{n,t}$ is the Bond duration. I approximate $y_{n-1,t+1}$ by $y_{n,t}$ and I compute the excess returns of Bonds in excess of the 3 months real T-bills. I calculate the duration $D$ at time $t$ as

$$D_{n,t} \approx \frac{1 - (1 + Y_{n,t})^{-n}}{1 - (1 + Y_{n,t})^{-1}}$$

Hedge Funds are represented by the Credit Suisse Tremont Hedge index. This global index is an asset weighted Hedge Fund index derived from TASS database of more than 5000 funds. Funds included in this index must have a minimum of 10 million under management and a current audited financial statement. The CS Tremont Hedge index is rebalanced monthly, and funds are reselected on a quarterly basis.

The Real Estate asset class is divided in two parts: public and private Real Estate. Public Real Estate is proxy by the composite total return REITs reported by the National Association of Real Estate Trusts (NAREIT). The Real Estate Investment Trust are listed on the NYSE, AMEX and Nasdaq and must fulfill minimum size and liquidity criteria. Private Real Estate returns are rather based on the NCREIF (National Council of Real Estate Investment Fiduciaries) Property Index.

This index is a quarterly time series composite total rate of return measure of investment performance of a very large pool of individual commercial real estate properties acquired in the private market for investment purposes only.

Estimations of private Real Estate returns are however problematic because most values are based on appraisals and not actual transactions. The volatility results too smooth and underestimated. Typically, high and smooth returns, low volatilities and low correlations with other asset classes characterize illiquid assets. Hedge funds and private Real Estate are good example of this. In a mean-variance framework, the model would allocate considerable weights to these categories due to their interesting but distorted properties. Following Geltner (1993) and Fischer, Geltner and Webb (1994), I attempt to correct this bias using autocorrelation models to unsmooth Hedge Funds and private Real Estate returns:

18 See section 9.3 to understand how illiquidity bias affect hedge funds return.

19 A real investment trust, or REIT, is a company that owns, and in most cases, operates income-producing Real Estate. Some REITs also engage in financing Real Estate. The shares of many REITs are traded on major Stock exchanges.

20 The property types in the NPI are apartments, hotels, industrial properties, office buildings, and retail only. All these properties have been acquired, at least in part, on behalf of tax-exempt institutional investors (the great majority being pension funds).
where $r_t^s$ is the smooth return at time $t$, $r_t^u$ is the unsmooth return at time $t$, $\rho$ is the first order autocorrelation (for Hedge funds $\rho$ is equal to 0.21, while for private Real Estate is 0.70).

It has been widely documented that Hedge Funds also suffer from backfilled bias. Among others, Posthuma and Van der Sluis (2003) show that historical Hedge Funds returns are too high on annual basis. Following Hoevenaars et al. (2008) I subtract an annual 2.15 percent from published return.

For Commodities, I consider S&P GSCI index which is a composite index of all world-production weighted commodity sector. For Gold, I use the sub-index S&P GSCI Gold. Both indices are unleveraged, long-only investment in fully collateralized nearby Commodity futures with full reinvestment.

### 7.3 Liability class

The correct valuation of pension fund’s liability is a task that belongs to actuaries. These professionals basically assess the present value of employee’s pensions through the use of two methods: Projected Benefits Obligations and Accumulated Benefits Obligations\(^{21}\). No matter which estimation model actuaries use, they will anyway face the problem of choosing the right discount interest rate. Financial theory tells that any future cash flows should be discounted using a term structure of discount rates that appropriately reflect the underlying the risk. Petersen (1996) states that:"the correct discount rate for pension plans should depend upon the type of risk inherent in the pension promise".

In December 1986, the U.S Financial Association Standards Board released the Statement N° 87 concerning “Employers’ Accounting for Pensions”, whose objective is to regulate the selection of the discount rate for discounting future cash flows. Prior FAS 87, the choice of the discount rate was completely discretionary and driven by strategic management purposes. Feldstein and Morck (1983) show that for large manufacturing companies in 1987, the discount rate ranged between 5 to 10.5 percent. These authors provide evidence that such diversity in discount rate was mainly driven by a pension plan’s trade-off between tax advantage of low discount rate and the cosmetic benefit to the annual report arising from the use of an high discount rate. The enforcement of FAS 87 definitely reduced such discretionary but it did not completely solve the problem. In fact, FAS 87 allows the company “to look to rates of return of high-quality fixed income investments currently

\(^{21}\) Projected Benefits Obligations is an estimate of the present value of future employee’s pension which assumes that employee continue to work and that pension contributions will increase due to the increase of employee’s salaries. Accumulated Benefits Obligations is also an estimate of the present value of future employee’s pension which assumes that employee ceases to work at the time of the estimation is made. (business dictionary.com)
available and expected to be available during the period to maturity of the pension benefits”. This grey zone has motivated Black and Inkmann (2007) to propose a new approach that utilizes the term structure of funding-risk-adjusted discount rates to valuate both liability and assets in an integrated manner.

Since the correct valuation of pension funds’ liability goes beyond the main objective of this paper, I proxy liability returns by Citigroup Pension Liability Index (former Salomon Brothers Pension Liability Index). This index is based on pricing each month a “typical” pension plan liability profile which allows to globally represent the behavior of a pension plan’s liability\textsuperscript{22}. In determining the current level of liability through the Projected Benefits Obligation model, each cash flow is discounted at its own corresponding yield of the Citigroup Pension Discount Curve\textsuperscript{23}. This yield curve is a composition of a Treasury model curve that reflects the entire Treasury coupon, STRIPS market and an OAS (option adjusted spread) curve of AA Corporate bond\textsuperscript{24}. The calculation of the quarterly return of index is given by the quarter-over-quarter change in the present value of liability.

Chun, Ciocchetti and Shilling (2000) provide evidence that the present value of liability, computed through a PBO model for 13 distinct industries in the US market, dramatically differ from one industry to another. This result suggests that using general benchmark as Citigroup Pension Liability index is appropriate only for academic research. For practical implementation, actuaries must be in charge of carefully valuating specific pension plan’s present value of liabilities.

7.4 State variables

Several economic factors (see 3.3) have predictive power in describing dynamics of returns. For instance, Campbell (1987) show that the Short-term nominal interest rate have good predicting power. Campbell and Shiller (1988, 1991) suggest that the Dividend yield and the Yield spread could be also add to this list.

To enhance the quality of the forecasts, different authors consider these variables in their Vector Autoregressive Return model (VAR). For example, Campbell and Viceira (2005) employ the Short-term-nominal Interest rate, the Yield spread and the Dividend yield. In the same way, Brandt and Santa Clara (2004) use the Term spread, the Credit spread and the 3 months nominal T-bills.

\textsuperscript{22} Citigroup’s index presentation does not clarify the term “typical”. I suppose that “typical” means that the liability index is a representative sample of pensioners retirement pensions, including partners’ vested pension rights. In order to identify the typical sample, it has been necessary to generate accurate forecasts for the age structure and the average width of pension payments relative to age.

\textsuperscript{23} The 30 years sport rate of the Citigroup Pension Discount Curve is applied to all cash flows beyond 30 years.

\textsuperscript{24} For further details about the construction of the Citigroup Pension Discount Curve: www.soa.org/files/pdf/salomon_1995.pdf
Similar to Hoevenaars et al. (2008), I use Dividend yield, Short-term nominal rate, Term spread and Credit spread in the VAR model. I proxy Short-term nominal rate by the log 3 months nominal T-bills. The Dividend yield corresponds to the weighted average of the ratio between dividend and price, for each firm belonging to a particular market. I consider the log Dividend yield for the S&P500 index. The Term spread represents the difference between the log 10-years zero-coupon yield and the log 3 months nominal T-bills rate. The Credit spread is calculated as the difference between the log BAA Moody’s yield and the log 10-years zero-coupon yield.

For sake of precision, 3 months nominal T-bills, the 20 years constant maturity yield, the BBA Moody’s yield are from the FRED website. Data on the stock index and on the Dividend yield are drawn from the “Irrational Exuberance” of Shiller. Public and private Real Estate data are from the official NAREIT website and from NCREIF website, respectively. The Hedge Funds data are from Hedge Index website, while all Commodity indices data are from Datastream. Citigroup Pension Liability Index data is from SOA (Society of Actuaries) Website.

8. Estimation results

8.1 Descriptive statistics

Table 2 reports the descriptive statistics. Due to the important difference in the starting date of time series, interpretations and comparisons must be taken with caution. Beginning with a classic comparison, Stocks have higher mean (6.2 percent vs. 4.4 percent) and higher volatility with respect to Bonds (14.2 percent vs. 9.9 percent). Their Sharpe ratios are however very close (0.44). T-bills is considered to be the risk-free asset class in this context. It has low volatility (1.2 percent), low mean (1.3 percent) and an interesting Sharpe ratio (1.08). Hedge funds remain the most attractive risky asset with a mean of 8.7 percent, a volatility of 8.6 percent and a corresponding Sharpe ratio of 1.01. Public Real Estate have similar volatility with respect to Stocks (14.2 percent), but they have higher mean (11.2 percent vs. 6.2 percent) which consequently results in a higher Sharpe ratio (0.79 vs. 0.44). Private Real Estate have lower mean with respect to public Real Estate (9.5 percent vs. 11.2 percent), but they are much less risky (9.5 percent vs. 14.2 percent). GSCI Commodity has higher mean and volatility compared to Stocks (10.30 percent and 19.3 percent). Gold has

25 http://research.stlouisfed.org/fred2/
27 www.reit.com
28 www.ncreif.com
29 http://www.hedgeindex.com
30 http://www.soa.org/professional-interests/pension/resources/pen-resources-pension.aspx
significant low mean (2.1 percent) but a quite high volatility (18 percent) that makes this asset class unattractive from a Sharpe ratio perspective (0.11).

8.2 Estimations of the Vector Autoregressive model

8.2.1 Estimations of the Core VAR

Table 3 reports the estimations of the core VAR model coefficients, $t$-statistics, R-square and F-statistics.

The first row of this table corresponds to the real T-bills equation. The lagged real T-bills rate and nominal T-bills rate have high positive coefficients (0.42 and 0.29) and statistically significant $t$-statistic (6.87 and 3.71). Term spread also has positive coefficient (0.49) with corresponding positive and significant $t$-statistic (3.49). Remaining variables are not statistically significant in predicting one-period ahead real T-bills return. The R-square of 35 percent shows that those variables sufficiently well explain the behavior of this asset class.

The second row corresponds to the equation of Stocks returns. The lagged nominal T-bills rate has high negative coefficient (-3.48) and statistically significant $t$-statistic (-3.27). In contrast, Dividend yield and Credit spread have high positive coefficient (5.51 and 9.49) and statistically significant $t$-statistics (3.72 and 2.63). These results are in line with Fama and French (1989) findings. The relatively low R-square (11 percent) reveals that Stocks returns are difficult to predict with this linear model.

The third row regards the equation of Bonds returns. The lagged Stocks return has high negative coefficient (-0.12) and statistically significant $t$-statistic (-2.53). Remaining variables are not statistically significant in predicting one-period ahead Bonds return. The low R-square (8 percent) reveals that Bond returns are even harder to predict than Stocks. Campbell and Thomson (2008) state that even small R-square can lead to large improvement in performance portfolio and therefore to be economically meaningful.

The last four rows concern the equations of the state variables. A common characteristic to all of these variables is the strong persistent dynamic. The nominal T-bills rate is predicted by its own lagged variable with positive coefficient (1.04) and high $t$-statistic (27.31). The Dividend yield is also explained by the lagged Dividend yield having positive coefficient (0.94) and extreme high $t$-statistic (61.61). The Term spread is predicted by its own lag with a positive coefficient (0.66) and relatively high $t$-statistic (12.35). The lagged Credit spread represents a variable with predicting power on Credit spread due to its positive coefficient (0.80) and its high $t$-statistic (19.95). These findings are in line with Chun, Campbell and Viceira (2003), Campbell and Viceira (2005) and Hoevenaars et al. (2008).
Table 4 reports the errors variance-covariance matrix. The entries above and below the main diagonal represent correlations and covariances, respectively. Estimations on the main diagonal are standard deviations which are expressed on a quarterly basis.

### 8.2.2 Estimations of the OLS model

Table 5 reports the estimations of the OLS model coefficients for alternative assets and liability, t-statistics and R-square in the last column. I set the values of the 3 months nominal T-bills and the Dividend yield equal to zero for both contemporaneous and lagged variables.\(^{31}\)

The first row refers to equation of Hedge funds. Contemporaneous Credit spread and the lagged Credit spread have negative and positive (-31.07 and 29.40) coefficients with significant t-statistics (-2.15 and 2.15). Other variables are considered to be not statistically significant. The R-square of 40 percent suggests that Hedge funds returns are far better predictable compared to Stocks and Bonds.

The second row concerns the equation of public Real Estate. The contemporaneous Stocks and Bonds returns and the lagged Credit spread are able to well explain public Real Estate dynamic. They have positive coefficients (0.52, 0.4 and 17.96) and significant t-statistics (7.57, 4.11 and 2.60). The R-square (48 percent) is also high.

The third row regards private Real Estate. The contemporaneous Term spread and the lagged own return have negative (-6.83 and -0.38) and statistically significant coefficients (-3.56 and -4.12). The lagged Term spread have positive (4.58) and statistically significant (2.55) forecasting ability. The R-square of 27 percent reveals that private Real Estate are more difficult to explain compared to public Real Estate.

The forth row corresponds to global Commodity GSCI. Only contemporaneous variables as 3 months real T-bills rate, Stock returns and Term spread are able to explain the behavior of GSCI. Those factors have negative (-5.01, -0.32 and -24.65) and statistically significant (-3.67, -2.94 and -2.31) coefficients. The R-square is about 20 percent.

The fifth row is the equation of Gold. The contemporaneous 3 months real T-bills rate and the lagged Term spread have negative (-5.35 and -9.91) and statistically significant (-4.24 and 3.03) impact on Gold dynamic. The contemporaneous Term spread has also predictive power with a positive (7.76) and significant (2.30) coefficient. The R-square of 26 percent is close to the one of public Real Estate.

The last row corresponds to the Citigroup Pension Liability index. The contemporaneous and the lagged Bonds variables have positive (1.14 and 0.49) and significant (6.52 and 2.42) coefficients.

---

31 Different authors show that 3 months nominal T-bills and Dividend yield are not good in predicting one-period ahead returns in an OLS model for alternative assets. As a result, I discard them and I set them equal to zero in (5).
The lagged own variable has negative (-0.36) and significant (-2.10) forecasting ability. The R-square of the Citigroup Pension Liability index is 83 percent.

### 8.2.3 Time diversification

Figure 1 shows the annualized conditional standard deviation of cumulative holding for 3 months real T-bills, Stocks and Bonds returns. I consider an investment horizon up to 25 years (in quarters). All slopes are not flat across time, reflecting the ability to capture predictability in returns. The annualized conditional standard deviation for Stocks displays a considerable decrease across time, meaning that this asset class is less volatile over the long-run than over the short-one. Its annualized standard deviation passes from 14 percent in the first quarters to less than 8 percent after 13 years. This effect is known as “mean-reversion” in returns. I mainly attribute this phenomenon to the predictability of the Dividend yield. The strong and negative correlation between Stocks and Dividend yield shocks (-0.98) and the positive significant coefficient of Dividend yield in the forecasting equation (5.51) engenders this “mean-reversion” effect. This effect is reinforced by the Credit spread which has a relatively low negative correlation with Stocks (-0.13) but a high positive coefficient in the forecasting equation (9.49).

The annualized conditional standard deviation of Bonds experiences a reduced “mean-reversion” effect with respect to Stocks. It starts at 10 percent in the first quarters, crosses the 6 percent level after 10 years and it finally stabilizes around 5.4 percent thereafter. This shape is mainly explained by two offsetting effects. On the one hand, the nominal T-bills has positive coefficient in the regression equation (0.71) and negative correlation with shocks in Bonds returns (-0.62). This causes per se “mean-reversion”. On the other hand, the Term spread positively forecasts Bonds returns (2.61) and has slight positive correlation with them (0.15). This causes per se “mean-aversion”. Overall, the nominal T-bills dominates the Term spread effect and therefore Bonds display a slight mean-reversion effect.

The 3 months real T-bills is a relative safe asset class in the short term, but it becomes riskier over longer horizons. Its annualized conditional standard deviation passes from 1 percent in the first quarter, crosses 2 percent after 10 years, and it remains constant at 2.9 percent thereafter. Although its volatility remains at the low level, the 3 months real T-bills experiences a “mean-aversion” effect. Campbell and Viceira (2005) explain this evolution by the persistent variation in real interest rate in the postwar period, which amplifies the volatility of returns when Treasury bills are reinvested over long horizons. This volatility does not have to be underestimated because, as Siegel

---

32 In line with Campbell and Viceira (2005), I highlight that nominal T-bills coefficient in the forecasting equation is not statistically significant, while Term Spread coefficient is highly significant.
(1994) shows, investing in Treasury bills can become even riskier than Stocks in the long-run horizon.

Figure 2 reports the annualized conditional standard deviation for Hedge Funds, public Real Estate and private Real Estate. Starting with Hedge funds, this asset class has a flat risk term structure. Its annualized conditional standard deviation is 9.62 percent in the short term and it minimally decreases to 8.78 percent in the long run. Public Real Estate experiences the same effect. In the first quarter the conditional annual standard deviation is 13.78 percent while after 25 years this value corresponds to 13.43 percent. No relevant variations of Hedge Funds and for public Real Estate’s standard deviation are congruent with Hoevenaars et al. (2008)’s findings. Conversely, private Real Estate display fast and high “mean reversion” effect. The annualized conditional standard deviation quickly passes from 22 percent to 17 percent after approximately 1 year horizon and it remains constant at the level of 18 percent thereafter.

Figure 3 reports the annualized conditional standard deviation of GSCI Commodity and Gold. I observe that GSCI experiences a strong increase in volatility. As soon as the investment horizon spreads, the volatility tends to increase. Its risk passes from 19.47 percent in the first quarter to 30.53 percent for a 10 years horizon. Conversely, Gold exhibits an almost flat term structure. Precisely, there is a slight increase from the short to the long term horizon, corresponding to a 21 percent in the first quarter to a 25 percent for a 25 years horizon.

Figure 4 reports the annualized conditional standard deviation of the Citigroup Pension Liability index. The term structure exhibits a decreasing pattern. The annualized conditional standard deviation passes from an initial value of 12.51 percent to 7.48 percent for a 25 years horizon.

8.2.4 Risk diversification

To representatively assess the risk diversification potentiality of the eight asset classes (in real terms) across different horizons, I only consider their correlations with Stocks and Bonds. The main purpose is to determine which asset category is suitable for the ROP portfolio construction. Figure 8 and 9 report these correlation structures.

The correlation between Stocks and Bonds is remarkably positive across all horizons, although its magnitude displays a hump-shape. Over short-term horizon the correlation is about 16 percent, it

---

33 According to Hoevenaars et al. (2008), the term structure of illiquid asset categories (hedge funds and private real estate) tends to be affected by the application of unsmooth filter. However, these authors do not explicitly state which consequences may cause applying such filter on the term structure.

34 The correlation and covariance patterns for the same asset classes do not have to be the same. This is due to the fact that correlation is the ratio of covariance to the product of standard deviations. Although I previously presented variances and covariances for the eight asset classes, I now consider correlations rather than covariances for its intuitive interpretation.
reaches its maximum value of 41 percent over a 7 years horizon and it finally decreases to 19 percent for the long-term horizon.

T-bills seem to be a good risk diversifier in a portfolio composed by Stocks and Bonds. On the one hand, the correlation with Stocks displays a hump-shape. It starts at 14 percent for short horizons, it quickly increases up to 26 percent but it sharply decreases thereafter. On the other hand, the correlation with Bonds exhibits a U-shape. It starts at 6 percent for short horizons, it comes down to -30 percent for the medium term and it rises again up to 15 percent over longer horizons.

Hedge funds correspond to the typical diversifying asset category. The correlations of Hedge Funds with both Stocks and Bonds are constantly close to 0 percent throughout the investment horizons. Martellini and Ziemann (2005) provide evidence that Hedge Funds are particularly suitable in an ALM framework for their risk diversification in a portfolio composed by Stocks and Bonds.

Public Real Estate are characterized by high correlations with both Stocks and Bonds. The correlation with Bonds displays an almost flat pattern across investment horizons floating around 40 percent. The correlation with Stocks begins at an incredibly high value of 60 percent on the short-term horizon, but it tends to slightly decrease as soon as the investment horizon spreads. These results clearly demonstrate the poor risk diversification quality of this asset class, which is in line with Brounen, Prado and Verbeek (2007) and Froot (1995)’s results. This author explains that similar factors, as productivity of capital and labor, drive both Stocks and Real Estate and that lots of corporate assets are invested in Real Estate. These are the possible reasons of why public Real Estate have high correlation with both publicly traded Stocks and Bonds indices.

Private Real Estate display an interesting risk diversification potentiality. Its correlations with both Stocks and Bonds are almost flat and close to 0 percent for all horizons. This evidence is also documented by Craft (2001 and 2005). Brounen, Prado and Verbeek (2007) also show that private Real estate have significant weak correlation with other asset classes.

The qualities of Commodity and Gold are rather ambiguous to evaluate since correlations with Stocks and Bonds have opposite effects. Precisely, both asset classes have a U-shape correlation with Stocks, while correlations with Bonds have a hump-shape. These values are however relatively low in absolute terms which leads to conclude that Commodity and Gold have discrete risk diversification properties. This evidence is supported by Froot (1995), who shows that risk averse investor is willing to bear an amount of Commodity risk. Gorton and Rouwenhorst (2006), Erb and Harvey (2005) and Nijman and Swinkels(2007) highlight the potential risk diversification properties of Commodity.
8.2.5 Liability-hedging property

To examine the liability-hedging property of the eight asset classes (in real terms), I analyze their correlations with liability across different horizons. The main purpose is to determine which asset category is suitable for the LHP portfolio construction. Figure 10 reports these correlations.

Without any doubts, Bonds represent the investment type that provides the best liability hedge. Correlation is about 90 percent for short-term horizon and about 85 percent for the long-run. This result is in line with the fact that Liability and Bonds are both strongly subject to interest rate risk. Public Real Estate also exhibit good liability hedging quality. Their correlation with liability varies around 35 percent for almost all horizons. Stocks reveal a discrete property in hedging liability risk. Their correlation with liability has a hump shape starting at 15 percent for the short term, achieving its highest value of 37 percent for the medium term, and ending at 16 percent for the long run. Although Gold displays similar hump-shapes as Stocks, its level of correlation is relatively lower. Specifically, its liability hedging property is appealing only for medium horizon, when its correlation achieves 11 percent. Hedge Funds and private Real Estate correlations with liabilities are almost 0 percent flat pattern across investment horizons. T-bills exhibit a U-shape liability-hedging property across time. The correlation is 5 percent for short term, it reaches -27 percent for the medium horizon and it returns to 12 percent for the long run. This result makes the T-bills a unattractive liability-hedger. An even worse liability-hedging is offered by Commodity GSCI. Its correlation with liability starts from -11 percent for short term, it quickly grows up to 6 percent and sharply decreases for longer horizons.

8.3 Estimations of Jorion-Stein model

Table 8 reports annual mean estimations of the Jorion-Stein shrinkage model presented in (9). There are also estimations of $Y$, $Y_0$ and $x$, which correspond to the sample mean, the GMVP mean and the shrinking factor. For a comparison purpose, historical mean estimations of the eight asset classes and liability are also added. Table 9 reports the variance-covariance matrix which is estimated on historical data. It is important to point out that all these estimations are based on a small sample of 48 observations (Q1:1994-Q4:2005). This is due to the short life of Credit Suisse Tremont Hedge Fund index.

Private Real Estate index is the most profitable asset class with a mean of 8.64 percent, a standard deviation of 2.6 percent and a Sharpe ratio of 3.27. This investment category has strongly positive correlation with public Real Estate, Stocks and Hedge Funds. Conversely, it displays significant negative correlation with T-bills.
Hedge Funds is the second most appealing investment class with a mean of 8.35 percent, a standard deviation of 4 percent, and a Sharpe ratio of 2.07. It shows considerable and positive correlation with public and private Real Estate and slight negative correlation with T-bills.

T-bills and public Real Estate have good risk-return tradeoff, with means of 1.63 percent and 10.52 percent, and standard deviations of 1 percent and 6.9 percent, respectively. T-bills display relatively important negative correlation with almost all asset classes. The exception is represented by the positive correlation with Stocks. Public Real Estate do not only exhibit important and positive correlation with Stocks, but also with Hedge Funds and private Real Estate.

Stocks and Bonds possess relatively modest investment characteristics with respect to means and standard deviations. Instead, Stocks display interesting diversification property thanks to their strong negative correlation with Gold and their significant negative correlation with Bonds and Commodity. Bonds only display interesting negative correlation with T-bills.

9. Strategic asset allocation

![Schema 6: the strategic asset allocation](image)

9.1 Conditional strategic allocation

To study the strategic asset allocation for pension funds under the conditional approach I use a time-varying variance-covariance matrix of cumulative returns and an unconditional full sample mean for assets and liability returns. I assume that the pension plan behaves like a buy-and-hold investor\(^{35}\) who has a 3, 5, 10, 15, 20 and 25 years investing horizons for a 15, 20, 30 and GMVP levels of risk aversion\(^{36}\). I solve the portfolio choice problem for ALM and AO strategies in a mean-variance framework.

\(^{35}\) The buy-and-hold investor is assumed to rebalance back its strategic asset allocation to the one designed at time \(t\).

\(^{36}\) When I refer to GMVP risk aversion, I mean a infinite risk averse agent.
9.1.1 ALM strategy

The ALM strategy’s asset allocation is based on the optimizing process (24). In (22) and (23) I describe how the optimal portfolio for the ALM strategy can be decomposed in two LHP and ROP portfolios. Asset classes having significant positive correlation with liability will obtain considerable importance for the constitution of LHP, while risk diversification and Sharpe ratio are central properties for the construction of ROP. Table 6 reports optimal weights for the typical US pension plan.

Bonds have a crucial role in the strategic asset allocation. They account for 57 percent, 70 percent and 83 percent in the 3 years investment horizon allocation for 15, 20 and 30 levels of risk aversion, respectively. Allocation percentage in Bonds tends to increase as soon as the level of risk aversion increases, but it clearly decreases for longer investment horizons. These results can be explained by the high but decreasing correlation between Bonds and Liability returns across time. This evidence is also encountered by Chun, Ciochetti and Shilling (2000), Craft (2001), Martellini and Ziemann (2005) and Hoevenaars et al. (2008).

Public Real Estate also represent a fundamental asset class in the ALM framework. They account for 27 percent, 20 percent and 13 percent in the 3 years investment horizon allocation for 15, 20 and 30 levels of risk aversion, respectively. Allocation to public Real Estate displays an increasing pattern across investment horizon for every level of risk aversion. These patterns are congruent with the relative high and constant correlation between public Real Estate and Liability returns across time. In support to my results, Chun, Ciochetti and Shilling (2000) report that pension plans should allocate between 20 percent and 30 percent of their wealth to Real Estate. Brounen, Prado and Verbeek (2007) also find that Real Estate occupy a significant role in the strategic asset allocation corresponding to approximately 25 percent. Both Public Real Estate and Bonds possess on the one hand great liability-hedging properties, but on the other hand they have extremely poor risk diversification qualities. As a result, I can conclude that high allocation to Bonds and public Real Estate are mainly attributable to the constitution of the LHP portfolio.

Allocation to Hedge Funds is also important. This asset class is particularly suitable for low risk averse agents with long investment horizons. Institutional investors with more than 10 years investment horizon should allocate between 10 percent and 30 percent of their wealth to Hedge Funds. Such allocation drastically decreases as soon as the agent becomes more risk averse. Knowing that the importance of the LHP portfolio in the asset allocation increases (to the detriment of ROP) when agent is getting more risk averse, I conclude that Hedge Funds are basically helpful to construct the ROP portfolio. This asset class has low correlation with other assets, high return and low variance which are characteristics particularly appealing for ROP. Karavas (2000) shows
that the addition of Hedge Funds improves risk-return opportunities when considered in a well-diversified portfolio. He specifies that such addition is especially beneficial during extreme market movement periods, when numerous asset classes, particularly Stocks and Bonds, see their correlations sharply increasing. Martellini and Ziemann (2005) conclude that suitably designed Hedge Funds portfolios can be particularly attractive in an ALM framework. This result is basically due to Hedge Funds’ benefits in terms of diversification properties.

Private Real Estate have an almost constant and modest relevance in the strategic asset allocation across investment horizons. The correlation pattern tends to decrease as soon as the agent becomes risk averse. However, private Real Estate has good diversification property, although its Sharpe Ratio is far from attractive compared to Hedge Funds. This implies that private Real Estate deserve only a limited asset allocation percentage.

Commodity have a discrete role in the strategic asset allocation for long horizon investors. Diversification allows Commodity to be constantly present with an allocation varying from 1 percent to 10 percent. This investment category experiences slight decreasing allocation both when the level of risk aversion increases and the investment horizon expands. These behaviors are basically explained by the high volatility generally characterizing Commodity.

In contrast to current industry practices, Stocks and T-bills do not seem to be interesting asset classes for pension funds. Although, Stocks experience a significant “mean-reversion” effect and have discrete liability-hedging feature, they are never considered in the strategic asset allocation. This could be possibly explained by the dominance of Bonds and public Real Estate in the composition of the LHP and by the poor risk diversification property of Stocks. T-Bills look interesting from a Sharpe ratio but absolutely unattractive from a liability-hedge perspective.

9.1.2 Asset-only strategy

The asset-only (AO) strategy relies on a classic Markowitz. I expect that investment categories having high expected returns, low volatilities and good risk diversification properties receive considerable attention. Table 7 reports optimal weights for the AO strategy.

It is straightforward to see that Hedge Funds and public Real Estate dominate the strategic asset allocation. Hedge Funds have high Sharpe ratio and low correlation with other assets. This investment category is always present with an allocation of more than 30 percent, no matter the investment horizon. Exposures to Hedge Funds do not vary significantly across time. This is

---

37 According to Swinkels (2004), when I deal with Hedge Funds I have to keep in mind that investment styles of these funds are highly diversified, which makes it hard to speak of a homogeneous asset class. In addition, Hedge funds suffer from several biases in collecting data as documented by Fung and Hsieh (2000). Even though I apply a unsmooth filter to Hedge Funds time series for reducing some potential biases, I must treat these results with caution.
basically due to the flat behavior of standard deviation and correlations with other assets across time.

Public Real Estate also obtains important allocation for different investment horizon and level of risk aversion. Unlike Hedge Funds, public Real Estate possess only discrete diversification property but interesting Sharpe ratio quality. This latter characteristic mostly determines the high exposure to public Real Estate. In an asset-only framework, Fugazza, Guidolin and Nicodano (2006) find that Real Estate ought to play a significant role in optimal portfolio choices, with allocation weights between 12 percent and 44 percent. Their findings are in line with my results.

Relatively good Sharpe ratio and risk diversification characteristics induce the optimization program to assign modest weights to GSCI Commodity. Such allocation exhibits a decreasing trend across time for every level of risk aversion. This pattern can be explained by the strong “mean aversion” effect experienced by this asset class.

Private Real Estate also obtain a modest allocation percentage in the strategic asset allocation. In spite of their relatively high volatility, private Real Estate show extraordinary risk diversification property which permits to private Real Estate to have high allocation no matter the level of risk aversion.

Thanks to its modest volatility in the short-term and its relatively good diversification risk quality, Gold seems to be particularly appropriate for risk averse agents with short-term investment horizon. As soon as the risk aversion decreases and investment horizon expands allocation to Gold tends to vanish. This is mainly due to “mean-aversion” effect exhibited by this asset class.

The classic asset menu composed of T-bills, Stocks and Bonds becomes a suitable investment choice only for risk averse investors. Despite its “mean-aversion” effect, 3 months Treasury bills is considered to be the safest asset class with an appealing Sharpe ratio and good risk diversification properties. From a volatility perspective, investing in Bonds is also one of the safest way of guaranteeing returns. This investment category becomes particularly attractive because of the “mean-reversion” effect that positively affects the Sharpe ratio. A similar but stronger effect is also experienced by Stocks, which allows this asset class to cut down its volatility across time. Overall, T-bills, Stocks and Bonds seem to be particularly interesting for risk averse agents with long-term investing horizons.

9.2 Unconditional strategic allocation

To study the strategic asset allocation for pension funds in an unconditional framework, I solve the portfolio choice problem for JS strategy in a single period. I also assume that the pension plan behaves like a buy-and-hold investor. Table 10 reports the optimal allocation weights for the Jorion-Stein strategy.
Private Real Estate dominates the strategic asset allocation. This investment type exhibits extremely low correlation with other assets, relatively high expected return and particularly low standard deviation. All these characteristics induce the Markowitz optimization to allocate important weight to this asset class. Hedge funds also display similar potentialities. However, lower mean and higher standard deviation do not permit to Hedge Funds to compete with private Real Estate. As a result, lower but anyway considerable amount of wealth is invested in this alternative asset class. Public Real Estate receives modest attention because of their interesting risk-return tradeoff but poor risk diversification property. Commodity obtains discrete but constant allocation for any level of risk aversion because of their low correlations with other assets. Stocks and Bonds receive partial attention in the strategic asset allocation due to their relative appealing risk-return tradeoff.

9.3 Hedge Funds bias

My results suggest that hedge funds play an important role in the strategic asset allocation for pension plans because of their great diversification property. However, these findings must be taken with caution. The main reason must be searched in the illiquidity bias affecting hedge funds. In fact, many of hedge funds portfolios contain various combinations of illiquid exchange-traded securities or difficult-to-price over-the-counter securities. Therefore, their monthly performances are either calculated based on the last available traded price or estimated from current market prices. These practices often induce hedge funds returns to be not perfectly synchronized with the market. As a result, reported returns appear smoother and highly serially correlated. Getmansky, Lo and Makarov (2004) show that such illiquidity bias causes significant downward bias on estimated variance and correlation (amplifying hedge funds diversification property), which have serious impact on investment decisions. Asness, Krail and Liew (2001) provide evidence that when hedge funds returns are adjusted to the non-synchronous effect the diversification effect vanishes. Thus, strategic asset allocations presented in section 9 must be analyzed considering that hedge funds returns may be subject to illiquidity bias.

10. Test out of sample

In order to compare ALM, AO, JS and 1/N strategies, I apply two methodologies presented in section 6 over a 3 years out-of-sample period (using quarterly returns from Q1:2006 to Q4:2008). The first approach consists of studying the pension plan’s funding ratio over time. I analyze the mean, the variance and the evolution of the Funding Ratio Return and I study the behavior of the funding status for a hypothetical pension plan having an initial value of 100 percent. The second methodology is based on a Sharpe ratio comparison using Jobson-Korkie statistic.
**10.1 Funding ratio return analysis**

Table 11 reports the cumulative funding ratio return of ALM, AO, JS and 1/N strategies for 15, 20, 30 and GMVP levels of risk aversion. Only three ALM (risk aversion of 20, 30 and GMVP) and one JS (risk aversion of 15) asset allocation obtain positive cumulative funding ratio return during the 3 years out-of-sample period. Conversely, AO and 1/N strategy yield deep negative funding ratio return.

Table 12 reports the realized funding ratio mean and variance of ALM, AO, JS and 1/N strategies for each level of risk aversion. The results show that ALM does not yield only positive funding ratio return for the majority of its strategies, but it successfully reduces the volatility of the funding status. For every level of risk aversion, ALM has a significantly low volatility compared to the others. Thus, I conclude that LHP provides an effective added value.

Figure 8 confirms the ability of ALM in minimizing the mismatch risk showing the evolution of the funding ratio return across the out-of-sample period (for a level of risk aversion of 30). Unlike AO, JS and 1/N strategies, the funding ratio return of ALM does not exhibit drastic variation (particularly during the 12th quarter when the financial market collapsed).

Figure 9 reports the evolution of the funding status of a hypothetical pension fund with a starting value of 100 percent (the level of risk aversion is 30). This figure confirms that ALM achieves its twofold objective. Over the entire period the funding status of this pension fund moderately grows with a reduced volatility. Conversely, JS, AO and 1/N strategies yield high return for the majority of the quarters, but as soon as the financial markets enter into the downturn, their funding status quickly drop. For AO and 1/N, the funding status dramatically falls from 120 percent to less than 90 percent, while for JS it passes from 147 to 102 percent during the last quarter.

**10.2 Sharpe ratio analysis**

Table 13 reports the mean, standard deviation, Sharpe ratio and the corresponding Jobson-Korkie statistic of asset returns for ALM, AO, JS and 1/N strategies. Jobson-Korkie statistics indicate that ALM’s Sharpe ratio is statistically higher than AO and JS’ ones for 15 and GMVP levels of risk aversion. Although the superiority of ALM is not encountered for every Sharpe ratio comparison, these findings tend to support the results of the first methodology.

**10.3 The appropriateness of ALM strategy**

The appropriateness of a model with respect to another depends on its ability to achieve the main purpose of a pension plan, which consists in guaranteeing the payment of current and future obligations on a continuing basis. In other words, this entity must assure that its funding ratio is
always kept at a level of 100 percent (or superior) without that this latter experiences drastic variations through time.

The heavy exposure to Bonds and Real Estate constitute an important hedge against liability movements. Even during extreme financial market conditions, the presence of LHP ensures that such variations are minimal compared to the ones experienced by the alternative models that mostly invest in Hedge Funds and alternative assets.

Despite the orientation of the ALM model to control the funding ratio, the presence of the ROP guarantees to pension plans a linear and smooth growth of the funding ratio. The discrete exposure to Hedge Funds and Commodities permits to diversify the portfolio and to yield risk adjusted return.

I conclude that the ALM model is the most suitable model for solving the strategic asset allocation assuming that pension plan accepts the mismatch risk.

10.4 ALM vs. strategic asset allocation in practice

In order to verify the superiority of ALM with respect to current investment decisions in practice, I compare the ALM strategic asset allocation to a typical European pension plan asset allocation (according to JPMorgan’s 2006 Survey, see appendix).

Figure 10 shows that the European asset allocation does not efficiently hedge liability risk and provokes high funding ratio volatility. This is principally due to the important portion of wealth invested in stocks that exposes the pension plan to market risk. Thus, I conclude that ALM is not only an appropriate portfolio choice model from a theoretical point of view, but it obtains successful results compared to current investment decisions in the pension fund business.

11. Conclusion

I define the strategic asset allocation of a typical US pension fund by applying an Asset-Liability Management model based on the funding ratio return perspective. This approach permits to decompose the portfolio allocation in a Liability Hedging Portfolio and a Return Optimizing Portfolio which aim to minimize the mismatch risk and to yield constant adjusted-returns. The use of a first order Vector Autoregressive method for modeling returns leads to study the strategic asset allocation for different investment horizons where volatilities and covariances are time-varying. In order to test out-of-sample the superiority of ALM, I also present a conditional AO and unconditional JS strategies both based on a classic Markowitz optimization and a further naïve 1/N strategy. I compare their performances by employing a funding ratio return analysis and a Sharpe ratios comparison (using Jobson-Korkie statistics).
My findings indicate that pension funds must adopt ALM approach when they design their strategic asset allocation. These entities must abandon classic Markowitz portfolio optimization because they are not appropriate for their purpose. In fact, by analyzing the funding ratio return I show that ALM clearly outperforms JS, AO and 1/N strategies. The second methodology confirms the superiority of ALM over AO and JS strategies.

My results also indicate that Bonds and public Real Estate are fundamental categories in which pension funds must heavily invest because of their important liability-hedging properties. Hedge Funds and private Real Estate are particularly appealing for their risk diversification and risk-return tradeoff qualities. Commodities must also be taken into consideration by pension managers because of their risk diversification potentiality. Surprisingly, T-bills, Stocks and Gold are ignored in the ALM framework. This last result is clearly in contrast with most of current investment decisions in practice that massively invest in stocks. I provide evidence that such allocations are evidently suboptimal because they do not provide an efficient liability hedging, but they rather increase the volatility of the funding ratio. A changeover to more effective ALM model is therefore necessary.

My findings are consistent with most of the literature regarding strategic asset allocation, asset-liability management, financial forecasts and alternative assets. One limitation of my article might however come from the shortness and extremeness of the period used to test the performance of the ALM strategy. Further researches could assess this superiority in longer and economical steadier lapses of time.

Viceira (2009) suggests that future academic researches should be concentrated on the development of “age-dependent” ALM models. A great challenge consists of designing investment options in the fund that appropriately reflect the investment horizon of pension fund participants. In that way, pension plans would no more be faced with the problem of choosing an adequate risk aversion and utility function. According to Hoevenaars and Steenkamp (2007), the ALM model presented in this article can integrate such evolution by accommodating the concept of age-dependent LHP and ROP portfolios.

---

38 Viceira L. presents “Pension fund design in Developing Economies” in the international conference organized by FIAP-ASOSFONDOS in Cartagena on the 23th-24th April 2009.

References


Inkmann J. and Blake D., 2007. Pension liability valuation and asset allocation in the presence of funding risk. 


Appendix

I. The European pension plans strategic asset allocation

JPMorgan’s Survey carried out in 2006 provides an overview regarding the strategic asset allocation of European pension plans:

<table>
<thead>
<tr>
<th>Strategic Asset Allocation</th>
<th>Tbls</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Gsci</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Gold</th>
<th>Ncreif</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>0.00%</td>
<td>59.20%</td>
<td>32.70%</td>
<td>3.10%</td>
<td>0.50%</td>
<td>4.50%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>NL</td>
<td>0.00%</td>
<td>36.90%</td>
<td>49.90%</td>
<td>3.90%</td>
<td>0.90%</td>
<td>8.40%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>DK/SE</td>
<td>0.00%</td>
<td>32.60%</td>
<td>57.00%</td>
<td>4.10%</td>
<td>1.40%</td>
<td>4.90%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Rest of Europe</td>
<td>0.00%</td>
<td>30.80%</td>
<td>52.50%</td>
<td>4.30%</td>
<td>1.10%</td>
<td>11.30%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Overall</td>
<td>0.00%</td>
<td>39.90%</td>
<td>48.00%</td>
<td>3.70%</td>
<td>1.00%</td>
<td>7.40%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>


II. The derivation of the ROP and LHP portfolios for ALM strategy

I represent the ALM portfolio optimization problem with log returns as follows:

\[
\max_{\alpha_t} E_t \left[ r_{F,t+T}^{(T)} \right] + \frac{1}{2} (1 - \gamma) \text{Var}_t \left[ r_{F,t+T}^{(T)} \right] \\
\text{s.t.} \\
\alpha_t^{(T)} = 1
\]

The Lagrangian of this problem is

\[
L = \left( \alpha_t^{(T)} \left( \mu_{A,t+T}^{(T)} + \frac{1}{2} \sigma_{A,t}^{2} \right) - \frac{1}{2} \alpha_t^{(T)} \Sigma_{A,t} \alpha_t^{(T)} \right) + \frac{1}{2} (1 - \gamma) \left( \sigma_{L,t}^{2} - 2 \alpha_t^{(T)} \sigma_{A,t}^{(T)} + \alpha_t^{(T)} \Sigma_{A,t} \alpha_t^{(T)} \right)
\]

Differentiating the Lagrangian by \( \alpha \) I obtain

\[
\frac{\partial L}{\partial \alpha} = \mu_{A,t} + \frac{1}{2} \sigma_{A,t}^{2} - \alpha \Sigma_{A,t} + \frac{1}{2} (1 - \gamma) \left[ -2 \sigma_{A,t} + 2 \alpha \Sigma_{A,t} \right] - \lambda 1 = 0
\]

\[
\mu_{A,t} + \frac{1}{2} \sigma_{A,t}^{2} - \alpha \Sigma_{A,t} + \frac{1}{2} (1 - \gamma) \left[ -\sigma_{A,t} + \alpha \Sigma_{A,t} \right] - \lambda 1 = 0
\]

\[
\alpha = \left[ \frac{1}{\gamma} \left( 1 - \frac{1}{\gamma} (\Sigma_{A,t} + \frac{1}{\gamma} \Sigma_{A,t}) \right) \right]^{-1} \left( \mu_{A,t} + \frac{1}{2} \sigma_{A,t}^{2} - (1 - \gamma) \sigma_{A,t} - \lambda 1 \right)
\]
Differentiating the Lagrangian by $\lambda$ I obtain

$$\frac{\partial L}{\partial \lambda} = 1 - \alpha' 1 = 0$$
$$\alpha' 1 = 1$$

Substituting the second differentiation into the first one, I define the optimal weights as:

$$\alpha = \left[ \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1} \left( \mu_{AA} + \frac{1}{2} \sigma^2_{A} - (1 - \gamma)\sigma_{AL} - \lambda 1 \right) 1 = 1$$

The optimal portfolio weights can be represented by three distinct portfolios:

$$\alpha_{ROP} = \left[ \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1} \left( \mu_{AA} + \frac{1}{2} \sigma^2_{A} \right)$$

$$\alpha_{LHP} = \left[ \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1} \left( (1 - \gamma)\sigma_{AL} \right)$$

$$\alpha_{ADJ} = \left[ \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1} \lambda 1$$

where the value of $\lambda$ can be expressed as

$$\lambda = \frac{\left[ 1 - \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1} \left( \mu_{AA} + \frac{1}{2} \sigma^2_{A} \right) + \left[ \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1} \left( (1 - \gamma)\sigma_{AL} \right)}{\left[ \frac{1}{\gamma} \left[ 1 - \frac{1}{\gamma} (\Sigma_{AA} + \frac{1}{\gamma} \Sigma_{AL}) \right] \right]^{-1}}$$

The sum of these three portfolio must sum up to one

$$\alpha_{ROP} + \alpha_{LHP} + \alpha_{ADJ} = 1$$

**III. The conditional variance-covariance matrix**

I start by presenting a set of equations representing the future value of the vector $z_t$ defined by the first order VAR in (1), I can forward substitute to obtain $z_{t+j}$
The cumulative returns of the vector $z_i$ over $k$ horizons can be compactly written as:

$$z_{i+1} + ... + z_{i+k} = \left[ \sum_{i=0}^{k-1} (k-i)\Phi^i \right] \Phi_0 + \left[ \sum_{j=0}^{k} \Phi^j \right] z_i + \sum_{q=1}^{k} \left[ \sum_{p=0}^{k-q} \Phi^p \sigma_{i+q} \right]$$

The conditional variance is given by

$$Var_i(z_{i+1} + ... + z_{i+k}) = Var_i \left[ \left( I + \Phi_1 + ... + \Phi_1^{k-1} \right) \sigma_{i+1} + \left( I + \Phi_1 + ... + \Phi_1^{k-2} \right) \sigma_{i+2} + ... + \left( I + \Phi_1 \right) \sigma_{i+k} + \right]$$

$$Var_i(z_{i+1} + ... + z_{i+k}) = \Sigma_i + \left( I + \Phi_1 \right) \Sigma_i + \left( I + \Phi_1 \right)' + ... + \left( I + \Phi_1 + \Phi_1^{k-1} \right) \Sigma_i + \left( I + \Phi_1 + \Phi_1^{k-1} \right)'$$

IV. The portfolio choice problem of AO and JS strategies

a. Asset-only (AO) strategy

I express the mean asset returns through a log-linear approximation form

$$E_i\left[ \mu_{i,t}^{(T)} \right] = T \left( \frac{\alpha_i^{(T)}}{2} \sum_{k,i} \alpha_i^{(T)} \right)$$

and the variance of asset returns through a log-linear approximation form as
\[ Var\left[ r^{(T)}_{A,t+T} \right] = T \left( \alpha^\prime \Sigma_{A} \alpha + \sigma^2_0 + 2 \alpha^\prime \sigma_{A,0}^{(T)} \right) \]

The optimization program can be written:

\[
\begin{align*}
\max_{\alpha^{(T)}} & \quad E \left[ r^{(T)}_{A,t+T} \right] + \frac{1}{2} \left( 1 - \gamma \right) Var \left[ r^{(T)}_{A,t+T} \right] \\
\text{s.t.} & \quad \alpha^{(T)} \mathbf{1} = 1 \\
& \quad \alpha \geq 0
\end{align*}
\]

(25)

Differentiating, the optimal weights vector becomes

\[
\alpha^{(T)} = \frac{1}{\gamma} \left( \left( 1 - \frac{1}{\gamma} \right) \Sigma^{(T)}_{A} + \frac{1}{\gamma} \Sigma_{A} \right)^{-1} \left( \mu^{(T)} + \frac{1}{2} \sigma^2_0 + \left( 1 - \gamma \right) \sigma_{A,0}^{(T)} \right)
\]

b. Jorion-Stein (JS) strategy

I express the mean of asset returns through a log-linear approximation form:

\[
E\left[ r_A \right] = \alpha^\prime \left( \mu_A + \frac{1}{2} \sigma^2_A \right) - \frac{1}{2} \alpha^\prime \Sigma_{A,0} \alpha
\]

where \( \alpha \) is the vector of optimal weights, \( \mu_A \) is the vector of unconditional expected asset returns calculated through the Jorion-Stein estimator, \( \sigma^2_A \) is the vector of unconditional variances of asset returns and \( \Sigma_{A,0} \) is the variance-covariance matrix between the eleven asset classes.

I express the variance of asset returns through a log-linear approximation form:

\[
Var\left[ r_A \right] = \left( \alpha^\prime \Sigma_{A,0} \alpha + \sigma^2_0 + 2 \alpha^\prime \sigma_{A,0} \right)
\]

where \( \alpha \) is the vector of optimal weights, \( \sigma^2_0 \) is the variance of real T-bills rate, \( \Sigma_{A,0} \) is the variance-covariance matrix of assets and \( \sigma_{A,0} \) is the vector of covariances between assets and real T-bills rate.

The optimization program can be written:

\[
\begin{align*}
\max_{\alpha} & \quad E\left[ r_A \right] + \frac{1}{2} \left( 1 - \gamma \right) Var\left[ r_A \right] \\
\text{s.t.} & \quad \alpha^\prime \mathbf{1} = 1 \\
& \quad \alpha \geq 0
\end{align*}
\]
V. The Principal Components Analysis procedure

Step 1: Calculate the mean for all data dimensions.

For the dimension $x$ I compute the mean $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$, where $x_i$ is the observation of $x$ at time $t$ and $N$ is the number of observations in the data set. The same is done for every dimension.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge</td>
<td>0.88%</td>
</tr>
<tr>
<td>Nareit</td>
<td>2.55%</td>
</tr>
<tr>
<td>Ncreif</td>
<td>3.27%</td>
</tr>
<tr>
<td>Gsci</td>
<td>2.33%</td>
</tr>
<tr>
<td>Gold</td>
<td>-1.63%</td>
</tr>
<tr>
<td>Agri</td>
<td>4.51%</td>
</tr>
<tr>
<td>Oil</td>
<td>3.84%</td>
</tr>
<tr>
<td>Energy</td>
<td>2.65%</td>
</tr>
</tbody>
</table>

Step 2: Subtract the mean from each of the data dimensions.

For PCA to work properly, I have to subtract the mean from each of the data dimensions. The mean subtracted is the average across each dimension. So, all the $x$ values have $\bar{x}$ (the mean of $x$ values calculated in Step 1) subtracted, and all the $y$ values have $\bar{y}$ subtracted from them. This produces a data set whose mean is zero.

Step 3: Calculate the variance-covariance matrix

There are no surprises here, so the variance-covariance matrix is calculated on the historical values.

<table>
<thead>
<tr>
<th></th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
<th>Agri</th>
<th>Oil</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge</td>
<td>0.017</td>
<td>-0.0001</td>
<td>0.0008</td>
<td>0.0036</td>
<td>0.001</td>
<td>-0.0003</td>
<td>0.0021</td>
<td>0.0001</td>
</tr>
<tr>
<td>Nareit</td>
<td>-0.0001</td>
<td>0.0016</td>
<td>0.0008</td>
<td>-0.0001</td>
<td>-0.0004</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>0.0002</td>
</tr>
<tr>
<td>Ncreif</td>
<td>0.0007</td>
<td>0.0008</td>
<td>0.0048</td>
<td>0.0008</td>
<td>0.0007</td>
<td>0.0036</td>
<td>0.0008</td>
<td>0.0004</td>
</tr>
<tr>
<td>Gsci</td>
<td>0.0105</td>
<td>-0.0001</td>
<td>0.0007</td>
<td>0.0017</td>
<td>0.0016</td>
<td>0.005</td>
<td>0.0155</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Gold</td>
<td>0.0016</td>
<td>-0.0004</td>
<td>0.0007</td>
<td>0.001</td>
<td>0.0056</td>
<td>0.0024</td>
<td>0.0008</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Agri</td>
<td>0.005</td>
<td>-0.0001</td>
<td>0.0036</td>
<td>-0.0003</td>
<td>0.0024</td>
<td>0.0255</td>
<td>0.0067</td>
<td>0.0008</td>
</tr>
<tr>
<td>Oil</td>
<td>0.0155</td>
<td>-0.0001</td>
<td>0.0008</td>
<td>0.0021</td>
<td>0.0008</td>
<td>0.0067</td>
<td>0.0242</td>
<td>-0.0003</td>
</tr>
<tr>
<td>Energy</td>
<td>-0.0001</td>
<td>0.0002</td>
<td>0.0004</td>
<td>0.0001</td>
<td>-0.0001</td>
<td>0.0008</td>
<td>-0.0003</td>
<td>0.0007</td>
</tr>
</tbody>
</table>
Step 4: Calculate the eigenvalues of the covariance matrix

Since the covariance matrix is square, I can calculate the eigenvalues for this matrix. These are rather important, as they tell us useful information about the data set.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge</td>
<td>0.0046</td>
</tr>
<tr>
<td>Nareit</td>
<td>0.0211</td>
</tr>
<tr>
<td>Ncreif</td>
<td>0.006</td>
</tr>
<tr>
<td>Gsci</td>
<td>0.0399</td>
</tr>
<tr>
<td>Gold</td>
<td>0.0027</td>
</tr>
<tr>
<td>Agri</td>
<td>0.0014</td>
</tr>
<tr>
<td>Oil</td>
<td>0.0006</td>
</tr>
<tr>
<td>Energy</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

Step 5: Selection of the most significant dimensions

Ranking in decreasing order the eigenvalues and their corresponding dimensions leads to list all components in significance order. I ignore the last three terms because their significance level.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Eigenvalues</th>
<th>Ranking</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gsci</td>
<td>0.0399</td>
<td>1</td>
<td>52.2%</td>
</tr>
<tr>
<td>Nareit</td>
<td>0.0211</td>
<td>2</td>
<td>27.6%</td>
</tr>
<tr>
<td>Ncreif</td>
<td>0.006</td>
<td>3</td>
<td>7.8%</td>
</tr>
<tr>
<td>Hedge</td>
<td>0.0046</td>
<td>4</td>
<td>6.0%</td>
</tr>
<tr>
<td>Gold</td>
<td>0.0027</td>
<td>5</td>
<td>3.5%</td>
</tr>
<tr>
<td>Agri</td>
<td>0.0014</td>
<td>6</td>
<td>1.8%</td>
</tr>
<tr>
<td>Oil</td>
<td>0.0006</td>
<td>7</td>
<td>0.8%</td>
</tr>
<tr>
<td>Energy</td>
<td>0.0002</td>
<td>8</td>
<td>0.3%</td>
</tr>
</tbody>
</table>
### Table 1

#### Principal Components Analysis: Eigenvalues and significance levels

This table reports the eigenvalues coefficients of the Principal Components Analysis, the corresponding significance percentages and the relative ranking for all of the eight alternative assets: Hedge Funds (Hedge), public Real Estate (Nareit), private Real Estate (Ncreif), Commodity (Gsci), Gold (Gold), Agriculture (Agri) and Oil crude (Oil).

<table>
<thead>
<tr>
<th>Asset</th>
<th>Eigenvalues</th>
<th>Ranking</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gsci</td>
<td>0.0399</td>
<td>1</td>
<td>52.2%</td>
</tr>
<tr>
<td>Nareit</td>
<td>0.0211</td>
<td>2</td>
<td>27.6%</td>
</tr>
<tr>
<td>Ncreif</td>
<td>0.006</td>
<td>3</td>
<td>7.8%</td>
</tr>
<tr>
<td>Hedge</td>
<td>0.0046</td>
<td>4</td>
<td>6.0%</td>
</tr>
<tr>
<td>Gold</td>
<td>0.0027</td>
<td>5</td>
<td>3.5%</td>
</tr>
<tr>
<td>Agri</td>
<td>0.0014</td>
<td>6</td>
<td>1.8%</td>
</tr>
<tr>
<td>Oil</td>
<td>0.0006</td>
<td>7</td>
<td>0.8%</td>
</tr>
<tr>
<td>Energy</td>
<td>0.0002</td>
<td>8</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

### Table 2

#### Descriptive statistics for assets, liability and state variables

This table summarizes the descriptive statistics for assets, liability and state variables. It includes quarterly and annualized mean and standard deviations, their corresponding Sharpe ratios, minimum and maximum quarterly returns, the Skewness and the Kurtosis values.

#### Panel A: Asset classes and Liability

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean St.Dev</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Annualized Mean</th>
<th>Annualized St.Dev</th>
<th>Annualized Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-bills</td>
<td>0.3% 0.6%</td>
<td>-1.7% 2.5%</td>
<td>-0.05 1.26</td>
<td>1.3% 1.2%</td>
<td>1.08 0.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>1.6% 7.1%</td>
<td>-28.6% 22.6%</td>
<td>-1.05 2.86</td>
<td>6.2% 14.2%</td>
<td>0.44 0.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>1.1% 4.9%</td>
<td>-15.9% 22.6%</td>
<td>0.36 2.20</td>
<td>4.4% 9.9%</td>
<td>0.44 0.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hedge</td>
<td>2.2% 4.3%</td>
<td>-11.3% 16.1%</td>
<td>-0.18 3.64</td>
<td>8.7% 8.6%</td>
<td>1.01 1.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nareit</td>
<td>2.8% 7.1%</td>
<td>-15.6% 19.6%</td>
<td>-0.16 0.11</td>
<td>11.2% 14.2%</td>
<td>0.79 0.79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ncreif</td>
<td>2.4% 4.8%</td>
<td>-21.9% 14.1%</td>
<td>-1.16 5.63</td>
<td>9.5% 9.5%</td>
<td>1.00 1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gsci</td>
<td>2.6% 9.6%</td>
<td>-22.2% 44.1%</td>
<td>0.36 2.02</td>
<td>10.3% 19.3%</td>
<td>0.54 0.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>0.5% 9.0%</td>
<td>-21.9% 34.8%</td>
<td>0.89 2.49</td>
<td>2.1% 18.0%</td>
<td>0.11 0.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liability</td>
<td>2.6% 5.3%</td>
<td>-10.1% 12.2%</td>
<td>-0.17 -0.38</td>
<td>10.3% 10.6%</td>
<td>0.98 0.98</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Panel B: State Variables

<table>
<thead>
<tr>
<th>State Variable</th>
<th>Mean St.Dev</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Annualized Mean</th>
<th>Annualized St.Dev</th>
<th>Annualized Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 months</td>
<td>1.3% 0.7%</td>
<td>0.2% 3.8%</td>
<td>1.08 1.72</td>
<td>5.1% 1.4%</td>
<td>3.66 4.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DY</td>
<td>-3.5% 0.4%</td>
<td>-4.5% -2.8%</td>
<td>-0.78 0.10</td>
<td>-3.5% 0.8%</td>
<td>-4.42 2.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term Spread</td>
<td>0.3% 0.3%</td>
<td>-0.9% 1.0%</td>
<td>-0.36 0.84</td>
<td>1.3% 0.6%</td>
<td>2.17 0.4%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Spread</td>
<td>0.4% 0.2%</td>
<td>0.1% 0.9%</td>
<td>0.38 -0.40</td>
<td>1.7% 0.4%</td>
<td>4.70 4.70</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 3
**Estimations of the core VAR coefficients**

This table reports the estimated coefficients of the core VAR \( y_{t+1} = \alpha + B y_t + \epsilon_{t+1} \). T-bills, Stocks, Bonds, 3 months nominal T-bills, Dividend yield (DY), Term Spread (Term), Credit Spread (Credit) are predicted by their own one-period lagged. The \( t \)-statistic for each coefficient is also reported. In the last two columns there are R-square and F-statistics.

<table>
<thead>
<tr>
<th>Lagged variables</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>3months</th>
<th>DY</th>
<th>Term</th>
<th>Credit</th>
<th>alpha</th>
<th>R-square</th>
<th>F-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-bills</td>
<td>0.42</td>
<td>0.01</td>
<td>0.01</td>
<td>0.29</td>
<td>-0.13</td>
<td>0.49</td>
<td>-0.50</td>
<td>-0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>6.87†</td>
<td>1.67</td>
<td>1.42</td>
<td>3.71†</td>
<td>-1.20</td>
<td>3.49†</td>
<td>-1.90</td>
<td>-1.49</td>
<td>0.35</td>
<td>15.45</td>
</tr>
<tr>
<td>Stocks</td>
<td>0.89</td>
<td>0.05</td>
<td>0.11</td>
<td>-3.48</td>
<td>5.51</td>
<td>-2.18</td>
<td>9.49</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.06</td>
<td>0.76</td>
<td>1.00</td>
<td>-3.27†</td>
<td>3.72†</td>
<td>-1.15</td>
<td>2.63†</td>
<td>3.83†</td>
<td>0.11</td>
<td>3.56</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.24</td>
<td>-0.12</td>
<td>-0.08</td>
<td>0.71</td>
<td>0.12</td>
<td>2.61</td>
<td>2.90</td>
<td>-0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>0.40</td>
<td>-2.53†</td>
<td>-1.05</td>
<td>0.94</td>
<td>0.11</td>
<td>1.93</td>
<td>1.13</td>
<td>-0.34</td>
<td>0.08</td>
<td>2.61</td>
</tr>
<tr>
<td>3months</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.01</td>
<td>1.04</td>
<td>-0.07</td>
<td>0.24</td>
<td>-0.47</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-1.35</td>
<td>1.91</td>
<td>1.55</td>
<td>27.31†</td>
<td>-1.28</td>
<td>3.49†</td>
<td>-3.66†</td>
<td>-0.82</td>
<td>0.88</td>
<td>217.68</td>
</tr>
<tr>
<td>DY</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
<td>0.94</td>
<td>0.02</td>
<td>-0.11</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-1.90</td>
<td>-0.45</td>
<td>-1.03</td>
<td>3.45†</td>
<td>61.61†</td>
<td>1.05</td>
<td>-3.03†</td>
<td>-3.49†</td>
<td>0.97</td>
<td>922.87</td>
</tr>
<tr>
<td>TERM</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.07</td>
<td>0.09</td>
<td>0.66</td>
<td>0.52</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.03</td>
<td>-0.98</td>
<td>-1.42</td>
<td>-2.28†</td>
<td>2.14†</td>
<td>12.35†</td>
<td>5.10†</td>
<td>1.86</td>
<td>0.64</td>
<td>51.11</td>
</tr>
<tr>
<td>CREDIT</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.06</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.80</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-1.66</td>
<td>-2.63†</td>
<td>-1.33</td>
<td>4.77†</td>
<td>-3.04†</td>
<td>2.46</td>
<td>19.95†</td>
<td>-2.73†</td>
<td>0.83</td>
<td>140.65</td>
</tr>
</tbody>
</table>

Significance level of 5% †
Significance level of 1% ‡

### Table 4
**Variance-covariance matrix of the core VAR model**

This table reports the errors variance-covariance matrix \( \sum_{\epsilon \epsilon} \) of the core VAR model \( y_{t+1} = \alpha + B y_t + \epsilon_{t+1} \). Diagonal entries are standard deviations. The upper triangular off-diagonal figures are covariances and down off-diagonal values are correlations.

<table>
<thead>
<tr>
<th></th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>3months</th>
<th>DY</th>
<th>Term</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-bills</td>
<td>0.005</td>
<td>4.5E-05</td>
<td>1.5E-05</td>
<td>5.8E-07</td>
<td>-7.5E-07</td>
<td>-1.2E-06</td>
<td>1.6E-08</td>
</tr>
<tr>
<td>Stocks</td>
<td>0.13</td>
<td>0.0679</td>
<td>5.4E-04</td>
<td>-1.9E-05</td>
<td>-4.7E-05</td>
<td>7.7E-06</td>
<td>-6.9E-06</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.06</td>
<td>0.16</td>
<td>0.0482</td>
<td>-7.3E-05</td>
<td>-5.7E-06</td>
<td>1.4E-05</td>
<td>1.5E-05</td>
</tr>
<tr>
<td>3months</td>
<td>0.05</td>
<td>-0.12</td>
<td>-0.62</td>
<td>0.0024</td>
<td>2.1E-07</td>
<td>-3.9E-06</td>
<td>-3.0E-07</td>
</tr>
<tr>
<td>DY</td>
<td>-0.21</td>
<td>-0.98</td>
<td>-0.17</td>
<td>0.12</td>
<td>0.0007</td>
<td>-7.0E-08</td>
<td>5.8E-08</td>
</tr>
<tr>
<td>Term</td>
<td>-0.13</td>
<td>0.06</td>
<td>0.15</td>
<td>-0.83</td>
<td>-0.05</td>
<td>0.0019</td>
<td>-3.4E-07</td>
</tr>
<tr>
<td>Credit</td>
<td>0.00</td>
<td>-0.13</td>
<td>0.400</td>
<td>-0.16</td>
<td>0.11</td>
<td>-0.24</td>
<td>0.0008</td>
</tr>
</tbody>
</table>
Table 5
Estimations of the OLS model coefficients

This table reports the estimated coefficients of the OLS model $x_{t+1} = c + D_y y^t + D_T y^{t+1} + H x^t + \eta_{t+1}$. Contemporaneous variables $Y^t$ are T-bills, Stocks, Bonds, 3 months nominal T-bills (3 months), Dividend yield (DY), Term spread (TERM) and Credit (CREDIT). Lagged explanatory variables $y_j$ are T-bills, Stocks, Bonds, 3 months nominal T-bills (3 months), Dividend yield (DY), Term spread (TERM), Credit (CREDIT) and Own variable (which consists of the lagged $x_2$ variable). The 3 months and DY coefficients are not used to predict alternative assets and their coefficients are set equal to zero for both contemporaneous and lagged variables. In each second line $t$-statistics are reported for each coefficient and in the last column there are R-square coefficients.

<table>
<thead>
<tr>
<th></th>
<th>Contemporaneous</th>
<th>Lagged</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>3months</th>
<th>DY</th>
<th>Term</th>
<th>Credit</th>
<th>OWN</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge</td>
<td>-0.46</td>
<td>0.18</td>
<td>0.13</td>
<td>0.00</td>
<td>0.00</td>
<td>-2.40</td>
<td>-31.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.50</td>
<td>1.75</td>
<td>0.67</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.34</td>
<td>-2.15†</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nareit</td>
<td>-0.84</td>
<td>0.52</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>-2.08</td>
<td>-12.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-1.01</td>
<td>7.57‡</td>
<td>4.11‡</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.93</td>
<td>-1.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ncreif</td>
<td>0.79</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-6.83</td>
<td>-9.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.08</td>
<td>-0.26</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>-3.56†</td>
<td>-1.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gsci</td>
<td>-5.01</td>
<td>-0.32</td>
<td>0.22</td>
<td>0.00</td>
<td>0.00</td>
<td>-6.23</td>
<td>-24.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-3.67‡</td>
<td>-2.94‡</td>
<td>1.35‡</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.66</td>
<td>-2.31†</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>-5.35</td>
<td>0.05</td>
<td>0.22</td>
<td>0.00</td>
<td>0.00</td>
<td>7.76</td>
<td>7.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-4.24‡</td>
<td>0.45</td>
<td>1.53</td>
<td>0.00</td>
<td>0.00</td>
<td>2.30†</td>
<td>0.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liability</td>
<td>1.54</td>
<td>0.00</td>
<td>1.14</td>
<td>0.00</td>
<td>0.00</td>
<td>6.00</td>
<td>-4.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.73</td>
<td>0.04</td>
<td>6.52‡</td>
<td>0.00</td>
<td>0.00</td>
<td>1.03</td>
<td>-0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Significance level of 5% †
Significance level of 1% ‡
Table 6
Strategic asset allocation of ALM strategy

This table reports the optimal weights of the ALM strategy. Six different investing horizons of 3, 5, 10, 15, 20 and 25 years and four levels of risk aversion of 15, 20, 30 and GMVP are taken into consideration. Asset classes considered in the strategic asset allocation are: T-bills, Stocks, Bonds, Hedge Funds (Hedge), public Real Estate (Nareit), private Real Estate (Ncreif), Commodities (Gsci) and Gold (Gold). In some cases the sum of weights differ from 1 due to rounding.

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>31.38%</td>
<td>13.88%</td>
<td>41.27%</td>
<td>4.86%</td>
<td>8.60%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>19.35%</td>
<td>19.90%</td>
<td>44.49%</td>
<td>6.17%</td>
<td>10.08%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>6.59%</td>
<td>29.00%</td>
<td>48.38%</td>
<td>8.23%</td>
<td>7.80%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>5.17%</td>
<td>32.00%</td>
<td>49.55%</td>
<td>8.83%</td>
<td>4.44%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>5.68%</td>
<td>32.95%</td>
<td>49.97%</td>
<td>9.08%</td>
<td>2.32%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>6.31%</td>
<td>33.27%</td>
<td>50.17%</td>
<td>9.24%</td>
<td>1.01%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>57.10%</td>
<td>7.24%</td>
<td>27.50%</td>
<td>2.74%</td>
<td>5.42%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>48.19%</td>
<td>11.55%</td>
<td>29.88%</td>
<td>3.72%</td>
<td>6.67%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>38.21%</td>
<td>18.36%</td>
<td>32.82%</td>
<td>5.35%</td>
<td>5.25%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>36.89%</td>
<td>20.69%</td>
<td>33.69%</td>
<td>5.85%</td>
<td>2.88%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>37.14%</td>
<td>21.45%</td>
<td>33.98%</td>
<td>6.07%</td>
<td>1.36%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>37.55%</td>
<td>21.72%</td>
<td>34.10%</td>
<td>6.22%</td>
<td>0.41%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>70.26%</td>
<td>3.87%</td>
<td>20.44%</td>
<td>1.62%</td>
<td>3.81%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>63.11%</td>
<td>7.23%</td>
<td>22.33%</td>
<td>2.42%</td>
<td>4.91%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>54.77%</td>
<td>12.77%</td>
<td>24.72%</td>
<td>3.78%</td>
<td>3.96%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>53.53%</td>
<td>14.72%</td>
<td>25.41%</td>
<td>4.23%</td>
<td>2.11%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>53.66%</td>
<td>15.38%</td>
<td>25.63%</td>
<td>4.43%</td>
<td>0.91%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>53.95%</td>
<td>15.63%</td>
<td>25.71%</td>
<td>4.56%</td>
<td>0.15%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>83.63%</td>
<td>0.45%</td>
<td>13.27%</td>
<td>0.47%</td>
<td>2.18%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>78.39%</td>
<td>2.83%</td>
<td>14.61%</td>
<td>1.05%</td>
<td>3.13%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>71.86%</td>
<td>6.99%</td>
<td>16.37%</td>
<td>2.12%</td>
<td>2.65%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>70.74%</td>
<td>8.54%</td>
<td>16.88%</td>
<td>2.50%</td>
<td>1.35%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>70.74%</td>
<td>9.09%</td>
<td>17.02%</td>
<td>2.67%</td>
<td>0.47%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>70.94%</td>
<td>9.20%</td>
<td>17.13%</td>
<td>2.73%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>GMVP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
### Table 7

**Strategic asset allocation of AO strategy**

This table reports the optimal weights of the AO strategy. Six different investing horizons of 3, 5, 10, 15, 20 and 25 years and four levels of risk aversion of 15, 20, 30 and GMVP are taken into consideration. Asset classes considered in the strategic asset allocation are: T-bills, Stocks, Bonds, Hedge Funds (Hedge), public Real Estate (Nareit), private Real Estate (Ncreif), Commodities (Gsci) and Gold (Gold). In some cases the sum of weights differ from 1 due to rounding.

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>Tbills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Risk aversion 10</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>37.92%</td>
<td>35.88%</td>
<td>11.61%</td>
<td>14.59%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>39.01%</td>
<td>37.50%</td>
<td>11.45%</td>
<td>12.04%</td>
<td>0.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>39.15%</td>
<td>40.75%</td>
<td>10.87%</td>
<td>9.23%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>38.40%</td>
<td>42.35%</td>
<td>10.41%</td>
<td>8.83%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>37.82%</td>
<td>43.20%</td>
<td>10.05%</td>
<td>8.93%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>37.47%</td>
<td>43.67%</td>
<td>9.77%</td>
<td>9.09%</td>
<td>0.00%</td>
</tr>
<tr>
<td><strong>Risk aversion 15</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>41.17%</td>
<td>32.38%</td>
<td>11.61%</td>
<td>13.08%</td>
<td>1.75%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>42.60%</td>
<td>34.22%</td>
<td>11.52%</td>
<td>10.76%</td>
<td>0.89%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>0.42%</td>
<td>0.00%</td>
<td>42.93%</td>
<td>37.47%</td>
<td>10.95%</td>
<td>8.23%</td>
<td>0.00%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>3.88%</td>
<td>0.00%</td>
<td>40.83%</td>
<td>37.34%</td>
<td>10.21%</td>
<td>7.75%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>6.45%</td>
<td>0.00%</td>
<td>39.17%</td>
<td>37.06%</td>
<td>9.62%</td>
<td>7.70%</td>
<td>0.00%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>8.17%</td>
<td>0.00%</td>
<td>38.05%</td>
<td>36.86%</td>
<td>9.16%</td>
<td>7.75%</td>
<td>0.00%</td>
</tr>
<tr>
<td><strong>Risk aversion 20</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>0.00%</td>
<td>0.79%</td>
<td>4.43%</td>
<td>40.15%</td>
<td>27.84%</td>
<td>11.14%</td>
<td>11.80%</td>
<td>3.86%</td>
</tr>
<tr>
<td>5 years</td>
<td>0.00%</td>
<td>1.90%</td>
<td>4.33%</td>
<td>41.16%</td>
<td>29.09%</td>
<td>10.97%</td>
<td>9.55%</td>
<td>3.00%</td>
</tr>
<tr>
<td>10 years</td>
<td>0.00%</td>
<td>4.54%</td>
<td>8.75%</td>
<td>39.01%</td>
<td>29.84%</td>
<td>9.86%</td>
<td>7.01%</td>
<td>0.99%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>9.02%</td>
<td>8.44%</td>
<td>36.68%</td>
<td>29.69%</td>
<td>9.03%</td>
<td>6.80%</td>
<td>0.35%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>12.96%</td>
<td>6.48%</td>
<td>35.29%</td>
<td>29.69%</td>
<td>8.48%</td>
<td>6.81%</td>
<td>0.28%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>15.95%</td>
<td>4.45%</td>
<td>34.50%</td>
<td>29.82%</td>
<td>8.08%</td>
<td>6.78%</td>
<td>0.43%</td>
</tr>
<tr>
<td><strong>Risk aversion 30</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>16.72%</td>
<td>1.53%</td>
<td>9.07%</td>
<td>30.63%</td>
<td>19.32%</td>
<td>8.28%</td>
<td>9.48%</td>
<td>4.96%</td>
</tr>
<tr>
<td>5 years</td>
<td>16.46%</td>
<td>2.37%</td>
<td>10.06%</td>
<td>31.04%</td>
<td>20.35%</td>
<td>7.97%</td>
<td>7.56%</td>
<td>4.18%</td>
</tr>
<tr>
<td>10 years</td>
<td>7.15%</td>
<td>6.14%</td>
<td>18.54%</td>
<td>31.32%</td>
<td>20.95%</td>
<td>7.73%</td>
<td>5.70%</td>
<td>2.47%</td>
</tr>
<tr>
<td>15 years</td>
<td>0.00%</td>
<td>11.54%</td>
<td>21.92%</td>
<td>30.79%</td>
<td>20.81%</td>
<td>7.51%</td>
<td>5.62%</td>
<td>1.81%</td>
</tr>
<tr>
<td>20 years</td>
<td>0.00%</td>
<td>15.87%</td>
<td>19.68%</td>
<td>29.22%</td>
<td>20.72%</td>
<td>6.91%</td>
<td>5.81%</td>
<td>1.80%</td>
</tr>
<tr>
<td>25 years</td>
<td>0.00%</td>
<td>19.16%</td>
<td>17.37%</td>
<td>28.32%</td>
<td>20.83%</td>
<td>6.48%</td>
<td>5.86%</td>
<td>1.99%</td>
</tr>
<tr>
<td><strong>Risk aversion GMVP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>77.21%</td>
<td>0.00%</td>
<td>9.66%</td>
<td>2.06%</td>
<td>1.23%</td>
<td>0.00%</td>
<td>3.56%</td>
<td>6.29%</td>
</tr>
<tr>
<td>5 years</td>
<td>71.91%</td>
<td>0.00%</td>
<td>13.90%</td>
<td>2.78%</td>
<td>2.24%</td>
<td>0.00%</td>
<td>3.12%</td>
<td>6.05%</td>
</tr>
<tr>
<td>10 years</td>
<td>60.42%</td>
<td>5.33%</td>
<td>18.48%</td>
<td>3.77%</td>
<td>2.07%</td>
<td>0.00%</td>
<td>4.08%</td>
<td>5.85%</td>
</tr>
<tr>
<td>15 years</td>
<td>54.13%</td>
<td>10.80%</td>
<td>17.96%</td>
<td>4.30%</td>
<td>1.92%</td>
<td>0.00%</td>
<td>4.81%</td>
<td>6.08%</td>
</tr>
<tr>
<td>20 years</td>
<td>50.54%</td>
<td>14.81%</td>
<td>16.48%</td>
<td>4.65%</td>
<td>2.05%</td>
<td>0.00%</td>
<td>5.06%</td>
<td>6.40%</td>
</tr>
<tr>
<td>25 years</td>
<td>48.37%</td>
<td>17.63%</td>
<td>14.92%</td>
<td>4.91%</td>
<td>2.30%</td>
<td>0.00%</td>
<td>5.12%</td>
<td>6.74%</td>
</tr>
</tbody>
</table>
Table 8
Jorion-Stein estimations of the sample mean

This table reports the Jorion-Stein estimations of sample mean for the eight asset classes and for liability. The following formula \( E_{JS}(R) = \hat{\mu}_{JS} = (1-x)Y + xY_0 1 \) permits to reduce the estimation error in the sample means and estimate Jorion-Stein means. In the first rows are reported the values of the \( Y \) sample mean, the \( Y_0 \) GMVP mean, and the \( x \) shrinking factor. For a comparison purpose, both historical and Jorion-Stein estimated means are presented.

<table>
<thead>
<tr>
<th>Delta</th>
<th>14.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shrinkage factor</td>
<td>0.24</td>
</tr>
<tr>
<td>Mean of the Prior GMVP</td>
<td>7.69%</td>
</tr>
</tbody>
</table>

Panel A: Historical Means

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-bills</td>
<td>1.34%</td>
<td>7.57%</td>
<td>5.58%</td>
<td>10.34%</td>
<td>13.29%</td>
<td>10.74%</td>
<td>9.44%</td>
<td>3.55%</td>
<td>10.47%</td>
</tr>
</tbody>
</table>

Panel B: Jorion-Stein Means

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-bills</td>
<td>1.63%</td>
<td>6.30%</td>
<td>4.82%</td>
<td>8.35%</td>
<td>10.52%</td>
<td>8.64%</td>
<td>7.68%</td>
<td>3.30%</td>
<td>8.44%</td>
</tr>
</tbody>
</table>

Table 9
Variance-covariance matrix of Jorion-Stein model

This table reports the variance-covariance matrix of the Jorion-Stein model. These estimations are based on historical data starting on Q1:1992. Diagonal entries are standard deviations. The upper triangular off-diagonal figures are covariances and down off-diagonal values are correlations.

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-bills</td>
<td>0.01</td>
<td>8.4E-05</td>
<td>-5.0E-05</td>
<td>-2.5E-05</td>
<td>-1.3E-04</td>
<td>-4.2E-05</td>
<td>-8.7E-05</td>
<td>-9.0E-05</td>
<td>-6.3E-05</td>
</tr>
<tr>
<td>Stocks</td>
<td>0.22</td>
<td>0.07</td>
<td>-5.2E-04</td>
<td>3.1E-04</td>
<td>-9.1E-04</td>
<td>3.7E-04</td>
<td>-1.2E-03</td>
<td>-1.8E-03</td>
<td>-1.3E-05</td>
</tr>
<tr>
<td>Bonds</td>
<td>-0.21</td>
<td>-0.16</td>
<td>0.04</td>
<td>2.0E-04</td>
<td>2.8E-04</td>
<td>6.6E-05</td>
<td>2.8E-04</td>
<td>-1.7E-04</td>
<td>-3.7E-05</td>
</tr>
<tr>
<td>Hedge</td>
<td>-0.12</td>
<td>0.11</td>
<td>0.11</td>
<td>0.04</td>
<td>7.4E-04</td>
<td>2.4E-04</td>
<td>-1.4E-04</td>
<td>-9.2E-05</td>
<td>-1.8E-04</td>
</tr>
<tr>
<td>Nareit</td>
<td>-0.35</td>
<td>-0.18</td>
<td>0.09</td>
<td>0.26</td>
<td>0.07</td>
<td>4.0E-04</td>
<td>6.6E-04</td>
<td>7.5E-04</td>
<td>-2.8E-04</td>
</tr>
<tr>
<td>Ncreif</td>
<td>-0.30</td>
<td>0.19</td>
<td>0.06</td>
<td>0.23</td>
<td>0.22</td>
<td>0.03</td>
<td>-1.4E-04</td>
<td>1.0E-04</td>
<td>-1.0E-04</td>
</tr>
<tr>
<td>Gsci</td>
<td>-0.16</td>
<td>-0.17</td>
<td>0.06</td>
<td>-0.03</td>
<td>0.09</td>
<td>-0.05</td>
<td>0.10</td>
<td>1.7E-03</td>
<td>-2.4E-04</td>
</tr>
<tr>
<td>Gold</td>
<td>-0.29</td>
<td>-0.41</td>
<td>-0.06</td>
<td>-0.04</td>
<td>0.18</td>
<td>0.06</td>
<td>0.27</td>
<td>0.06</td>
<td>-1.9E-04</td>
</tr>
<tr>
<td>Liability</td>
<td>-0.23</td>
<td>0.00</td>
<td>-0.02</td>
<td>-0.08</td>
<td>-0.08</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.06</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 10
Strategic asset allocation of JS strategy

This table reports the strategic asset allocation of JS strategy for four levels of risk aversion of 15, 20, 30 and GMVP. Asset classes considered in the strategic asset allocation are: T-bills, Stocks, Bonds, Hedge Funds (Hedge), public Real Estate (Nareit), private Real Estate (Ncreif), Commodities (Gsci) and Gold (Gold). In some cases the sum of weights differ from 1 due to rounding.

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>T-bills</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge</th>
<th>Nareit</th>
<th>Ncreif</th>
<th>Gsci</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.00%</td>
<td>4.99%</td>
<td>0.00%</td>
<td>12.87%</td>
<td>9.63%</td>
<td>66.02%</td>
<td>6.50%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20</td>
<td>0.00%</td>
<td>5.77%</td>
<td>3.03%</td>
<td>13.14%</td>
<td>7.00%</td>
<td>64.93%</td>
<td>6.14%</td>
<td>0.00%</td>
</tr>
<tr>
<td>30</td>
<td>0.00%</td>
<td>6.80%</td>
<td>10.15%</td>
<td>12.15%</td>
<td>4.28%</td>
<td>61.15%</td>
<td>5.48%</td>
<td>0.00%</td>
</tr>
<tr>
<td>GMVP</td>
<td>90.25%</td>
<td>0.35%</td>
<td>5.05%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>4.35%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
Table 11
Funding ratio return of ALM, AO, JS and 1/N strategies

This table reports the Funding Ratio Return of the four ALM, AO, JS and 1/N strategies. I compute the Funding ratio cumulative returns over the 3 years out-of-sample period (Q1:2006 to Q4:2008) for 15, 20, 30 and Global Minimum Variance Portfolio (GMVP) risk aversions.

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>ALM strategy</th>
<th>AO strategy</th>
<th>JS strategy</th>
<th>1/N strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-2.2%</td>
<td>-18.6%</td>
<td>-1.5%</td>
<td>-9.3%</td>
</tr>
<tr>
<td>20</td>
<td>3.8%</td>
<td>-16.4%</td>
<td>-0.1%</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>9.8%</td>
<td>-11.4%</td>
<td>2.1%</td>
<td></td>
</tr>
<tr>
<td>GMVP</td>
<td>19.1%</td>
<td>-6.8%</td>
<td>-8.4%</td>
<td>-9.3%</td>
</tr>
</tbody>
</table>

Table 12
Realized Mean and Variance of ALM, AO, JS and 1/N strategies

This table reports the realized funding ratio return Mean and Variance of the four ALM, AO, JS and 1/N strategies for each level of risk aversion. The ALM strategies display inferior variances with respect to additional strategies. This evidence suggest that LHP portfolio provides a good mismatching risk minimization.

<table>
<thead>
<tr>
<th>Risk aversion</th>
<th>ALM strategy</th>
<th>AO strategy</th>
<th>JS strategy</th>
<th>1/N strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-0.75%</td>
<td>-6.21%</td>
<td>-0.51%</td>
<td>-3.10%</td>
</tr>
<tr>
<td>20</td>
<td>1.26%</td>
<td>-5.47%</td>
<td>-0.04%</td>
<td>7.42%</td>
</tr>
<tr>
<td>30</td>
<td>3.27%</td>
<td>-3.79%</td>
<td>0.69%</td>
<td>6.94%</td>
</tr>
<tr>
<td>GMVP</td>
<td>6.36%</td>
<td>-2.28%</td>
<td>-2.81%</td>
<td>6.12%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.53%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 13
Sharpe ratios and Jobson-Korkie statistics for ALM, AO, JS and 1/N strategies

This table reports the Sharpe ratio of ALM, AO, JS and 1/N strategies during the 3 years out-of-sample test. I compare the Sharpe ratio of ALM for 15, 20, 30 and Global Minimum Variance Portfolio (GMVP) levels of risk aversion to the performance of AO, JS and 1/N strategies. Mean is the average excess realized asset return. Excess asset return is defined as the return in excess of the 3 months T-bills rate (considered to be the benchmark). Standard deviation is also defined in excess of 3 months T-bills rate. In order to statistically test the superiority of ALM strategy, I use the Jobson-Korkie statistics.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mean</th>
<th>St.Deviation</th>
<th>Sharpe ratio</th>
<th>J-K statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALM strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.7%</td>
<td>4.1%</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.2%</td>
<td>3.3%</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.7%</td>
<td>3.6%</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>GMVP</td>
<td>2.5%</td>
<td>5.2%</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>AO strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-0.6%</td>
<td>9.7%</td>
<td>-0.06</td>
<td>1.83*</td>
</tr>
<tr>
<td>20</td>
<td>-0.4%</td>
<td>9.3%</td>
<td>-0.05</td>
<td>0.60</td>
</tr>
<tr>
<td>30</td>
<td>0.0%</td>
<td>7.1%</td>
<td>0.00</td>
<td>0.32</td>
</tr>
<tr>
<td>GMVP</td>
<td>0.6%</td>
<td>0.9%</td>
<td>0.61</td>
<td>3.91***</td>
</tr>
<tr>
<td>JS strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.8%</td>
<td>6.5%</td>
<td>0.12</td>
<td>1.66*</td>
</tr>
<tr>
<td>20</td>
<td>0.9%</td>
<td>6.0%</td>
<td>0.15</td>
<td>-0.15</td>
</tr>
<tr>
<td>30</td>
<td>1.1%</td>
<td>5.2%</td>
<td>0.21</td>
<td>0.04</td>
</tr>
<tr>
<td>GMVP</td>
<td>0.5%</td>
<td>1.0%</td>
<td>0.48</td>
<td>8.71***</td>
</tr>
<tr>
<td>1/N strategy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2%</td>
<td>6.5%</td>
<td>0.03</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.21</td>
</tr>
</tbody>
</table>

Significance level of 10% *
Significance level of 1% ***
Figures

Figure 1: Annualized Standard Deviation for real T-bills, Stocks and Bonds
Annualized standard deviation (on y-axis) of T-bills, Stocks and Bonds asset classes across 100 quarters investment horizons (on x-axis).

Figure 2: Annualized Standard Deviation for Hedge Funds, public and private Real Estate
Annualized standard deviation (on y-axis) of Hedge Funds, public real estate (NAREIT) and private real estate (NCREIF) across 100 quarters investment horizons (on x-axis).

Figure 3: Annualized Standard Deviation for Global Commodity index and Gold index
Annualized standard deviation (on y-axis) of Global Commodity index (GSCI) and Gold sub-index (GOLD) across different investment horizons (on x-axis). The time line is expressed on quarterly basis up to 100 quarters.
Figure 4: Annualized Standard Deviation for Citigroup Pension Liability index
Annualized standard deviation (on y-axis) of Citigroup Pension Liability index (LIABILITY) across different investment horizons (on x-axis). The time line is expressed on quarterly basis up to 100 quarters.

Figure 5: Correlation of asset classes with Stocks
Correlations (on y-axis) with Stocks across different investment horizons (on x-axis). The asset classes are T-bills, Stocks, Bonds, Hedge Funds, Nareit, Ncreif, Gsci, Gold. All asset returns are expressed in real terms. The time line is expressed on quarterly basis up to 100 quarters.
Figure 5: Correlation of asset classes with Stocks (continuation)
Correlations (on y-axis) with Stocks across different investment horizons (on x-axis). The asset classes are T-bills, Stocks, Bonds, Hedge Funds, Nareit, Ncreif, Gsci, Gold. All asset returns are expressed in real terms, in order to discover their risk diversification properties. The time line is expressed on quarterly basis up to 100 quarters.

Figure 6: Correlation of asset classes with Bonds
Correlations (on y-axis) with Bonds across different investment horizons (on x-axis). The asset classes are T-bills, Stocks, Bonds, Hedge Funds, Nareit, Ncreif, Gsci, Gold. All classical and alternative assets are expressed in real terms, in order to discover their risk diversification properties. The time line is expressed on quarterly basis up to 100 quarters.
Figure 6: Correlation of asset classes with Bonds (continuation)
Correlations (on y-axis) with Bonds across different investment horizons (on x-axis). The asset classes are T-bills, Stocks, Bonds, Hedge Funds, Nareit, Ncreif, Gsci, Gold. All asset returns are expressed in real terms, in order to discover their risk diversification properties. The time line is expressed on quarterly basis up to 100 quarters.

Figure 7: Correlation of asset classes with Citigroup Pension Liability index
Correlations (on y-axis) with the Citigroup Pension Liability Index across different investment horizons (on x-axis). The asset classes are T-bills, Stocks, Bonds, Hedge Funds, Nareit, Ncreif, Gsci, Gold. All asset returns are expressed in real terms, in order to discover their hedge properties against liability risk. The time line is expressed on quarterly basis up to 100 quarters.
Figure 7: Correlation of asset classes with Citigroup Pension Liability index (continuation)
Correlations (on y-axis) with Citigroup Pension Liability Index across different investment horizons (on x-axis). The asset classes are T-bills, Stocks, Bonds, Hedge Funds, Nareit, Ncreif, Gsci, Gold. All asset returns are expressed in real terms in order to discover their hedge properties against liability risk. The time line is expressed on quarterly basis up to 100 quarters.

Figure 8: Out of sample Funding Ratio Return
Funding Ratio Return (on y-axis) of the four ALM, AO, JS and 1/N strategies across 3 years investment horizons (on x-axis) for a risk aversion of 30. The time line is expressed on quarterly basis.
Figure 9: Funding ratio evolution for an hypothetical pension plan
Funding ratio (on y-axis) of the hypothetical pension plan having an initial funding ratio of 100. The development of the funding status is studied across 3 years investment horizons (on x-axis) for a risk aversion corresponding to 30. The time line is expressed on quarterly basis.

Figure 10: ALM strategy vs. current practices
Funding ratio (on y-axis) of the hypothetical pension plan having an initial funding ratio of 100 percent. The development of the funding status is analyzed across 3 years investment horizons (on x-axis). ALM strategy corresponds to the strategic asset allocation with a risk aversion of 30. Current practices rely on the statistics drawn from the JPMorgan’s 2006 Survey. The time line is expressed on quarterly basis.