Nelson-Plosser revisited: the ACF approach

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ABSTRACT

We detect a new stylized fact that is common to the dynamics of all macroeconomic series, including financial aggregates. Their Auto-Correlation Functions (ACFs) share a common four-parameter functional form that arises from the dynamics of a general equilibrium model with heterogeneous firms. We find that, not only does our formula fit the data better than the ACFs that arise from autoregressive models, but it also yields the correct shape of the ACF, thus explaining the lags with which macroeconomic variables evolve and the onset of seemingly-sudden turning points. This finding puts a premium on quick and decisive macroeconomic policy interventions at the first signs of a turning point, in contrast to gradualist approaches. (JEL C22, E32, E52, E63).

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1. INTRODUCTION

Is the economy about to plunge into a recession? If so, will it happen quickly? Could we have predicted this pattern some time ago or is it truly due to a recent “surprise”? Such questions about macroeconomic dynamics are of paramount importance, especially at times when an economy is changing course. The same reasoning will be relevant to answering questions about exiting a recession.

The dynamics of a series can be depicted by its Auto-Correlation Function (ACF). ACFs evaluate the correlation of a series with its past; hence showing how much persistence there is in a series, but also what shape the decay of memory takes as the past becomes more distant. Auto Regressive (AR) models generate ACFs that can only contain patterns of gradual changes in correlation, but we will show that the evidence suggests otherwise.

We detect a new stylized fact about the common dynamics of macroeconomic series, including financial aggregates. All their ACFs share a common four-parameter functional form that we derive from the dynamics of a general equilibrium model with heterogeneous firms introduced in Abadir and Talmain (2002), to be referred to henceforth as AT. We find that, not only does our formula fit the data better than the ACFs that arise from AR models, but it also yields the correct shape of the ACF. The shape of an ACF is important. Getting the ACF shape right means that we will be able to understand the lags with which macroeconomic variables evolve and the onset of seemingly-sudden turning points.

This result provides an answer to our opening questions. For example, if an economy is starting to slow down, our ACFs predict that it will produce a long sequence of small signs of a slowdown followed by an abrupt decline. These features
have also been noted by Barro (2006) and Barro and Ursúa (2008) within a context of crises, and we show here that similar dynamics extend also to quiet periods. When only the small signs have appeared, no-one fitting a linear (e.g. AR) model would be able to guess the substantial turning point that is about to occur. An implication is that any stimulus that is applied to the economy should be timed to start well before the abrupt decline of the economy has taken place, and will take a long time to have an impact (and will eventually wear off unlike in unit root models). Consequently, a gradualist macroeconomic policy will not yield the desired results because it will be a case of too little and too late. In other words, a gradualist approach can be compatible with linear models but will be disastrous in the context of the ACFs that arise from macroeconomic data and that are compatible with the nonlinear dynamics generated by the general equilibrium model of AT.

The origins of our work are as follows. AT’s model implied that the ACF of real GDP per capita should exhibit an initial concave shape, followed by a sharp drop, a prediction which they validated empirically for the UK and the US. They showed that linear Auto Regressive Integrated Moving-Average (ARIMA) models, as well as their three separate components (including the special case of random walks), all exhibit different types of decays of memory from the one they found. This simple yet accurate shape for GDP invites us here to investigate the shape of the ACFs for all the main macro variables since, in that general equilibrium model, the evolution of all the variables are linked through the common dynamics set by GDP. We study more than twice the number of variables in Nelson and Plosser (1982), including all of theirs. Henceforth, we refer to that seminal paper as NP.

The ACF of AT was the leading term of an expansion of an elaborate integral,
and was suitable as a rough approximation of the broad features of GDP’s ACF. Another novel feature here (apart from considering all main macro series in addition to GDP) is that we go beyond the 1-term asymptotic approximation of the ACF of AT, taking into account the remaining terms of the ACF expansion. The resulting functional form typically combines the original shape in AT (plateau plus drop-off) with a cycle. As we shall see, this augmented version of the ACF shape fits closely the ACF of all of the variables studied by NP, and this fit is better than the one produced by AR processes, including the special case of the unit root. In addition, it also fits very well the ACFs of variables not considered by NP, some of them known to have notoriously difficult dynamics; e.g. investment, components of the trade and fiscal deficits.

One of the legacies of NP was the unified modelling of the process generating many macroeconomic data. If anything, our paper reinforces this message by offering a parsimonious functional form of only 4 parameters that can model the ACF of most economic aggregates. This empirical regularity is truly impressive. Our simple functional form is rich enough to produce a variety of observed shapes. We find that most of the variables can be classified into only two broad types. The shape of the ACF of most level variables is dominated by the plateau-shape. The ACFs of the rate variables are dominated by an attenuated cycle, the original AT form providing the attenuation. Interestingly, the length of the estimated cycles matches those of the medium run cycles proposed by Comin and Gertler (2006). One feature of the data that comes in strongly when studying ACFs is the presence of a (business) cycle, whether by our method or the more standard ones.

Section 2 reviews briefly the broad developments of the topic. Section 3 presents
our estimation procedure. Section 4 applies it to macro variables and the results are compared to the traditional ones. We show how our estimation method can be augmented to incorporate checks for structural breaks and other deterministic trends. Our earlier results turn out to be robust and accurate. Section 5 considers the implications for the implementation and timing of macroeconomic policy. Section 6 concludes. The Appendix covers some technicalities relating to testing for the difference between ACFs from different models.

2. DEVELOPMENT OF THE LITERATURE

According to NP, most macroeconomic time series become stationary after differencing once, that is, they are integrated of order 1, denoted I(1). The econometric implication of NP’s result is that trends are stochastic, rather than deterministic and predictable, and that all shocks to trends are permanent. The economic implication is that the fluctuations of the business cycle can no longer be dissociated from long run growth. Another implication is to invalidate the traditional idea that the conduct of stabilization policy could be separated and had no implication for policies aimed at economic growth.

Models with an AR root close to but less than 1 still produce I(0) series, where the memory decays exponentially, whereas a unit root leads to I(1) series having infinite (permanent) memory. To bridge the gap between I(0) and I(1) models, fractionally-integrated models have been introduced and denoted by I(d) where d is not necessarily an integer; see Granger and Joyeux (1980) and Hosking (1981). A larger d indicates higher persistence (more memory), but for d < 1 the effect of
shocks to the series eventually decays, unlike when $d = 1$. Such models have been applied to macroeconomic series by Diebold and Rudebusch (1989), Baillie and Bollerslev (1994), Gil-Alaña and Robinson (1997), Chambers (1998), Michelacci and Zaffaroni (2000), Abadir, Distaso, and Giraitis (2005). The results show that $d$ is generally less than 1. However, fractionally-integrated models imply convex hyperbolic decay rates for the ACFs. They also do not allow for long cycles, because the peak of the spectrum is at the origin. Some progress has been made on this aspect by Giraitis, Hidalgo, and Robinson (2001) and Hidalgo (2005). I$(d)$ models give a good indication of the rate of decay of ACFs in the tails (distant past), but do not tell us what happens in the interim, an information that is of great interest to policymakers and that we shall be modelling in the next sections.

Apart from I$(d)$ models, there is an even larger area of support for the view that macroeconomic series can indeed be stabilized if necessary. It is a nonlinear model that was successfully introduced by Perron (1989), showing that most of these series are best represented as stationary around deterministic trends, with infrequent structural breaks in the trend. This is confirmed by recent evidence in Andreou and Spanos (2003).

Both trend and difference stationary models are linear. The two extensions that followed, I$(d)$ and breaks, tackled intermediate memory and nonlinearity, respectively. In the rest of this paper, we present a method that combines a different type of long-memory and nonlinearity in a simple yet accurate way, arising from the economic model of AT.
3. ESTIMATION PROCEDURE

There are two traditional ways to look at macroeconomic time series: the time domain and the frequency domain. Here, we introduce estimation in the related ACF domain. Many papers, including NP, have looked at autocorrelations from a descriptive perspective, but reporting them only for a few lags and not estimating their pattern. In this paper, we are going to evaluate whether an extension of the functional form in AT represents the ACF of macroeconomic times series better than traditional AR processes, even when structural breaks are allowed for.

The ACF $\rho_1, \rho_2, \ldots$ of a process $\{z_t\}_{t=1}^T$ is defined as the sequence of correlations of the variable with its $\tau$-th lag:

$$\rho_\tau := \frac{\text{cov}(z_t, z_{t-\tau})}{\text{var}(z_t) \text{var}(z_{t-\tau})},$$

where $\rho_0 \equiv 1$ and $\text{cov}(z_t, z_{t-\tau}) := \mathbb{E}[(z_t - \mathbb{E}z_t)(z_{t-\tau} - \mathbb{E}z_{t-\tau})]$. This general definition allows for a time-varying expectation of $z_t$, but without imposing any parametric structure on the evolution of $\mathbb{E}(z_t)$. This definition was also used in AT.

We begin with the simplest setup of an AR($p$) process, which we will show in (6) to also covers the random walk as a special case of the AR(1). We will also deal with adding deterministic trends and/or breaks at the end of next section. To start, consider a stationary AR($p$) process

$$x_t = a_0 + a_1 x_{t-1} + \cdots + a_p x_{t-p} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ is a sequence of IID($0, \sigma^2$) residuals. The ACF of this process is denoted
by $\rho^\text{AR}_\tau$. The first $p$ values are given by the Yule-Walker equations

$$
\begin{pmatrix}
1 & \rho^\text{AR}_1 & \rho^\text{AR}_2 & \cdots & \rho^\text{AR}_{p-1} \\
\rho^\text{AR}_1 & 1 & \rho^\text{AR}_1 & \cdots & \\
\rho^\text{AR}_2 & \rho^\text{AR}_1 & 1 & \cdots & \rho^\text{AR}_2 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\rho^\text{AR}_{p-1} & \cdots & \rho^\text{AR}_2 & \rho^\text{AR}_1 & 1
\end{pmatrix}
\begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
a_p
\end{pmatrix}
=
\begin{pmatrix}
\rho^\text{AR}_1 \\
\rho^\text{AR}_2 \\
\rho^\text{AR}_3 \\
\rho^\text{AR}_p
\end{pmatrix};
$$

(2)
e.g. see Granger and Newbold (1986). This is a linear system of $p$ equations in the $p$ values $\{\rho^\text{AR}_1, \ldots, \rho^\text{AR}_p\}$ that can therefore be determined uniquely; e.g. see Abadir and Magnus (2005). Since the system is linear, it is numerically straightforward to evaluate the first $p$ values of the ACF of the AR($p$). The remaining values are given by the recursive relation

$$
\rho^\text{AR}_\tau = a_1\rho^\text{AR}_{\tau-1} + \cdots + a_p\rho^\text{AR}_{\tau-p},
$$

(3)
for all $\tau > p$.

The alternative to the AR is an extension of the ACF functional form proposed in AT. The new ACF is

$$
\rho^\text{AT}_\tau = \frac{1 - a [1 - \cos (\omega \tau)]}{1 + b \tau^c},
$$

(4)
where we have the parameters $a$, $b$, $c$, $\omega$. The extension of the AT form into (4) is explained as follows. The model of AT showed that GDP and other variables driven by it follow a new type of long-memory process that reverts to a possibly time-varying mean. The functional form of the ACF in AT is just the denominator of (4), obtained from using only the leading term of an expansion with alternating
where this formula extends (A.3) of AT (in their notation) by means of Kummer’s function; see Abadir (1999). The subsequent derivations of AT would become intractable by taking these higher-order oscillating terms, but a heuristic approximation can be obtained by capturing their effect with the numerator of (4) subject to it tending to 1 as $\tau \to 0$ (since $\rho_0 \equiv 1$). The denominator still controls the decay of memory. The parameter $a$ controls the amplitude of the oscillations. When $a = 0$, we are back to the old form of the ACF and omitting the higher-order terms.

We want to decide which of the two models best represents the ACF data. First, we will need to start by selecting the order $p$ of the AR model for each time series under consideration. The AR model has $p$ parameters and our model has 4. Since the two models do not necessarily have the same degrees of freedom, we need to use an information criterion to determine which model fits best in the ACF domain. We use the Schwarz information Criterion (SC), which is known to be consistent. The alternatives are the Akaike criterion and the Hannan–Quinn criterion. The former was shown by Nishii (1988) to be inconsistent. The latter is designed to determine the orders $p$ and $q$ of ARMA($p,q$) processes and, since the AT process does not belong to this class, we use the broader Schwarz criterion instead. Note that one can also estimate the parameters of an AR process from the time domain. By construction, the resulting fit for the ACF would not be as good as fitting directly the ACF. Our choice to fit ACFs for both models keeps an even field for

\begin{align}
\int_0^1 \alpha^{b-1} (1 - \alpha)^{h_\alpha - 1} \exp(\nu \mu t \alpha) \, d\alpha \\
\approx \frac{\Gamma(h_\alpha)}{\nu \mu t)^{h_\alpha}} \exp(\nu \mu t) \left( 1 - \frac{h_\alpha (b - 1)}{\nu \mu t} + \frac{h_\alpha (h_\alpha + 1) (b - 1) (b - 2)}{2 (\nu \mu t)^2} - \cdots \right),
\end{align}
the comparison.

Second, the functional forms of the two ACFs are different, so the models to be fitted and compared are nonnested. This does not affect the ranking of models by SCs, but it does affect standard significance testing, e.g. to check if the SCs are significantly different for the AT and AR models. For this, we use the Vuong (1989) test \( V \) as detailed in our Appendix, such that \( V \sim N(0, 1) \) if both models are equally good, negative significant values of \( V \) if AT is better than AR, or positive significant ones if the opposite is true. The reader is referred to Brockwell and Davis (1991) where Bartlett’s formula gives the required distribution theory for empirical ACFs and their consistency as estimators of \( \rho_r \). The corresponding distributional results for the ACF parameters follows by the Delta method. This is covered in Caggiano (2006), as are resampling and subsampling approaches to estimating confidence bands for ACFs.

Given the empirical ACF of a series, we can estimate by nonlinear least squares the two ACF formulae seen earlier. We find that \( p \leq 4 \) in all the cases at hand. In the ACFs that we fitted for the AT form, it was often the case that \( a \approx 1 \), so we could have done away with one more parameter and reduced the penalty in SC to only 3 parameters instead of 4. We preferred not to do so, in order to give AR models their best shot.

Only a few data points from \( \{ z_t \} \) contribute to the calculation of the tail end of the empirical ACF. Consequently, the tail of the empirical ACF is typically very erratic and is not a reflection of the true ACF. A common practice in time series is to discard a proportion of the end lags of the empirical ACF; see for instance Box and Jenkins (1976). Here, we discard the last 1/4 of these lags and use the rest for
fitting the ACFs. This leaves plenty of data points to estimate the ACF parameters that measure the initial slope, curvature, amplitude, and first turning point (hence frequency); which can be inferred from the early part of the ACFs.

4. ESTIMATION RESULTS

4.1. Comparison of the AT and AR models.

We obtained annual data for all our macro variables from the Bureau of Economic Analysis (BEA), the Bureau of Labor Statistics (BLS), and the Federal Reserve Economic Data (FRED®). In order to minimize the possibility of error due to data manipulation, we do not splice series. Although the lack of splicing means that some of our series start later than the corresponding ones in NP, all of our series end in 2004 which adds 34 years of data over the 1970 end date in NP. With so much additional data, we had enough observations in the high quality datasets provided by the BEA, the BLS, and the FRED to conduct our analysis. This also allowed us to stick to annual data, rather than a higher frequency, so that our conclusions are in no way affected by treatments of seasonality. From these series, we calculated the real counterparts of the nominal variables, and the growth rates of the level variables. We then computed the ACFs of the logarithm of the level variables, and the ACFs of the rate variables. The programs and data files that we use are available online at:

http://www.gla.ac.uk/departments/economics/ourstaff/professorgabrieltalmain/linktothedatafileandmatlabroutines/

Table 1 presents the Schwarz criterion for the AT and the best AR model, the order $p$ of the best AR($p$), the $R^2$ of the two models, and the Vuong test of
significance between the two models. In terms of $R^2$, the fit of the AT model is always superior. When taking into account the number of degrees of freedom through the Schwarz criterion, the AT fit is superior in all cases except for the nominal money stock and bond yields, where AR has a slight advantage that is statistically insignificant according to the Vuong test. The fit of the two models for these cases is basically the same, and the AR has a better SC only because it has one parameter less than AT. However, comparing the fit for the growth rate of the money stock, AT clearly dominates AR by SC and $R^2$. Also, the AR fit for the real money stock is nowhere near as good as the AT fit. There is not a single series for which the $V$ test indicates a significantly better performance of AR than AT, whereas in almost all the cases the opposite is true, even at the 1% significance level.

This impressive fit for money is particularly striking in Figure 1, where we also report the fiscal components (government expenditure and tax), wages, and prices (CPI and GDP deflator). The blue (darker), the pink (square-marker and heavy) and the red (smooth and thin) lines depict the actual data and their fit by the AT and the AR models. We see that the fit for prices is also outstanding. From the figure, we see a broad picture emerging whereby the memory of macro variables is of neither of the two types that AR models can produce: exponential speed of decay for I(0) or approximately linear for I(1). For example, the best AR approximation for real money is basically a unit root with the implied linear ACF (clearly not the pattern displayed by the empirical ACF in the graph) and, for real wages, it is a cycle which dampens too fast because the roots of the AR are stationary. Note that the ACF of a unit-root process is $(1 + \tau/t)^{-1/2} \approx 1 - k\tau$ where $k := 1/(2t)$ is a
small constant when the process started in the distant past; see AT for details. For
a given sample, this ACF can be approximated numerically by the stationary AR’s
ACF since

\[ \alpha^r \equiv \exp \left( (\log \alpha) \tau \right) \approx 1 + (\log \alpha)\tau \]  

(6)

by the exponential expansion when \( \log \alpha \approx 0 \) (i.e. \( \alpha \approx 1 \)).

In Figure 2, we see that GDP has dynamics that are much better approximated
by AT than AR. This is true for nominal, real, and per capita GDP. The same is true
also for employment and industrial production. In the case of nominal industrial
production, we can see an unusual pattern of dynamics in the data: cycling that
persists for a long time (does not decay fast), but that starts with an early drop
in memory that misleads linear models (such as ARMA) into thinking that the
memory will continue to decay fast. This type of persistent cyclical behavior is
picked up by our ACF, but not by the ACF of the autoregressive model which
produces cyclical but stationary roots (exponentially-fast decay of memory).

Figure 3 illustrates a series that has given so much difficulty to macroeconomic
modelers, and which is not in the original NP dataset. Investment, both in nominal
and real terms, evolves along the lines suggested here, not as AR models would
imply. Notice how closely the ACF of investment resembles the ACF of nominal
industrial production seen in Figure 2, which is not surprising given the economic
link via production functions. Notice also that investment shares the same func-
tional form of ACFs as the other variables analyzed here, which is in agreement
with Barro’s (1990) finding of a common dynamics between investment and the
stock market.

Figure 4 displays common stock prices, a variable that was in the NP dataset.
Our ACFs show that there is no stochastic trend of the unit-root type, but rather a long and asymmetric cycle. The memory drops off very rapidly after some point, unlike the prediction of unit-root models. The high autocorrelation at low lags will force a root close to one when AR models are fitted. However, inspections of the ACF indicates that this is not appropriate. Our findings are in line with the results, for individual stocks, that were first noted by De Bondt and Thaler (1985, 1987, 1989).

Figure 5 contains the remainder of the ACFs from Table 1. These include components of the trade deficit, which are so eagerly followed by practitioners because of their impact on policymakers’ decisions. Again, our dynamics are much more accurate than the ones arising from ARs.

One final observation can be made. An AR(1) with a positive AR root has a globally-convex ACF, while an AR(2) or AR(3) with complex-conjugate roots has a locally concave ACF within each half-cycle (although the ACF decays at an exponential rate, hence “convexly” in the long run). This is why ACF estimation produces few AR(1) models in Table 1.

4.2. Comparison of the two models after accounting for structural breaks.

In this part, we show that our results are not an artifact of the presence of a structural break. We show that, for a dataset in which there are no structural breaks, the information criterion and Vuong test for the AT model is still better than for the AR model. We now switch to the original NP dataset which has been extensively studied. Perron (1989) did not detect any structural breaks in the period 1946-1970 for velocity, and in the period 1930-1970 for all of his other series. We apply the previous analysis to these periods. Note that it is also possible to
estimate ACFs for series with an identified break, by a procedure similar to the one to be introduced in Section 4.3. This can be done for AR and AT models, but it does not add much to the analysis here, and it was therefore omitted.

Table 2 compares the two models. We find that the AT model produces a better information criterion than AR models, for all the variables, even bond yields and the money stock. In the previous dataset (Section 3.1), the two models were hard to tell apart for these two variables. However, we now find that our model still fits very well, even better than before, while the AR fit for these two variables has worsened. As before, in terms of the Vuong test, there is no case where the AR model outperforms AT, and in most cases the opposite is true.

4.3. Comparison of the two models for data that may contain deterministic trends

It is possible to incorporate deterministic trends in the analysis. If a series is suspected of having a trend, then the data can be detrended and the procedure of Section 3.1 repeated. In addition, we can compare the models with and without trend by adjusting the penalty factor of SC when using detrended data. For example, if a simple linear trend is removed, then one more parameter is added to the penalty factor of SC. The intercept is the mean which is always estimated by definition in (1), and so it does not require an additional penalty. The comparison of models with and without trends should be in terms of SC but not $R^2$, unless $R^2$ is augmented to incorporate the trend’s contribution to the explained sum of squares (normal SC does not depend on explained sum of squares).

Table 3 compares the two models when a linear trend may be present. Variables in rates, such as unemployment rates, are excluded from this table, since their generating process cannot possibly contain a simple linear trend. The only case
where AR has a better SC than AT is for detrended log of real exports, with $-3.24 < -3.14$. However, this is a case where a model with trend is worse than a model without. This is evidenced by comparing the four SCs of real exports in Tables 1 and 3: the best of the four models is the AT without a trend, which has the best SC of $-7.20$. Incidentally, comparing the SCs of Tables 1 and 3, the only instances where accounting for a linear trend improves the AT fit (in the sense of SC) are the cases of real industrial production and real wages. Finally, the same comment about the uniformly-superior significance performance of AT relative to AR applies here, like in the previous tables.

5. IMPLEMENTATION AND TIMING OF MACROECONOMIC POLICY

This paper does not concern itself with welfare, so we cannot study directly optimal economic policy. However, our study is still helpful in the implementation of economic policy because it reveals the dynamics of macroeconomic series. Our model predicts that changes in economic policy take time to work through the system, but not in a gradual way as was previously thought: the result is seeming inertia in the direction taken by the economy, followed by a seemingly-sudden turning point. But this pattern is predictable with a good degree of confidence. Our ACFs’ patterns have been substantiated by past events and have relevance for current and future debates on the timing and magnitude of macroeconomic policy interventions. They are different from existing models that misinterpret the inertia, projecting it into the future, hence missing these sudden turns.

From the previous section, the shape of the ACF of level variables (such as GDP) indicates that any impulse will decay only very slowly until the end of the ACF’s
plateau is reached, and that the course of these variables takes a long time to alter. Hence, economic policy should be guided by the long lags over which it operates. For instance, if the size of an economic intervention is enough to turn around GDP quickly, the momentum imparted to it will lead to a period of overheating. Likewise, an economic policy that imparts, period after period, a stimulus to the economy will eventually build up momentum. Therefore, if a policy intervention is needed to counter the signs of a slowdown, it should:

1. occur as soon as possible to give time to the policy to operate;

2. impart a stimulus sufficient to achieve the objective, taking into account the increments that will keep occurring afterwards due to inertia; and

3. revert to a neutral stance well before the objective is achieved, letting the economy ease onto its intended path.

This is particularly relevant to the debate about the current stimulus packages and their aftermaths, in addition to the earlier debate about policy responses to recessions.

Interestingly, a number of recent policy oriented papers have advocated policies which react promptly to new information; see Mishkin (1999), Clarida, Galí and Gertler (1999), Bernanke and Gertler (2001). Similarly, recent speeches from Fed governors have started to favour the recommendations that we enumerated earlier; e.g. see Mishkin (2008) on the observed nonlinear macroeconomic dynamics and Bernanke (2008) on the sudden turning points in the economy and the need for quick reactions. Since the end of 2007, Fed actions have been more aggressively expansionary to counter the threat of a recession, and our recommendations show
that this is the right course of action but they also warn of overheating in the aftermath.

Mishkin (2007), speaking from an empirical perspective, stresses that “what drives many macroeconomic phenomena that are particularly interesting is heterogeneity of economic agents”. It is worth recalling that our new ACF’s functional form arose from solving explicitly a general equilibrium model with heterogeneous agents.

6. CONCLUDING COMMENTS

By extending AT’s ACF beyond the 1-term asymptotic approximation, we are able to find a common parsimonious functional form for the ACFs of macroeconomic and aggregate financial time series. Its fit is impressive and does at least as well as AR models, usually substantially much better than them. We do robustness checks and show that the fit of this functional form is not an artifact of omitted deterministic trends, including possibly breaking ones.

There are two dominant features of our fitted ACFs: a plateau and drop off (already noted in AT) and a decaying cycle. The first feature tends to dominate the ACF of level series, the second those of rate series (such as interest rate, growth rate, etc.). It is worth noting that, even among AR processes, the ACF approach favours in many cases cyclical (those including some complex roots) AR processes, a feature that has not been prominent in the time-domain empirical literature. We discuss the implications of our ACFs’ findings for policymaking, and the problems that arise from using models with the wrong ACF. Our functional form allows
an early detection of shocks and the determination of their future impact on the economy.

By design, this enquiry into the nature of the individual dynamics of macroeconomic and aggregate financial series is a first step. Our single-variable analysis has revealed that these series are not of the integrated type. The next step in our endeavour will be to model the co-movement of such series that are not integrated, hence not co-integrated (even though they are co-moving).

APPENDIX

The errors in the estimation of an ACF \(\{\rho_\tau\}\) by a correlogram are, in finite samples, correlated and heteroskedastic for various \(\tau\). By the consistency of the correlogram as an estimator of the ACF, we will omit these non-i.i.d. features in the asymptotic analysis below at the cost of a loss in finite-sample efficiency that arises from using OLS as opposed to GLS. Note that this applies to both the AR and AT models, which are being fitted to the correlogram as opposed to the true ACF.

We will use the Fisher-transformed

\[
 u_t := \tanh^{-1}(v_t/2),
\]

where \(v_t \in (-2, 2)\) are the estimation errors by the models, and we assume that they are symmetrically-distributed around 0. The transformation is known to improve inference for correlation coefficients, which are bounded in \((-1, 1)\), by mapping them into \(\mathbb{R}\) and making their distribution closer to the limiting one; e.g. see Muirhead.
(1982). The joint quasi likelihood for the sequence \( \{ u_t \} \) is
\[
L := \prod_{t=1}^{T} \frac{\exp\left(-\frac{u_t^2}{2\sigma^2}\right)}{\sigma \sqrt{2\pi}} = \exp\left(\frac{-\frac{1}{2\sigma^2} \sum_{t=1}^{T} u_t^2}{\sigma^T (2\pi)^{T/2}}\right),
\]
because \( \tanh^{-1} \) being an odd function and \( v_t \) being symmetric around 0 imply
\[
E(\tanh^{-1}(v_t/2)) = E(\tanh^{-1}(-v_t/2)) \equiv -E(\tanh^{-1}(v_t/2)) \implies E(\tanh^{-1}(v_t/2)) = 0.
\]
The maximized quasi likelihood is
\[
\hat{L} := \frac{\exp\left(\frac{-1}{2\hat{\sigma}^2} \sum_{t=1}^{T} \hat{u}_t^2\right)}{\hat{\sigma}^T (2\pi)^{T/2}} = \exp\left(\frac{-T}{2}\right) \propto \hat{\sigma}^{-T},
\]
where \( \hat{\sigma}^2 := T^{-1} \sum_{t=1}^{T} \hat{u}_t^2 \). Notice that this MLE of \( \sigma^2 \) does not require de-meaning of \( \{ \hat{u}_t \} \).

We will write \( \hat{\ell}_{\text{AR}} \) and \( \hat{\ell}_{\text{AT}} \) to denote the maximized quasi log-likelihoods obtained for the AT and AR methods, respectively; and similarly for \( \hat{\sigma}_{\text{AR}}, \hat{\sigma}_{\text{AT}} \) and \( \hat{\sigma}_{\text{AR}}^2, \hat{\sigma}_{\text{AT}}^2 \). The test of Vuong (1989) is
\[
V := \frac{Tl + (4-p) \log T}{2s \sqrt{T}},
\]
where \( l := \log(\hat{\sigma}_{\text{AT}}^2/\hat{\sigma}_{\text{AR}}^2) \) and
\[
s^2 := \frac{1}{4T} \sum_{t=1}^{T} \left( \frac{\hat{u}_{t,\text{AR}}^2}{\hat{\sigma}_{\text{AR}}^2} - \frac{\hat{u}_{t,\text{AT}}^2}{\hat{\sigma}_{\text{AT}}^2} - l \right)^2 = \frac{l^2}{4} = \frac{1}{4T} \sum_{t=1}^{T} \left( \frac{\hat{u}_{t,\text{AR}}^2}{\hat{\sigma}_{\text{AR}}^2} - \frac{\hat{u}_{t,\text{AT}}^2}{\hat{\sigma}_{\text{AT}}^2} \right)^2.
\]
We have \( V \sim N(0, 1) \) if both models are equally good, negative significant values of \( V \) if AT is better than AR, or positive significant ones if the opposite is true.
ACKNOWLEDGEMENTS

We thank, for their comments, seminar participants at the Bank of England, Bank of Italy, Federal Reserve Board (Washington DC); at Boston U., Cambridge, Cardiff, Essex, Glasgow, Marseille, National Taiwan U., Queen Mary, Rochester, St Andrews, U. Michigan, U. Pennsylvania, U. Zürich; at the ESRC Workshop on Nonlinearities in Economics and Finance (Brunel), European Meeting of the Econometric Society (Vienna), Far Eastern Meeting of the Econometric Society (Beijing), Imperial College Financial Econometrics Conference (London), Knowledge, Economy, and Management Congress (Kocaeli), London-Oxford Financial Econometrics Workshop (London), Macromodels International Conference (Zakopane), Society for Nonlinear Dynamics and Econometrics Meeting (St Louis), Symposium on Financial Modelling (Durham). We are grateful for ESRC grants RES000230176 and RES062230790.

REFERENCES


Table 1. Comparison of the AT and AR models.

<table>
<thead>
<tr>
<th>In NP? Series</th>
<th>Schwarz criterion</th>
<th>$R^2$</th>
<th>Vuong statistic</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$T$ $n$</td>
<td>AT</td>
<td>AR</td>
</tr>
<tr>
<td>Yes GDP (nom.)</td>
<td>76 55</td>
<td>-11.93</td>
<td>-8.66</td>
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<td>Yes GDP (real)</td>
<td>76 55</td>
<td>-10.27</td>
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<td>Yes GDP per capita (nom.)</td>
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<td>-11.54</td>
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<td>Yes Industrial production (nom.)</td>
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<td>No Industrial production (real)</td>
<td>76 55</td>
<td>-3.57</td>
<td>-2.80</td>
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<td>Yes Employment</td>
<td>57 41</td>
<td>-11.06</td>
<td>-10.59</td>
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<tr>
<td>Yes Unemployment rate</td>
<td>57 41</td>
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<td>Yes GDP deflator</td>
<td>76 55</td>
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<td>-6.58</td>
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<td>-6.57</td>
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<td>Yes Velocity</td>
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<td>76 55</td>
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<td>-8.34</td>
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<td>-7.80</td>
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<td>No Current tax receipts (real)</td>
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<td>-7.14</td>
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<td>No Inflation (growth rate of CPI)</td>
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<td>-2.44</td>
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<td>No Growth rate of wages (real)</td>
<td>40 28</td>
<td>-3.72</td>
<td>-3.19</td>
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</table>

Note: $T$ is the sample size, $n$ is the number of ACF lags used for fitting, and $p$ is the number of AR lags selected by SC. The Vuong statistic is the likelihood ratio based statistic proposed by Vuong (1989), to test whether the AT or AR models are closer to the truth. A negative number indicates that the Vuong test rejects the AR model. Vuong showed that the asymptotic distribution of this statistic is an $N(0, 1)$:

* significant at the 10% level
** significant at the 5% level
*** significant at the 1% level
Table 2. Comparison of the AT and AR models, time spans in which no structural breaks were detected.

<table>
<thead>
<tr>
<th>Series</th>
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<th>Vuong statistic</th>
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<tbody>
<tr>
<td></td>
<td>AT model</td>
<td>AR model</td>
</tr>
<tr>
<td>GDP (nominal)</td>
<td>42 29</td>
<td>-7.29</td>
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<tr>
<td>GDP (real)</td>
<td>42 29</td>
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<td>42 29</td>
<td>-5.89</td>
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<tr>
<td>Industrial production (nominal)</td>
<td>42 29</td>
<td>-6.99</td>
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<tr>
<td>Employment</td>
<td>42 29</td>
<td>-6.27</td>
</tr>
<tr>
<td>GDP deflator</td>
<td>42 29</td>
<td>-5.66</td>
</tr>
<tr>
<td>Consumer prices (CPI)</td>
<td>42 29</td>
<td>-5.41</td>
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<tr>
<td>Wages (nominal)</td>
<td>42 29</td>
<td>-7.80</td>
</tr>
<tr>
<td>Wages (real)</td>
<td>42 29</td>
<td>-8.25</td>
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<tr>
<td>Money stock (nominal)</td>
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<td>-8.86</td>
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<tr>
<td>Velocity</td>
<td>25 17</td>
<td>-5.23</td>
</tr>
<tr>
<td>Bond yield (nominal)</td>
<td>42 29</td>
<td>-5.54</td>
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Note: $T$ is the sample size, $n$ is the number of ACF lags used for fitting, and $p$ is the number of AR lags selected by SC. For the Vuong statistic, see the note to table 1.

Table 3. Comparison of the AT and AR models, detrended data.

<table>
<thead>
<tr>
<th>Series</th>
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<tr>
<td></td>
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<tr>
<td>GDP (nom.)</td>
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<td>Industrial production (real)</td>
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<td>-2.80</td>
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<tr>
<td>Employment</td>
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<tr>
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<td>-2.51</td>
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<tr>
<td>Wages (nom.)</td>
<td>-7.10</td>
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<td>Wages (real)</td>
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<td>Money stock (nom.)</td>
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<td>Money stock (real)</td>
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<td>-2.17</td>
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<tr>
<td>Velocity</td>
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<td>-3.48</td>
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<tr>
<td>Stock prices (S&amp;P 500) (nom.)</td>
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<td>-3.95</td>
</tr>
<tr>
<td>Investment (nom.)</td>
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<td>Investment (real)</td>
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<td>-2.19</td>
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<tr>
<td>Exports (nom.)</td>
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<tr>
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<td>Government expenditures (real)</td>
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<td>-2.07</td>
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<td>Current tax receipts (nom.)</td>
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<td>-3.01</td>
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<tr>
<td>Current tax receipts (real)</td>
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<td>-4.23</td>
</tr>
</tbody>
</table>

Note: $n$ is the number of ACF lags used for fitting, and $p$ is the number of AR lags selected by SC. For the Vuong statistic, see the note to table 1.
Figure 1. Actual ACFs, and their fits by AT and AR models: real government expenditure and tax, real money, real wages, CPI, and GDP deflator.
Figure 2. Actual ACFs, and their fits by AT and AR models:

GDP (real, nominal, and real per capita), employment,
and industrial production (real and nominal).
Figure 3. Actual ACFs, and their fits by AT and AR models: investment (real and nominal).

Figure 4. Actual ACF and its fit by AT and AR models: S&P 500.
Figure 5. Actual ACFs, and their fits by AT and AR models: all the other variables.