The welfare consequences of irrational exuberance: Stock market booms, research investment, and productivity

Michał Jerzmanowski a,*, Malhar Nabar b

a Department of Economics, Clemson University, 221B Sirrine Hall, Clemson, SC 29634, United States
b Department of Economics, Wellesley College, 106 Central Street, Wellesley, MA 02481, United States

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Abstract

This paper studies the effects of stock market valuation on research investment, the rate of innovation, and welfare. In the presence of financing constraints for R&D investment, episodes of high market valuation can ease these constraints and raise the economy-wide investment in R&D and the rate of innovation. If the decentralized equilibrium rate of innovation is inefficiently low, then such episodes may lead to an increase in aggregate welfare even if the higher valuation is not entirely justified by fundamentals. We present a Schumpeterian-style growth model with a costly financial intermediation process to characterize the relationship between market value, entry of new firms, and the aggregate rate of innovation. We use the model to measure the welfare consequences of a stock market run-up that may only partly be justified by fundamentals. In particular, we apply the model to the US economy in the 1990s and calibrate the impact of the NASDAQ boom on the rate of innovation, growth and welfare. The welfare effect depends on the underlying change in fundamentals. We find that with an acceleration in US trend productivity growth from a pre-1995 rate of 1.4% to a rate of 2.0% per annum, the NASDAQ boom will have resulted in a net welfare gain of 0.55%. If the new growth rate is as high as 3%, the net gain was 1.35% of the present discounted value of consumption.

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* Corresponding author. Tel.: +1 864 656 0551; fax: +1 864 656 4192.
E-mail addresses: mjerzma@clemson.edu (M. Jerzmanowski), mnabar@wellesley.edu (M. Nabar).
1. Introduction

Episodes of run-ups in asset prices are often thought to be associated with costly misallocations of resources. However, when the assets subject to the boom are claims on innovating firms, the aggregate welfare effect could be positive in net terms if the run-up in asset prices eases financing constraints on new research projects and raises the economy-wide rate of innovation and growth. In this paper, we provide a method for evaluating the welfare consequences of such episodes. In an application of the method to the NASDAQ boom of the 1990s, we show that the aggregate welfare effects may have been positive even if much of the optimism about the “new economy” was unjustified.

Since the NASDAQ episode involved a rapid run-up in the value of innovating firms, we employ an innovation-based endogenous growth model to study its welfare consequences. In order to establish a link between asset prices and the supply of research (R&D) funding, we incorporate imperfections in the market for research funding that restrict entry into the research sector. The cost of the boom is clearly the over-investment relative to what would have taken place based on a correct perception of fundamentals in the decentralized equilibrium. However, there is also a potential bright side. Endogenous growth models exhibit externalities associated with knowledge and usually imply that the decentralized level of innovation is below the social optimum (Aghion and Howitt, 1998; Jones and Williams, 2000). The aggregate welfare effect depends on whether the investment in research and the growth rate were suboptimal prior to the run-up, and the extent to which the increase in asset prices leads to the entry of new firms, more innovation and faster growth. We calibrate the model to the path of the NASDAQ index over the period 1995–2000 and examine whether, given a realistic degree of under-investment in (and externalities associated with) R&D, the run-up in the prices of innovating firms improved aggregate welfare by raising research investment closer to the socially-optimal rate.

Not surprisingly, the welfare consequences of the surge in R&D financing turn out to depend on the impact that innovation has on the trend growth rate of labor productivity. There is growing evidence that the trend growth rate of productivity in the US has increased as a result of new IT-related technologies (Oliner and Sichel, 2003; Gordon, 2004), but there is uncertainty about the size of this increase. Given this uncertainty, different scenarios for the increase in the trend growth rate are considered. For each scenario presented in the simulation, we separate the part of the run-up consistent with the fundamentals for that scenario from the part which is investment over and above this level (driven by over-optimistic expectations) and we evaluate the costs and benefits of this over-investment component of the run-up.

1 For example, Rajan and Zingales (2003) note that the stock market bubble of the 1990s may have been quite costly due to excessive investment, especially by the telecommunications sector. They cite examples such as the Finnish company Sorena spending over $30 billion on a third generation mobile telephone license only to give it away for free later, or the enormous overcapacity in transatlantic bandwidth left behind by the investment boom. They conduct some “back-of-the-envelope” calculations which suggest almost half a trillion dollars were lost to over-investment and conclude that: “[R]esources can be grossly misallocated even in countries with developed financial markets”.

2 Olivier (2000) demonstrates the theoretical possibility of the growth-enhancing effects of bubble on productive assets formally in an endogenous growth model. Our emphasis here is on providing a quantitative assessment of the welfare consequences of the NASDAQ episode of the 1990s.
Pre-1995, the trend productivity growth rate for the US is estimated to be 1.36% per annum during 1970–1991 and 1.54% per annum during 1991–1995 (Oliner and Sichel, 2003). The model is therefore simulated assuming a growth rate of labor productivity of 1.40% prior to the bubble. The analysis finds that with an acceleration in the growth rate of productivity from 1.40% to a rate of 2% per annum, the NASDAQ boom will have resulted in a net benefit of 0.55% of the present discounted value of consumption. If the new growth rate is as high as 3%, the bubble will have resulted in a net gain of 1.35% of the present discounted value of consumption. This is not a very large number although it is much larger than, for example, Lucas’ estimate of welfare gains from elimination of business cycles (Lucas, 2003). Perhaps the sign of the result is more important than its magnitude because it shows that the NASDAQ episode, even if it was based in part on “irrational exuberance”, may in fact have been welfare-improving. This has important implications for the optimal policy response to a stock market bubble.

Three recent papers that have studied the relationship between the stock market boom, economic expansion and the productivity revival are Jermann and Quadrini (in press), Caballero et al. (2006), and Beaudry and Portier (2003). These papers study similar questions to ours and are complementary in some ways to our analysis. However, our focus and methodology differ substantially from these other studies.

Both Jermann–Quadrini and Caballero et al., provide expectations-based explanations for the events of the late 1990s. Beaudry and Portier (2003) focus on the importance of news about future technological opportunities – shocks that are correlated with productivity in the long run but may not be in the short run – as a force behind short run fluctuations.

In contrast to Jermann–Quadrini and Caballero et al., we do not set out to provide an expectations-based account of the NASDAQ episode. Furthermore, although our model studies how an upward shift in expected profitability of research affects stock market valuation and the financing of innovation, unlike Beaudry–Portier, we are not interested in studying the implications of this shock for short run fluctuations. Instead, we provide a framework for evaluating the welfare consequences of the late 1990s.

The theme of the paper is also related to Datta and Dixon (2002) and Michelacci and Suarez (2004) who look at the role of the stock market in encouraging the creation of businesses and the financing of innovation. While the current paper develops an alternative framework for thinking about these relationships, its main contribution is to provide a method of welfare analysis that resolves the tension between, on the one hand, the benefits of an investment run-up in activities with positive economy-wide spillovers and, on the other, costly over-investment beyond a level justifiable by the fundamentals.

The paper is organized as follows. Section 2 presents the theoretical model and characterizes the steady state. Section 3 discusses in more detail how a stock market run-up may affect the rate of innovation and welfare. Section 4 outlines the calibration exercise and Section 5 presents the quantitative results. The last section concludes.

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3 These results hold under the baseline specification. As discussed below, we also present results for alternative specifications that differ in their degree of “under-provision” of R&D in the decentralized equilibrium. The extent of the under-provision affects the welfare effect of the boom.
2. Model

The model is a modified version of the multisector quality-improvement model of Aghion and Howitt (1992, 2005). The major modification is the introduction of frictions in the financing of R&D. The frictions ensure that the supply of research capital is imperfectly elastic. Hence, in response to a positive shock to investors’ expectations regarding the profitability of research investment, higher demand for research capital raises both the market price of existing capital and the volume of new investment, generating a positive association between the two. In the absence of these frictions, increases in demand for research capital would simply lead to a higher volume of research investment, leaving the price of installed research capital unchanged.

2.1. Consumers

Consumers are infinitely lived. The lifetime utility for a representative household is given by

\[ U_0 = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} c_t. \]

This implies that in every time period along the optimal consumption path, the interest rate in this economy adjusts to equal the exogenous rate of time preference \( \rho \). Utility maximization leads to a perfectly elastic supply of loanable funds at \( r = \rho \) and the equilibrium rate of investment is determined in each time period by the demand for loanable funds at that interest rate.

2.2. Production

Final output is produced by perfectly competitive firms using \( m \) varieties of intermediate goods and labor according to

\[ Y_t = \left( \sum_{i=1}^{m} A_{it} x_{it}^a \right) \left( L/m \right)^{1-a}, \]

where \( A_{it} \) is the productivity of the latest vintage of intermediate good \( i \), \( x_{it} \) is the amount of good \( i \) used in production in period \( t \), and \( L \) is the total supply of labor in the economy. Labor is assumed to be supplied inelastically in a competitive market.

Technological progress takes the form of improvements in the productivity of intermediate goods. These increases are random with the probability dependent on research effort. All \( m \) sectors are symmetric and thus the equilibrium research effort will be identical. However, because of the random nature of innovation, at any point in time the sectors may differ in the level of productivity \( A_{it} \). A successful innovator supplies the latest vintage of intermediate good \( i \) until she is replaced with a more productive version. Intermediate good monopolists produce using capital and a linear technology

\[ x_{it} = K_{it}/A_{it} \equiv k_{it}, \]

reflecting the increasing capital requirement for more sophisticated (newer vintage) intermediates. The monopolist’s marginal cost is thus \( A_{it} \xi \), where \( \xi = \rho + \delta \) is the time-invariant rental rate.
We assume that there exists an imitation technology which allows competition to supply the latest vintage of intermediate good at marginal cost \(1/\epsilon\) times greater than that of the original innovator, where \(\epsilon < 1\). The monopolist chooses the price \(A_{it}/\epsilon\) so as to eliminate fringe competitors. Using the inverse demand for intermediate good \(i\),

\[ aA_{it} x_{it} \left( \frac{L}{m} \right)^{1-a} \]

we can show that equilibrium output for all intermediate sectors is constant \(x_{it} = k\), and that profits \(\pi_{it}\) are proportional to the productivity level \(A_{it}\). Furthermore, using \(x_{it} = k\) for all sectors and for all time periods, we have

\[ Y_t = \left( \sum_{i=1}^{m} A_{it} x_{it} \right) \left( \frac{L}{m} \right)^{1-s} = mk^s \left( \sum_{i=1}^{m} A_{it} L^{1-s} = m\bar{A}_t k^s \left( \frac{L}{m} \right)^{1-s} , \right. \]

where \(\bar{A}_t\) is the average level of productivity across sectors.

In what follows we will therefore describe the equilibrium in a generic sector \(i\) and later appeal to the Law of Large Numbers to describe the entire economy as the expected value of sector \(i\).

2.3. R&D sector

Improvements in the quality of intermediate goods occur through successful research. There is a separate research sector for each intermediate good. At any point in time there exist many potential projects, or ideas, on improving the current vintage of the intermediate good. Specifically, we assume that the number of potential research projects (ideas) is proportional to the population and evenly distributed across sectors – there are potentially \(L/m\) projects in sector \(i\) at every moment in time. Individuals with ideas need to obtain funding for their projects before they can be researched. Each funded research project incurs a constant research cost per period \(\mu_{it}\) in units of final output and has a fixed probability of successfully designing a new generation of intermediate goods. Each new generation of intermediate goods has productivity level \(c\) times higher than the previous generation. That is \(A_{G+1} = c A_G\), where \(G\) denotes the generation of intermediate goods and \(c > 1\).

Success in research follows a Poisson process with the arrival rate \(\lambda\), i.e. in every period a research project with funding has a flow probability \(\lambda\) of resulting in an innovation. This arrival rate is the same in every sector. The expected rate of innovation in sector \(i\) is \(\lambda N_i\), where \(N_i\) is the number of research firms in sector \(i\) and the average growth rate of the sector is given by \((c-1)\lambda N_i\).

2.4. Frictions in the financing of innovation

Frictions in the financing of research – and hence the rationing of credit to research projects looking for finance – arise on account of asymmetric information between individual investors and entrepreneurs/innovators.\(^4\) As a result, specialized financial interme-

\(^4\) There is a vast literature, going back to Fazzari et al. (1988), that documents the effect of financing frictions on investment. In particular, Himmelberg and Petersen (1994) find that due to frictions in raising external finance for research, there are significant cash flow and retained earnings effects on R&D investment in a panel of 179 small firms in high-tech industries, matching the profile of firms that deliver a high proportion of innovations in the US. Mulkay et al. (2000) compare investment in R&D and “ordinary” investment for samples of US and French manufacturing firms. In both countries, they find that R&D investment is affected by cash flow and internal finance in a similar manner to ordinary investment, pointing to constraints in raising external finance for R&D.
diaries such as venture capital (VC) firms play a prominent role in the financing of innovation.

In our model, potential innovators must obtain funding from VC firms in order to initiate research on their projects. This financing is not modeled here explicitly. Instead it is assumed that an appropriate match between a VC partnership and a potential innovator involves costly search. The motivation for this assumption comes directly from observations about the nature of the VC market which provides funds to young and start-up R&D firms. Due to informational asymmetries, a VC firm will only be willing to finance a project if it can evaluate its potential and monitor its progress. VC firms differ in their characteristics (possibly due to differences in expertise or areas of specialization) and therefore not every VC firm will be appropriate to evaluate and finance a given project. A search and matching framework is adopted to conveniently summarize these frictions. Thus VC firms must search for profitable investment opportunities matching their evaluation and monitoring capabilities. It is assumed that VC firms are sector specific, i.e. they must search among the potential projects in their sector. This search is costly. In particular, every VC firm without a research project but looking for one incurs a flow cost, for example a direct cost of screening projects, equal to $A_{it}$. We assume that this cost increases with the complexity of the projects ($A_{it}$).

Firms and research projects are matched according to the following matching function:

$$M_{it} = F_{it}^\phi (\eta L/m)^{1-\phi},$$

where $F_{it}$ is the number of VC firms searching for projects to fund in period $t$ in sector $i$, $\eta$ captures both the availability of projects for the given population size and the efficiency of the search technology (perhaps a function of communication infrastructure and agglomeration) and $M_{it}$ is the number of matches that occur in sector $i$.

We assume that if a researcher–financier match successfully innovates it breaks up, i.e. the project is exhausted. We also assume that a constant fraction $\zeta$ of research firms go out of business every period. Thus the number of R&D firms on average evolves according to

$$N_{it+1} = (1 - \zeta - \lambda)N_{it} + M_{it}.$$  

In other words, the number of R&D firms in sector $i$ increases by the number of new matches and decreases by the exit of firms that have made an innovation (on average $\lambda N_{it}$) and by the exogenous break-up of unsuccessful matches. By the Law of Large Numbers, the above equation describes the dynamics of the aggregate number of firms. At the sector level, however, the actual number of firms will vary slightly depending on how many firms successfully innovate. In what follows we approximate each sector around a path where the number of research firms is equal to the average.

A successful innovator gains the monopoly rights to supply the new vintage of intermediate good $i$. This monopoly right is permanent in principle but in practice ends when a new innovation replaces the current generation of intermediates. The value of these monopoly rights in period $t$ satisfies the following arbitrage equation

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5 The term venture capital is used for emphasis although it should be interpreted as referring to a broader spectrum of financial intermediaries involved in financing innovation.

6 When multiple firms innovate the assignment of monopoly rights is random.
\[ V(A, N) = \pi + \frac{1}{1 + \rho} \{(1 - \lambda N) V(A, N_{t+1})\}, \]

where as above \(\pi\) is the flow profit for an incumbent monopolist in sector \(i\).

The term in brackets follows from the fact that if there is an innovation (an event that occurs with probability \(\lambda N\)) the value of the previous vintage of technology is zero, and if there is no innovation, the incumbent continues as the monopoly supplier of the intermediate good and the technology index remains at \(A\). Dividing through by \(A\) and denoting the productivity-adjusted values by a tilde, we have

\[ \tilde{V}(N) = \tilde{\pi} + \frac{1}{1 + \rho} \{(1 - \lambda N) \tilde{V}(N_{t+1})\}, \]

where \(\tilde{\pi}\) is the time-invariant productivity-adjusted flow profit.

Similarly, using the fact that \(A_{t+1} = \gamma A\) when an innovation occurs, the productivity-adjusted values of a financial intermediary firm searching for a research project in sector \(i\), \(S_i\), and a firm with a research project, \(J_i\), are given by

\[ \tilde{S}(N) = -\kappa + \frac{1}{1 + \rho} \left\{ \frac{M}{F} \tilde{J}(N_{t+1}) + \left(1 - \frac{M}{F} \right) \tilde{S}(N_{t+1}) \right\}, \]

\[ \tilde{J}(N) = -\mu + \frac{1}{1 + \rho} \left\{ \lambda \gamma \tilde{V}(N_{t+1}) + \lambda N \gamma \tilde{J}(N_{t+1}) + (1 - \lambda - \gamma - \lambda N) \tilde{J}(N_{t+1}) \right\}. \]

Both equations are simple arbitrage pricing formulas.\(^7\) Eq. (8) says that a firm searching for a research project has to pay a flow cost of \(\kappa\) and has a probability \(\frac{M}{F}\) of being matched with an appropriate research project, which has a value of \(\tilde{J}(N_{t+1})\). With complementary probability the firm remains in searching state in the next period.\(^8\) Eq. (9) says that a research firm has a flow probability of making an innovation that gives it the property rights to the next vintage of intermediate good, worth \(\tilde{V}(N_{t+1})\). With probability \(\lambda N\) some other research firm in sector \(i\) innovates, the research firm starts working on the next innovation, and its value becomes \(\tilde{J}(N_{t+1})\). It is also possible that no innovation occurs in sector \(i\) and the firm continues pursuing the same research project. In this case, its value is \(\tilde{J}(N_{t+1})\). The research firm also incurs flow research cost \(\mu\) per time period. Note that without adjusting for productivity the research cost is \(\mu A\), i.e. we assume it is proportional to the complexity of the research project as captured by the productivity level.

We assume free entry into the venture capital sector. This implies that the equilibrium value of a searching firm must be zero, i.e. \(\tilde{S}(\cdot, \cdot) = 0\). Using Eq. (8) this implies

\[ \frac{M}{F} = \frac{(1 + \rho) \kappa}{\tilde{J}(N_{t+1})}. \]

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\(^{7}\) Note that we approximate by omitting any possibilities that require simultaneous events with small probabilities. For example, a searching firm could find a match at the same time as a new innovation occurs in the economy. The probability of that is equal to \(\frac{M}{F} \times \lambda N\), which is second order relative to other terms in the equation. This is done to simplify exposition and does not alter the main properties of the model. The derivation of equilibrium conditions with the omitted terms is available upon request. When solving the model numerically and simulating it, we use all the terms.

\(^{8}\) In this formulation, matched VC firms appropriate the entire surplus from any future research successes. An alternative formulation would allocate a fraction of the surplus to VC firms as an outcome of a bargaining process. However, this would create another parameter for us to track in the calibration without adding further insights to the analysis. We therefore stick to the current formulation.
Using the matching function (4) we can rewrite this condition as

\[ M_i = \eta(L/m) \left[ \frac{\tilde{J}(N_{i+1})}{(1+\rho)\kappa} \right]^{\frac{1}{\gamma-1}}. \]  

(11)

Eq. (11) shows the key link of the model. Entry into the business of funding R&D in sector \( i \) depends on the expected value of sector \( i \) research firms, \( \tilde{J}_i \). It follows that in response to a stock market boom more funding firms will enter. In turn the matching process will relax the financing constraints and generate a higher inflow of R&D firms, resulting in an acceleration of technological progress in sector \( i \). If the stock market boom is economy-wide (i.e. affects all sectors), then technological progress will accelerate in all sectors.

Conditions (5), (7), (9) and (11) determine the equilibrium of the economy. Formally,

**Definition:** The equilibrium is a sequence of allocations \( \{c_i\}_{i=0}^\infty, \{k_{it}, x_{it}, F_{it}, N_{it}\}_{i=0}^\infty \) for \( i = 1, \ldots, m \) and prices \( \{\xi_t, p_t\}_{i=0}^\infty, \{V_{it}, J_{it}, S_{it}\}_{i=0}^\infty \) for \( i = 1, \ldots, m \) such that individuals maximize consumption, producers of intermediate goods maximize profits, the capital market clears, for each sector \( i = 1, \ldots, m \) arbitrage conditions (7)–(9) determine the entry of venture capital firms, the number of research firms is determined by (5) and an intertemporal budget constraint is satisfied.

2.5. Steady state

The steady state in sector \( i \) is defined as one in which the productivity-adjusted values of the incumbent monopolist and research firm remain constant. In other words, the unadjusted values grow at the same rate as labor productivity (i.e. with each innovation that occurs in steady state, they get scaled up by the factor \( \gamma \)).

The steady state number of research firms is constant and is given by the fixed point of (7) and (9). Thus, from the dynamic Eq. (5), in steady state we have

\[ N_{is} = \frac{M_{is}}{\tilde{\lambda} + \tilde{\lambda}}. \]  

(12)

The average steady state growth rate of productivity will be \( \tilde{\lambda} N_{is}^{\gamma}(\gamma - 1) \).

We can solve Eq. (7) for the steady state value of the incumbent monopolist

\[ \tilde{V}_{is} = \frac{(1 + \rho)\tilde{\pi}}{\rho + \tilde{\lambda} N_{is}^{\gamma}}. \]  

(13)

This simply states that the value of the blueprint of the latest innovation is the discounted profit stream that this innovation is expected to generate. The profit stream is an increasing function of the capital stock. The relevant financial discount rate is the obsolescence-adjusted interest rate (Aghion and Howitt, 1992), which is equal to the risk free interest rate \( \rho \) plus the probability that the current vintage will be replaced by the next innovation, \( \tilde{\lambda} N_{is}^{\gamma} \). This also shows that with a larger number of research firms working on the next

\[ \gamma \]

To ensure that the consumers’ maximization problem is well-defined we make the standard assumption \( \gamma > 1 \), i.e. that the growth rate does not exceed the discount rate.
innovation, the expected duration as incumbent monopolist is shorter and the blueprint of the current leading technology becomes less valuable.

Similarly, we can solve for the steady state value of a research firm

$$J_{s}^{i} = \frac{\lambda (\gamma V_{s}^{i} - J_{s}^{i}) - (1 + \rho)\mu}{\rho + \chi + \lambda N_{s}^{i} - \gamma \lambda N_{s}^{i}}.$$ 

By assumption, the denominator of the above expression is always positive. The value of a research firm is the discounted value of the research project when it is successful minus the value of the project in research. The probability of success is \(\lambda\). The discount rate is slightly different than in the equation for \(V_{s}^{i}\). In fact, because \(c > 1\), the discount rate may be less than the risk free rate. This is a consequence of research spillovers. Each time another firm makes an innovation a research firm switches to working on the next generation of technology, which is more productive. Therefore, the number of research firms decreases the discount rate in proportion to the expected productivity increase, \(\gamma \lambda N_{s}^{i}\). On the other hand, the exogenous probability of break-up \(\chi\) increases the discount rate.

The steady state flow of new R&D firms follows from Eq. (11):

$$M_{s}^{i} = \eta(L/m) \left[ \frac{J_{s}^{i}}{(1 + \rho)\kappa} \right] ^{\frac{1}{\phi}}. $$ 

The steady state values are defined by Eqs. (12)–(15). These relationships can be summarized by combining Eq. (12) with (15) and (13) with (14) to obtain the following system of two equations:

$$N_{s}^{i} = \eta L/m \left[ \frac{J_{s}^{i}}{(1 + \rho)\kappa} \right] ^{\frac{1}{\phi}}, $$ 

$$J_{s}^{i} = \frac{(1 + \rho)\gamma \lambda N_{s}^{i} / (\rho + \lambda N_{s}^{i}) - (1 + \rho)\mu}{\zeta - \lambda N_{s}^{i}(\gamma - 1)}, $$

where \(\zeta = \rho + \lambda + \chi\).

Eq. (FE) is the steady state version of the relationship between stock market valuation of existing research firms in sector \(i\), \(J_{i}\), and the entry of new research firms \(M_{i}\) (and thus their steady state number \(N_{i}\)). The (FE) relationship is upward sloping in the \((J_{s}^{i}, N_{s}^{i})\) space, reflecting the fact that higher valuation of existing firms induces entry.

Eq. (JJ) is the steady state relationship between the value of research firms and the number of such firms. This relationship reflects both the negative effect of the number of firms on the value of a representative firm through lowering the expected duration (and hence the present value) of the stream of profits from the innovation (creative destruction), as well as the positive effect through research spillovers. An appendix available on request from the authors shows that under suitable assumptions about the step-size of innovation \(\gamma\) there is an \(\bar{N}\) such that for \(N < \bar{N}\) the creative destruction effect is stronger and the (JJ) relationship is downward sloping. With additional reasonable assumptions about the parameters there is a steady state with \(N_{s}^{i} < \bar{N}\) and relationship (JJ) is downward sloping in the relevant range of \(N\) as shown in Fig. 1. These assumptions are maintained for the comparative statistics below and the calibration that follows. The steady state is determined by solving the system of Eqs. (FE) and (JJ).
2.6. The aggregate economy

In the steady state all sectors will have the same productivity adjusted value of research firms, \( J^*_i \), as well as the same number of these firms, \( N^*_i \). Recall that output per capita is given by:

\[
Y_t = \frac{L}{m} A_t k^\rho (L/m)^{1-\rho} = \frac{\varepsilon A_t}{\rho + \delta} \left( \frac{\varepsilon A_t}{\rho + \delta} \right)^{1/\rho}
\]

and so it is proportional to average productivity. Thus, with a large number of sectors, output per capita will grow at approximately the rate \((\gamma - 1) \lambda N^*_i\).

Consider what happens when the productivity of new innovations, \( \gamma \), rises. An increase in \( \gamma \) shifts out the \((JJ)\) curve in every sector. This happens because, for a given number of research firms \((N^*_i)\), greater improvement in technology with each innovation translates into a higher value of a blueprint for a new innovation (numerator of Eq. \((JJ)\)) and greater spillovers from other firms’ research (denominator of \((JJ)\)). The \((FE)\) curve is unchanged. The increase in \( \gamma \) is reflected in movement up and to the right along the \((FE)\) curve. This is illustrated in Fig. 1.

In the new steady state, research firms are worth more in every sector (higher \( J^*_i \)), which results in a greater inflow of financing firms, more research firms in every sector (higher \( N^*_i \)) and thus a higher aggregate rate of innovation. This implies that the growth rate, equal to \( \lambda N^*_i (\gamma - 1) \), increases for two reasons: each new innovation increases productivity by more \((\gamma' > \gamma)\), and there are more R&D firms so that the rate of discovery of new blueprints increases \((\lambda N^*_i > \lambda N^*_i)\). Finally, the value of the stock market where research firms are listed \((mN_i J_i)\) goes up.

3. Stock market booms and welfare

This section discusses the welfare effects of a stock market boom in our model. In particular, we ask how a stock market boom not justified by fundamentals changes the actual investment in R&D relative to the socially optimal investment.
The optimal research investment in this economy would be chosen by a social planner by maximizing the present discounted value of output net of investment in physical capital, R&D, and search costs, subject to the resource constraint. As in the standard Schumpeterian growth model, while comparing the social optimum with the private equilibrium level of R&D we need to consider two types of externalities. On the one hand, since an innovating firm fails to internalize the “intertemporal spillover” benefit it provides by raising the productivity of all future innovations, there is too little decentralized research. Furthermore, anticipated creative destruction effects (or the fact that future innovation will eventually destroy monopoly rents) lowers the current rate of innovation relative to the socially optimal rate.

On the other hand, there are effects leading a laissez-faire economy to undertake too much research since potential innovators do not internalize the fact that they destroy the rents of the current monopolist. In addition, duplication of research effort, either accidental or a purposeful byproduct of patent races, leads to an inefficiently high level of R&D. This effect is not directly incorporated in the model as the aggregate rate of innovation is proportional to the sum of research effort. However, a similar role is played here by the search externality. Venture capital firms do not internalize the cost they impose on other VC firms (by lowering their chances of finding a matching project) when they enter the market. This effect reinforces the business stealing effect in leading to too much equilibrium funding for research and thus too much R&D.

If the first group of effects dominates the combination of business stealing and search congestion effects, then there is an inefficiently low level of R&D in the decentralized equilibrium. In this case, a stock market boom that is not entirely justified by the fundamentals may have a net positive effect on the economy by channelling more funds towards research, making the rationing of research projects less severe, and bringing the R&D rate closer to the social optimum. Despite the over-investment from the private point of view, the social welfare consequence of these innovations can be positive due to the spillover effects described above. These effects can, of course, go too far. If the degree of misperception about new productivity is very high, and thus the stock market boom and surge in R&D investment are very large, it may turn out that the resources that have been channelled into research may be too much even from the point of view of a social planner, leading to an overshooting of the social optimum.

In order to allow for the possibility that expectations may deviate from the fundamentals, in the simulation below we will allow for periods when \( \gamma^{\text{perceived}} \neq \gamma^{\text{true}} \). In terms of the value Eqs. (8) and (9), the value of a research firm as well as that of a searching firm

---

10 This negative congestion externality parallels the externalities imposed in the labor market by firms with vacancies on other searching firms and by unemployed workers on other workers searching for employment (Hosios, 1990). Note, however, that since research projects are supplied inelastically, the fact that VC funds appropriate the entire surplus from a successful innovation does not introduce additional inefficiencies in entry of research projects.

11 Actual episodes of stock market booms may have various causes. Proposed explanations include models with rational investors who face complex data extraction problems (Meltzer, 2002; Banerjee, 1992) as well as departures from rationality in the form of over-optimistic expectations about the future (e.g. irrational exuberance). Since we are focusing on the effects of the run-up in stock prices, we do not complicate the analysis by studying why shocks to expectations take place. For an example of how imperfect information and learning about the fundamentals can trigger changes in expectations, see Nabar, 2006.
will be determined instead by the *perceived* step-size of the next innovation, $\gamma^{\text{perceived}}$, while productivity will evolve according to $A^{G+1} = \gamma^{\text{true}} A^{G}$.

The effect on stock prices of this misperception is to raise the market value of research firms. Starting in steady state, an increase in market value above $J_{SS}$ leads to entry of venture capitalists, an increase in the number of matches and research investment, which subsequently raises the aggregate rate of innovation and productivity growth. When the boom ends, the economy is left with a permanently higher level of productivity than it would have had in the absence of the boom. Furthermore, for some time after the market valuation boom ends, productivity may continue to grow at a rate higher than the new post-boom steady state rate. This is because the sunk cost of search has been borne and the excess research-funding matches that were created during the boom continue to exist until they innovate or break up.

The welfare effect of the boom will depend on two variables: the degree of under-provision of R&D in a decentralized equilibrium, $N^{\text{planner}} / N^{\text{decent}}$, and the degree of over-investment, $N^{\text{boom}} / N^{\text{decent}}$ (which depends on the magnitude of the stock market boom). We refer to the latter as the degree of “exuberance”. Fig. 2 illustrates how the welfare effects change with the degree of exuberance, $N^{\text{boom}} / N^{\text{decent}}$. A value of one for this ratio ($N^{\text{boom}} = N^{\text{decent}}$) implies no exuberance and thus no welfare change relative to the decentralized equilibrium. As exuberance increases, the amount of R&D gets closer to the social optimum and welfare increases. This effect peaks at $N^{\text{planner}} / N^{\text{decent}} = N^{\text{boom}} / N^{\text{decent}}$, that is when the boom is just the right size to induce the socially optimal level of innovation. Beyond that point welfare gains decline and eventually become negative as an excess of resources is directed towards R&D from the social point of view. Fig. 2 also adds a plot of the relationship for a higher level of $N^{\text{planner}} / N^{\text{decent}}$, i.e. a greater under-provision of R&D in the decentralized equilibrium. In this case an even greater stock market boom is required to bring the economy to the social optimum, and it also takes a greater stock market boom to lower welfare by overshooting the social optimum.

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*Fig. 2.* Welfare effects of irrational exuberance. The horizontal axis is $N^{\text{boom}} / N^{\text{decent}}$. A value of one implies $N^{\text{boom}} = N^{\text{decent}}$, i.e. no exuberance. The welfare effect peaks at $N^{\text{planner}} / N^{\text{decent}} = N^{\text{boom}} / N^{\text{decent}}$. The dashed line represents an economy with a greater degree of under-provision of R&D in a decentralized equilibrium.
4. Calibration of the 1990s episode

Our objective in this section is to calculate the welfare effects of the run-up in the NASDAQ index for a reasonably calibrated version of the model presented above. As stressed above we remain agnostic about the cause of the NASDAQ run-up. We also use the terms “run-up”, “boom”, and “bubble” interchangeably.

The behavior of this index in the 1990s is illustrated in Fig. 3a. As discussed in the previous section, we allow for the possibility that this behavior is driven, in part, by over-optimistic expectations regarding the profitability of research and the acceleration in the trend growth rate of productivity. In terms of the model, there is a consensus expectation of $\gamma$, say $\gamma_{\text{perceived}}$, which deviates from the true value $\gamma$. The realignment of expectations with reality may not be entirely straightforward because greater expected $\gamma$ leads to higher value of the stock market, entry of firms, more innovation, and thus faster growth. If individuals cannot distinguish between the frequency and size of innovation, the higher observed growth will serve as justification of higher expected $\gamma$ even if the expectation is incorrect. Since our interest is in tracing out the effects of the episode on research investment, we do not get into a discussion of the details of the mechanism behind the observed exuberance.

We are interested in comparing two scenarios. In the first case an unexpected change in $\gamma$ occurs in 1995, agents know what the new value of $\gamma$ is, and adjust their decisions accordingly. In the second scenario, we calibrate the market value of innovating firms to follow the path of the NASDAQ index in Fig. 3a. Agents make research investment decisions based on the inflated values of asset prices. This irrationally exuberant (IE) economy discovers the true size of productivity change in 2000 and the stock market adjusts.

The parameters used in the calibration are chosen either based on the literature or picked to match specific statistics from the US economy. We match the rise of the productivity adjusted stock market index of 4.9 based on Fig. 3a. The parameters $z$, $\rho$, $\epsilon$ are set based on the literature. We choose $z$ to be 0.3 to match the capital share of income. For $\rho$, the risk free rate of interest, we choose 4%. We follow Jones and Williams (2000) in choosing the monopoly mark-up $1/\epsilon$ to be less than the inverse of the capital share. For the preferred calibration we use a value of 1.25. Typical estimates in the literature range from 1 to 1.4 (Norrbin, 1993).

For the cost of R&D, $\mu$, we choose a value that yields R&D investment share of GDP equal to 2.5% following Howitt (1998) who observes that this is the average for the US economy over the period 1959–1998 (according to NSF data on total R&D) and Jones and Williams (2000) who use 2.2%.

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12 As stressed above we remain agnostic about the cause of the NASDAQ run-up. We also use the terms “run-up”, “boom”, and “bubble” interchangeably.

13 The figure plots NASDAQ/GDP per hour, the variable that will be used in the calibration below. In the present model, the corresponding variable is $(J \times N)/y$ – the market value of innovating firms relative to output per worker.

14 One interpretation of such a shock to investor expectations is in Beaudry and Portier (2003). News about technological opportunities is reflected first in elevated stock prices, long before the TFP series reflects acceleration in technological progress. The authors argue that this type of shock to expectations regarding future technological opportunities does a better job of explaining properties of macro time series (including stock market data) than other standard sources of fluctuations.
We set $k$ and $g$ to match the average life of a patent and the ratio of decentralized to optimal R&D. In terms of the average life of a patent, we follow Jones and Williams (2000) in choosing a value of 10 years. For the ratio of decentralized to optimal research expenditure, we choose to match a value of 1.4. We also provide results for alternative values of 1.2 and 1.6. These are consistent with the values calculated by Jones and Williams (2000), who provide several values for the ratio of decentralized to optimal research expenditure ranging from about 1 to a factor of 4.15

The exogenous break-up rate of good matches, $\chi$, is set at 0.06 to match data on death rate for firms. The US Small Business Administration gives the 2002 death rate of firms in Information, Finance & Insurance, Professional, Scientific, & Technical Services, Management of Companies & Enterprises, and Health Care & Social Assistance sectors at about 8%.16 Within Professional, Scientific, & Technical Services, it is about 11%. In our model, research firms exit either because of a match break up or because they successfully innovate. Thus in the model the rate at which research firms disappear is $\lambda + \chi = 0.04 + 0.06 = 10\%$. We set $\delta = 0.05$, which pins down $K/Y = 2.7$.

15 This ratio is important for the results since the welfare effect depends on how close the decentralized level of R&D is to the social optimum to begin with. We therefore present three different scenarios within the range calculated by Jones and Williams.

We pick a value for \( c \) to match the average pre-boom growth rate of productivity at 1.4%.\(^{17}\) This average pre-boom growth rate of productivity implies an investment rate of about 18% of GDP.

The amount \( jF_{ss} \) corresponds to the total expenditure on finding, evaluating and monitoring potential research projects by financiers (venture capitalists) in each sector. The value of \( j \) used produces a value of \( m_kF_{ss}/\text{GDP} \) of 1%. We feel this is a reasonable number. Venture capital funds are a part of non-depository financial institutions, which make up only about 0.6–0.9% of GDP (based on data from the Bureau of Economic Analysis, Gross Domestic Product by Industry, 1994–2001). However, we are trying to capture much more than just venture capital as the source of funding for R&D. There are also costs of evaluation of in-house research projects as well as expenditures on marketing, legal and other business services associated with research projects. All these costs taken together make up \( kF_{ss} \).

We choose the size of a sector \( L/m \) to be such that the stock market value of all research firms \( mJ_{ss}N_{ss}/Y_{ss} \) is about 10%. This is reasonable given that in 1995 the NASDAQ’s total market capitalization was about 15% of GDP and that of the top 100 firms (presumably those with the greatest influence on the aggregate rate of innovation) was about 10% of GDP.\(^{18}\)

The last parameter to choose is \( \phi \), or the elasticity of matches with respect to VC firms. This is an important parameter because it determines the responsiveness of entry to asset prices. In Appendix A, we show that the free entry condition (11) can be used to derive the following relationship between the rate of economic growth and the stock market index:

\[
\hat{\gamma}_t = \beta_0 + \beta_1 \gamma_t + \beta_2 \log(L) + \beta_3 \log(S&P500/y) - \beta_4 \log(1 + r_t) + \epsilon_t, \tag{16}
\]

where \( r_t \) is the long term interest rate, \( \hat{\gamma} \) is the trend growth of GDP per worker and \( \phi = \beta_3/(0.02 + \beta_3) \). We estimate this equation for the period 1955–2000. For trend growth we use growth of HP-filtered GDP per worker from the Penn World Tables. We use the real S&P500 series as well as the long term interest rate from Shiller (2001). A time effect is added for each five-year period to capture changing \( c \). We also estimate a specification where we allow for autocorrelation by including up to four lags of \( \hat{\gamma} \). In some specifications we include a time trend to allow for rising sophistication of financial markets \( \eta \). The estimates are reported in Table 1. In each case we focus on the magnitude and statistical significance of \( \beta_3 \).

The OLS results most likely over-estimate \( \beta_3 \) because of reverse causality from the trend growth rate of productivity to the S&P index. Furthermore, the value of the S&P500 index may be a noisy proxy for the expected value of innovating firms (captured by \( J \) in the model), and we assume in the regressions that the other parameters of interest remain unchanged over the period of our sample. Nevertheless, these estimates are indicative of

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\(^{17}\) According to Oliner and Sichel (2003), during the period 1974–1990 this value was 1.36% and during the period 1991–1995 it was 1.54%.

\(^{18}\) The numbers we use are the averages for 2005 – 20% for NASDAQ and 13% for top 100 firms. The ratio of the NASDAQ index to GDP has increased by 33% between 1995 and 2005. Hence the market capitalization in 1995 was approximately 20%/1.33 = 15% (or 13%/1.33 = 10% for top 100 firms). Note that the choice of \( m \) does not matter since once the size of the sector is fixed, \( m \) becomes simply a scaling factor. This means that increasing the size of the economy by increasing the number of sectors (or increasing population and sectors proportionally) does not affect the growth rate, i.e. the strong scale effect is not present in this model. The model is thus a “second generation” or product-proliferation Schumpeterian model. See Jones (2005) for a discussion.
a reasonable range of values for \( \phi \) that may be used in the calibration. Our preferred value for \( \phi \) is 0.15. Using this value along with the other parameters selected as described above, we replicate two patterns of the data for the second half of the 1990s: initial rapid entry of innovating firms (start-ups) as the NASDAQ index rose dramatically, followed by a collapse in investment in the aftermath of the NASDAQ crash.19

The baseline parameters are presented in Table 2. These parameters correspond to the ratio of optimal to decentralized R&D of 1.4. We later allow this ratio to vary by changing the mark-up \( 1/\epsilon \).

### Table 2
Calibration – baseline parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) Capital share</td>
<td>0.30</td>
</tr>
<tr>
<td>( \rho ) Discount rate</td>
<td>0.04</td>
</tr>
<tr>
<td>( \delta ) Depreciation rate</td>
<td>0.05</td>
</tr>
<tr>
<td>( L/m ) Sector size</td>
<td>20</td>
</tr>
<tr>
<td>( \lambda ) Success probability</td>
<td>0.04</td>
</tr>
<tr>
<td>( \mu ) Cost of research</td>
<td>0.36</td>
</tr>
<tr>
<td>( 1/\epsilon ) Mark-up</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Model one includes trend, model two includes trend and four lags of growth rate and model three includes four lags as well as time dummies. All models include a constant term. A (*) denotes significance at 5% level and (**) denotes significance at 1% level.

5. The welfare effects of the 1990s episode

As discussed in Section 3, the welfare effects of the stock market boom depend on the degree of over-optimism of expectations and the initial under-provision of R&D by the decentralized economy. These two values depend on the pre-1995 parameters of the economy and the increase in productivity of research after 1995 (increase in \( \gamma \)), if it occurred. We want to consider the possibility this increase took place (i.e. part of the stock market appreciation was justified by fundamentals). Since data on the post-1996 productivity revi-

19 We believe that a lower value than this is much less reasonable. The reasoning is as follows. With a low \( \phi \), the matching process is slow and it takes much longer for new firms to enter following the run-up in stock prices. In most of the scenarios for productivity acceleration that we study below, this implies that entry continues after the crash. We do not believe that this is consistent with the evidence. Results for a lower value of \( \phi = 0.05 \) are available upon request.
val are still preliminary, we do not yet have a consensus on what the true numbers for new trend productivity growth are (Oliner and Sichel, 2003; Gordon, 2004). In the following exercise we consider different possibilities for the new value of $\gamma$, and thus the new steady state productivity growth after 1995. We also consider several values for the initial equilibrium under-provision of R&D. For each possible true productivity growth rate and for each initial planner/decentralized R&D ratio, we calculate the welfare change for the associated degree of over-optimism as described in Section 3.

Before analyzing the welfare effects we present the time paths of several key variables for a simulated economy. In these figures we assume that the growth rate of productivity has accelerated from 1.40% to 2.3%.


Fig. 3b presents the path of $mJ_iN_i/y_i$, the model equivalent of NASDAQ/GDP during the 1990s. The behavior of the index is matched to Fig. 3a. Initially there is a gradual appreciation of the index. Around 1998 a sharp jump occurs, caused by a sudden increase in the expected value of productivity of new research firms.\textsuperscript{20}

Fig. 3b compares the IE economy to one without over-optimistic expectations. In both cases the value of the stock market increases immediately after expectations about productivity of new innovations change. This reflects the fact that research is believed to be more productive right away, boosting expected profits of existing research firms. In the long run these firms will face more competition, which will drive profits down. However, since the process of entry is costly and time consuming, increased competition does not arrive immediately. In the case of over-optimistic investors, the value of existing firms jumps by much more than is justified by the true fundamentals. Once investors correct their expectations (some time in the year 2000), the stock market crashes. In the figure, the solid line represents the path for the irrationally exuberant economy while the dashed line represents the path for the non-IE economy.

Fig. 3c plots the evolution of the number of research firms. In both economies new firms enter the research sector in response to the increased value of existing firms. In the economy with correct expectations (dashed line), there is a gradual inflow of new firms. This process is not immediate because of the costly and time consuming nature of matching research ideas with funding. In the IE economy (solid line), the entry is much faster due to the very large jump in the stock market valuation of existing research firms. Following the crash, the number of research firms declines gradually to the new steady state. The number of R&D firms does not crash since existing firms have already obtained funding and, as long as their value is positive, there is no voluntary exit from the sector. The slow decline is a consequence of a drop in entry of new firms. However, the number of venture capital firms (not illustrated) plummets as their value turns negative and there is exit from the business of funding research. Fig. 3d plots the logarithm of consumption for the two economies. The IE economy (solid line) undergoes a large drop in consumption as funds are shifted towards search for new R&D projects and their financing. The cost of investment in research pays off in the form of faster technological progress than in the non-IE economy.

\textsuperscript{20} Note that in the model innovations arrive in discrete steps. However, with a large enough number of sectors the Law of Large Numbers ensures a smooth evolution of aggregate variables. In the simulation we therefore present the expected values.
Both economies have the same productivity of new innovations, but in the IE economy there are many more firms with funding working on projects and therefore more projects are successful. Note that because of the increase in the number of R&D firms during the boom, the research intensity and productivity growth remain higher well after the boom has subsided. This is consistent with the experience of the US economy as emphasized by Gordon (2004).

In the long run both economies grow at the same rate. The short period of faster growth may however allow the IE economy to attain a higher consumption path as in Fig. 3d. This is not necessarily always going to be the case. If the drop in consumption is very large, and the extra R&D financed by it is not very productive, the IE economy may end up on a lower path. In addition, the welfare comparison between the two economies depends on whether the present discounted value of extra consumption exceeds the cost of extra investment today. We now turn to this comparison.

5.2. Welfare

As described above, we present the welfare calculations for different values of the actual new $\gamma$ (and thus new steady state growth rate of productivity) and for different optimal/decentralized R&D ratios. The results are reported in Table 3 and plotted in Fig. 4. This corresponds to the solution for $\omega$ in the equation

$$\sum_{t=1995}^{\infty} \frac{1}{(1+\rho)^t} (1+\omega)c_t = \sum_{t=1995}^{\infty} \frac{1}{(1+\rho)^t} c^\text{IE}_t,$$

where $c$ is the consumption path without over-investment and $c^\text{IE}$ is the consumption path in an economy with over-investment.

Consider first the baseline case where the initial R&D effort is about 70% (=1/1.4) of the optimum. If the bubble reflected pure “irrational exuberance”, and the underlying labor productivity growth rate remained unchanged at 1.4% (first row of the table), then the welfare effect of the bubble was a 0.07% gain in terms of present discounted value of consumption. Note that the growth rate is expected to be higher during the boom due to the increased number of research firms, but the extra innovations generated by the boom

<table>
<thead>
<tr>
<th>New steady state growth</th>
<th>$\frac{N_{\text{planner}}}{N_{\text{decentralized}}}$ ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.2</td>
</tr>
<tr>
<td>1.40</td>
<td>−0.54</td>
</tr>
<tr>
<td>1.45</td>
<td>−0.49</td>
</tr>
<tr>
<td>1.75</td>
<td>−0.18</td>
</tr>
<tr>
<td>2.00</td>
<td>0.07</td>
</tr>
<tr>
<td>2.25</td>
<td>0.33</td>
</tr>
<tr>
<td>2.50</td>
<td>0.59</td>
</tr>
<tr>
<td>2.75</td>
<td>0.85</td>
</tr>
<tr>
<td>3.00</td>
<td>1.10</td>
</tr>
<tr>
<td>3.25</td>
<td>1.36</td>
</tr>
<tr>
<td>3.50</td>
<td>1.62</td>
</tr>
<tr>
<td>3.75</td>
<td>1.88</td>
</tr>
</tbody>
</table>
lead to a small welfare gain since the underlying growth rate of productivity has remained unchanged. However, as discussed above, the welfare benefits of the investment boom increase with the productivity of innovations. If the new steady state growth rate is 2.5% the boom will have a more substantial positive effect, raising the present discounted value of consumption by about 0.95%. Finally, a 3% new steady state growth implies a welfare gain of 1.35%. These changes are summarized in the solid line in Fig. 4. The horizontal axis denotes the new steady state rate of growth, which is a function of the size of the new productivity of innovations $c$. The pre-bubble growth rate is equal to 1.4%. The vertical axis measures the present discounted value of the difference between the two consumption paths depicted in Fig. 3.

The effect of different values of the optimal/decentralized R&D ratios can be seen by looking at the other two lines in Fig. 4. The welfare effects display the pattern discussed in Section 3, i.e. higher initial under-provision of R&D results in bigger gains due to the bubble (or smaller losses). If there was only about 62% of the optimal level of R&D to begin with ($N_{planner}/N_{decent} = 1.6$; in the figure, this corresponds to the dotted line), then the bubble had a positive welfare effect of 0.51% increase in the present discounted value of consumption even if the steady state growth rate did not change at all. If the productivity growth went up to as much as 3%, the welfare improvement was 1.5%. On the other hand, if the shortfall of decentralized R&D was only about 17% ($N_{planner}/N_{decent} = 1.2$; in the figure, this corresponds to the dashed line), then the decentralized equilibrium was rela-

$^{21}$ Gordon (2004) estimates the trend growth rate of labor productivity from the fourth quarter of 1995 through the second quarter of 2003 to be 2.48%. Even if only 75% of this revival can be attributed to information technology, (implying that of the 1.1 percentage point increase in the growth rate of labor productivity, close to 0.85 percentage points can be attributed to IT), the welfare gain associated with a new growth rate of 2.25% is 0.75% of the present discounted value of consumption.
tively close to the socially optimal outcome to begin with. In this case, a relatively small degree of exuberance would lead to an overshooting of research investment beyond the socially optimal rate.

Finally, it is important to note how the welfare costs and benefits are distributed over time. As Fig. 3 d makes clear, the costs of the bubble, in the form of over-investment in research, are always incurred in the short run. The positive side of the run-up in the form of a faster rate of innovation (and thus higher level of productivity and consumption) is relatively small in any given period. The benefits accumulate over time, however, as the productivity advantage is permanent.

The present value calculation presented above takes this into consideration. For an infinitely lived agent this calculation reflects the actual welfare difference. In a world of finitely lived agents, who discount the utility of their offspring at a higher rate than their own, the timing (and not just the present value of welfare changes) matters. To get a sense of the temporal pattern of the welfare effect we calculate the change in present discounted utility at different finite horizons. 22

Fig. 5 presents the results for the baseline case. The solid line represents the percentage change in present discounted value of consumption from 1996 until time $t$ marked on the horizontal axis.

![Fig. 5. Welfare effects at different time horizons for the baseline case. The solid line represents the percentage change in present discounted value of consumption from 1995 until time $t$ marked on the horizontal axis.](image)

22 In other words, we calculate $\sum_{t=1995}^{T} \frac{1}{(1+r)^t} (1+\omega)c_t = \sum_{t=1995}^{T} \frac{1}{(1+r)^t} c_t^{IE}$ for different values of $T$. 

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The horizontal axis marks years after the stock market boom begins. The vertical axis measures the percentage change in present discounted value of consumption from 1995 until the time marked on the horizontal axis. The solid line represents the case of change in steady state productivity growth to 3%, i.e. the case when an infinitely lived agent experiences a 1.35% welfare gain. The dashed line
represents an increase to 2% steady state growth, corresponding to an infinite horizon welfare gain of 0.55%. Finally, the dotted line corresponds to no change in steady state productivity growth and a 0.07% welfare gain. Consider the intermediate case of 2% steady state growth rate. In this case, the run-up results in a welfare gain of 0.55% for an infinitely lived consumer. However, for the welfare change to be positive, the time horizon of the individual must be more than 60 years after the start of the bubble. In the case of no steady state growth acceleration, it takes an even longer horizon. A stock market boom of this nature may have positive welfare effects, but the benefits accumulate slowly over time while the cost is borne today.

6. Conclusion

The literature on innovation-based growth strongly suggests that R&D investment in the decentralized equilibrium may not be socially optimal (since innovators fail to internalize the externalities associated with the creation of ideas). We have evaluated the proposition that, given a sub-optimal amount of investment in R&D, a stock market boom raises the aggregate rate of innovation and lifts the rate of technological progress closer to the socially-optimal rate by relaxing financing constraints for new firms. The novelty of our approach lies in the framework presented for conducting a quantitative assessment of the welfare effects of a stock market boom. By introducing financing frictions into the basic Aghion and Howitt model we were able to link aggregate investment in R&D to asset prices. This allowed us to quantitatively assess the NASDAQ episode of the late 1990s.

Gordon (2004) estimates that the trend growth rate of labor productivity over the period 1995–2003 is 2.48%. He also reports that the trend growth rate is possibly accelerating even further, reaching 3.04% at the end of 2003. The results of our baseline calibration suggest that if the long run growth rate of the US economy has indeed increased to about 3%, the NASDAQ episode had a positive effect on present discounted value of consumption, raising it by about 1.35%. It must be noted, however, that the benefits of the episode accumulate over time while the costs of over-investment are borne by the current generation.

One interesting implication of the results is for the optimal policy of the Federal Reserve in the face of a stock market run-up. While Alan Greenspan did use the phrase “irrational exuberance” in reference to the rise of the NASDAQ index during the early stages of the episode, he later argued that the stock market appreciation was based on solid fundamentals of faster productivity growth. Many critics have subsequently argued that the Fed should have taken actions to deflate the run-up and its chairman should not have made claims fuelling the episode. For a plausible range of parameter values in our model, the calibration presented suggests that even if the Fed knew that the perceptions about the productivity of the new economy were exaggerated, it was optimal from a long intertemporal horizon perspective not to argue against them.

23 The pattern is similar in the case when productivity growth accelerates to 3%, although this time there is a gain for agents who live only through the first 3 years of the bubble. In this case, the stock market does not overshoot the fundamentals in the early years and so there is no over-investment.
Acknowledgements

We thank David Weil, Peter Howitt and Tom Krebs for comments and feedback.

Appendix A. Derivation of Eq. (16)

Recall the free-entry equation (in steady state)

\[ N_{ss} = \frac{\eta L}{\lambda + \chi} \left[ \frac{\tilde{J}(N_{ss})}{(1 + \rho) \kappa} \right]^{\frac{\lambda - \chi}{\lambda}}. \]

The (average) growth \( \dot{y} \) rate is given by

\[ \dot{y} = (\gamma - 1) \lambda N_{ss}. \]

Taking logs this yields

\[
\log(\dot{y}) = \log((\gamma - 1) \lambda) + \log(N_{ss}) + \log\left(\frac{\eta L}{\lambda + \chi}\right) + \frac{\phi}{1 - \phi} \log(\tilde{J}(N_{ss})) - \frac{\phi}{1 - \phi} \log((1 + \rho) \kappa)
\]

\[ = C + \Omega(\gamma) + \log(\eta L) + \frac{\phi}{1 - \phi} \log(\tilde{J}(N_{ss})) - \frac{\phi}{1 - \phi} \log((1 + \rho)). \]

Now approximating \( \log(\dot{y}) \) around 2% we get

\[
\log(\dot{y}) \approx \log(0.02) + \frac{1}{0.02} (\dot{y} - 0.02). \]

Combining the two equations we get

\[
\dot{y} = C' + \Omega'(\gamma) + 0.02 \log(\eta L) + 0.02 \frac{\phi}{1 - \phi} \log(\tilde{J}(N_{ss})) - 0.02 \frac{\phi}{1 - \phi} \log(1 + \rho),
\]

which can be mapped into the regression equation used in the text:

\[
\dot{y} = \beta_0 + \beta_1 \gamma_t + \beta_2 \log(L) + \beta_3 \log(S&P500/y) - \beta_4 \log(1 + r_t) + \epsilon_t.
\]

References


Jermann, Urban, Quadrini, Vincenzo, in press. Stock market boom and the productivity gains of the 1990s. Journal of Monetary Economics.


