Should day care be subsidized?

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Abstract

In an economy with distortionary taxes on labor, can subsidies on day care, financed by an increase in taxes, raise welfare by encouraging women with small children to work? We approach this question in three stages. First we consider a stylized life-cycle model where we prove analytically that, under some assumptions, the Ramsey optimal policy consists in making day care expenses tax deductible. Then we explore the quantitative implications of subsidizing day care in the context of this stylized model. This exploration reveals that the welfare gains from day care subsidies are larger the greater extent to which child-rearing is confined to a small part of the life-cycle. Finally, we construct a calibrated stochastic overlapping generations model of household decision-making designed to capture some key facts about fiscal policy and labor supply in Germany. We find that the welfare gains associated with subsidizing day care are considerable, and that the maximum gains are realized when day care is subsidized to an even greater extent than what tax deductibility would imply.

Keywords: Female labor force participation, Germany, day care subsidies
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1 Introduction

In both Europe and North America, public support for preschool day care is on the rise. At its Barcelona meeting in 2002, the European Union encouraged member countries to supply subsidized full-day places for one-third of 0- to 3-year-olds, and for over 90% of all 3- to 6-year-olds by the year 2010. In the United States from 2002 to 2007, the fraction of four-year-olds attending state-funded preschool education rose from 14 to 22 percent. At the same time, U.S. state funding for preschool education rose from 2.4 to 3.7 billion dollars.\(^1\) In Canada, partly inspired by the $5-a-day day care policy introduced in Québec in 1997, all major parties in the 2006 federal election campaign put forward proposals for a national day care programme.

Several possible justifications can be found for government funding of preschool day care. One is that it might raise fertility and thereby contribute to solving the problems generated by an aging population. Another is that it may promote early learning, especially among disadvantaged children. Finally, day care subsidies tend to encourage female labor force participation, which might be desirable either because it promotes equality within the household\(^2\) or because it leads to a more efficient allocation of resources. Here we focus entirely on the consequences of day care subsidies for the allocation of resources. We approach this issue from an optimal taxation perspective, essentially in the spirit of Ramsey (1927).

In this paper, we assess the merits of public funding for day care, and we address this question in three stages. First we develop a highly stylized one-agent life-cycle framework designed to give a qualitative answer to the question whether there is a case for a positive subsidy rate on day care. The demand for day care in this environment arises from the assumption that every hour of work requires the agent to purchase an hour of day care per child. The optimal taxation problem arises from the assumption that the government


\(^2\) Knowles (2006) shows that rising women’s wages relative to men’s wages has strengthened the relative bargaining position of women within U.S. households. Since the introduction of day care subsidies raises the effective wage for women, it is likely to have the same effect.
has an exogenous spending requirement and that lump-sum taxes are ruled out but linear taxes and subsidies are allowed. Thus the consumption/leisure choice is distorted and the problem is to minimize the total welfare cost of this distortion. Our main finding in this stylized environment is that the optimal policy is to make day care expenses tax deductible. The intuitive reason why this is the optimal policy is that it equalizes the ratio of marginal private to marginal social returns to working across different time periods; by doing that, the total distortion is minimized. This is essentially a tax smoothing argument; by spreading the distortion equally over periods, the total distortion is reduced.

The second stage is to examine, in the context of this stylized environment, the quantitative significance of optimal day care subsidies. We find that the size of the welfare gains is quite sensitive to details of the calibration. For instance, we find that welfare gains from day care subsidies are larger the greater extent to which child-rearing is confined to a small part of the life-cycle. Also, the level of pre-reform tax distortions and the size of the unit resource cost of day care are crucial for the size of the welfare gains.

This of course suggests that a more serious calibration exercise is in order. The third and final stage, then, is to provide exactly that. For this purpose, we consider Germany. There are several strong reasons for studying Germany in the context of day care policy. One, as emphasized in a recent OECD survey of Germany (OECD, 2008), is that “fewer mothers with small children are employed in Germany than in other countries and those who work do so for fewer hours”. Merz (2004) studies female labor supply trends in western Germany and finds that although participation has risen somewhat, hours per female employee actually fell between 1980 and 2000, the net result being almost no change in weekly market hours worked per woman.³

Another reason to consider Germany is that, until recently, availability of subsidized day care is very limited there, especially in western Germany; see Kreyenfeld and Hank (2000), Wrohlich (2005) and OECD (2006). On average, across the OECD countries for which

³ It is worth stressing that Merz (2004) only studied west German data; once the new eastern Bundesländer are taken into account, female weekly or annual market hours worked per person did increase markedly after 1990, but that is largely because of the addition of new women from the east who were already working more hours.
data are available, 23 percent of 0-3 year-olds are in formal day care. In Germany, that number is just 9 percent, and in western Germany it is less than 3 percent.\(^4\)

A further reason to study Germany is that its day care policy is currently in transition. In 2004, the SPD-Green coalition government enacted the *Tagesbetreuungsausbaugesetz*; this legislation requires that, each year, 1.5 billion euros have to be invested in day care, and local authorities are responsible for supplying sufficient day care for children under the age of three. The new CDU/CSU-SPD coalition government that took office in 2005 continued in the same spirit. In 2007 the German government announced that it plans by 2013 to triple the number of subsidized day care spots for young children to 750000.\(^5\) In 2008, the *Kinderförderungsgesetz* was passed in the *Bundestag* (federal parliament), establishing a right to a day care spot for every child from the age of one.\(^6\) Our work contributes to an evaluation of this and other reforms expanding the availability of subsidized day care in Germany.

If the conclusions from our calibration exercise are to be taken seriously, several departures from the stylized environment are necessary. In the first place, it is clear that individuals will be affected differently by day care subsidies depending on their marital status, their gender and the number of children they have over their lifetime; this means that the model must allow for heterogeneity in these dimensions.\(^7\) Specifically, we use an overlapping generations model where individuals live for seven 6-year periods plus an 18-year retirement period and are either permanently single or permanently cohabiting. Fertility is exogenous and children arrive in such a way as to generate a realistic distribution of children across households of different characteristics.

Moreover, a guiding principle in designing our model has been that it should replicate

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\(^4\) Sources: Wrohlich (2005) and OECD (2007).

\(^5\) Source: Deutsche Welle, [http://www.dw-world.de/dw/article/0,2144,2430472,00.html](http://www.dw-world.de/dw/article/0,2144,2430472,00.html), retrieved on November 22, 2008.

\(^6\) For more details on these reforms and also appropriate references to German legislation, see the website of the German federal ministry of families, seniors, women, and youth, [http://www.bmfsfj.de/](http://www.bmfsfj.de/).

\(^7\) When talking about *marital status*, we are not interested in whether anyone is legally married, only whether they are living in a relationship that is economically equivalent to marriage. Therefore we will use the terms “married” and “cohabiting” interchangeably throughout the paper.
some key features of the German data. In particular, we want the model to match average hours worked across individuals of different marital status, age, gender and number of children. For this purpose, we allow heterogeneity with respect to the disutility of labor, and we also require households to do an amount of housework that is allowed to depend on household characteristics.

Finally, the current German system of taxes and transfers is very far from the linear tax scheme of the stylized environment. In order to make our analysis applicable to the current German situation, we want to make sure that the model is broadly consistent with the existing framework of German fiscal policy. This allows us to focus sharply on the effects of subsidizing day care, keeping other aspects of the tax-transfer system unchanged, subject only to government solvency.

Our findings can be summarized as follows. Subsidized day care leads to substantial increases in the labor supply of mothers with small children and to sizeable gains in welfare for everyone. More specifically, tax deductibility for day care expenses leads to an increase in the labor supply of single mothers with small children by 96 percent and there is a noticeable (27 percent) increase for married mothers of small children too. The welfare gain for couples corresponds to a 0.9 percent increase in consumption; for single men this number is 1.0 percent, and for single women it is 0.7 percent. In contrast to the stylized environment, deductibility of day care expenses is not the optimal policy. In fact, welfare increases for everyone as a function of the subsidy rate up to a rate of about 90 percent. At that rate, the welfare gain for couples corresponds to a 1.8 percent increase in consumption; for single men the number is 1.6 percent, and for single women it is 1.9 percent.

Our work is related to several strands of the literature. First, there is an empirical literature documenting that subsidized day care may encourage labor supply of mothers. The seminal work of Heckman (1974) presents strong evidence from the United States that day care subsidies do indeed increase female labor supply; more recently, in a case more directly relevant to this paper, Wrohlich (2006) finds, using an econometric approach, that expanding availability of subsidized day care increases maternal employment in Germany.
Another strand considers the role of public policy in determining the choice between home and market production. Lindbeck (1982) argues that subsidies to day care in many countries has contributed to the increased labor supply of women, counteracting the rise in tax rates that have been required to finance these subsidies. The reason for Lindbeck’s conclusion is that he thinks, as we do, of market provided day care as a close substitute for day care at home. Rosen (1997), on the other hand, considers, in the context of a model conceptually similar to Lindbeck’s, whether high day care subsidies can be justified and concludes that they cannot. This conclusion hinges on imposing a rather low upper limit on the degree of substitutability between market and home produced day care. Another important reason why our conclusion differs from Rosen’s is that the tax smoothing considerations central to our analysis are absent from his static model.

A third strand of the literature is concerned, as we are, with understanding female labor supply, especially its remarkable rise since the 1950s in the United States. Prominent contributions to that literature include Jones et al. (2003), Greenwood et al. (2005), and Olivetti (2006). More recently, Attanasio et al. (2008) have emphasized the importance of reduced day care costs in explaining observed changes in U.S. female participation rates. Other related papers consider the role of fiscal policy in explaining differences in labor supply across time and across countries, starting with Prescott (2004). Of particular relevance to the present paper is the idea that differences in labor-supply-promoting public spending may be an important factor in accounting for differences in labor supply across countries, especially the difference between Scandinavia and (the rest of) continental Europe; this point is explored in Olovsson (2004), Ragan (2005), and Rogerson (2006).

Our work is also closely related to the growing literature on quantitative evaluation of fiscal policy reforms in dynamic models with heterogeneity. A particularly relevant part of the literature are those papers that deal with multi-member households, such as Chade and Ventura (2002) and Güner et al. (2008) who look at the effects of income tax reforms on labor supply and Erosa et al. (2008) who look at the effects of parental leave policies.

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8 Bergstrom and Blomquist (1996) argue that the effect on labor supply may be so large that subsidizing day care is self-financing; a similar result is found in Lundholm and Ohlsson (1998).
The paper is organized as follows. Section 2 presents the stylized model, proves the optimality of day care expense deductibility and explores some quantitative implications of this optimal policy. Section 3 presents the model that we use to evaluate policy reform in Germany, describes the calibration and discusses the effects of day care subsidies on output, labor supply and welfare. Section 4 concludes.

2 A stylized model

Consider a $T$-period one-agent life-cycle model where at age $s$ the agent has $b_s$ small children. The demand for day care arises from the following assumption: for every unit of time that the agent works, it needs to purchase day care for each small child. The social cost of day care is $d$ per unit of time and child. The government levies age-dependent linear taxes on labor income (at rates $\tau_s$) to finance exogenous government purchases $G$ and possibly age-dependent linear subsidies on day care (at rates $\theta_s$). There is a “storage” technology (which can be thought of as a world capital market) that enables the agent to transform one unit of the age $s$ good into $1 + r$ units of the age $s + 1$ good and vice versa without any non-negativity restrictions on the amount “stored”. Capital income is not taxed and factor prices are exogenous.

Notice that agents in the model are indifferent to the presence of children; time spent with children is like leisure spent without children, no better and no worse. The only significance of children in the model is that they require constant supervision and so day care is required when the parent is working.

The representative agent solves

$$\max \sum_{s=1}^{T} \beta^s [u(c_s) + v(s, h_s)],$$

where $u: R_+ \rightarrow R$ is a differentiable and concave function, and $v(s, \cdot): R_+ \rightarrow R$ is a differentiable and convex function for each $s$, subject to the life-time budget constraint
(associated with the Lagrange multiplier $\lambda$)
\[
\sum_{s=1}^{T} (1 + r)^{-s} c_s + \sum_{s=1}^{T} (1 + r)^{-s}(1 - \theta_s)db_s h_s = \sum_{s=1}^{T} (1 + r)^{-s}(1 - \tau_s)w_s h_s,
\]
where $c_s$ denotes consumption, $h_s$ denotes hours worked, $w_s$ denotes age-specific productivity, and $r$ is the (exogenous) interest rate. Assume that $(w_s - db_s) > 0$ for all $s$, i.e. that the wage exceeds the cost of day care. This is the rationale for the production of day care outside the family. Notice that we allow the disutility of labor, $v(s, h)$, to depend on age; this enables us to establish a slightly more general result than otherwise.

Assuming an interior solution for $h_s$, labor supply is characterized by the agent’s first order conditions which are
\[
\beta^s u_{c,s} - \lambda (1 + r)^{-s} = 0
\]
and
\[
\beta^s v_{h,s} + \lambda (1 + r)^{-s} [(1 - \tau_s)w_s - (1 - \theta_s)db_s] = 0.
\]

The resource constraint is
\[
\sum_{s=1}^{T} (1 + r)^{-s} [c_s + db_s h_s + G] = \sum_{s=1}^{T} (1 + r)^{-s} w_s h_s,
\]
where $G$ denotes government consumption.

### 2.1 Ramsey government

The Ramsey government maximizes (1) subject to (5) (associated with the Lagrange multiplier $\mu$) and the implementability constraint (with Lagrange multiplier $\varphi$)
\[
\sum_{s=1}^{T} \beta^s [u_{c,s} c_s + v_{h,s} h_s] = 0.
\]

The first order conditions are
\[
\beta^s u_{c,s} [1 + \varphi R_{c,s}] - \mu (1 + r)^{-s} = 0
\]
and
\[ \beta^s v_{h,s} [1 + \varphi R_{h,s}] + \mu (1 + r)^{-s} [w_s - db_s] = 0, \]
where
\[ R_c = 1 + \frac{u_{cc^c}}{u_c} \]
and
\[ R_h = 1 + \frac{v_{hh^h}}{v_h}. \]

Notice that \( R_h \) is one plus the Frisch elasticity of labor supply.

### 2.2 Ramsey policies

If we divide the first order condition with respect to hours at two different ages \( s \) and \( t \) for the household (Equation 4) and for the Ramsey government (Equation 7) respectively, we get
\[ \frac{\beta^{t-s}}{(1 + r)^{(t-s)}} \frac{v_{h,t}}{v_{h,s}} = \frac{(1 - \tau_t)w_t - (1 - \theta_t)db_t}{(1 - \tau_s)w_s - (1 - \theta_s)db_s}, \]
\[ \frac{\beta^{t-s}}{(1 + r)^{(t-s)}} \frac{v_{h,t}}{v_{h,s}} = \frac{1 + \varphi R_{h,s} w_t - db_t}{1 + \varphi R_{h,t} w_s - db_s}. \]

Evidently the left hand sides of these equations are identical. Comparing the right hand sides, we see that if \( R_{h,s} = R_{h,t} \), then the equations are satisfied if \( \tau_s = \theta_s = \tau \) for all \( s \).

This establishes the following Proposition.

**Proposition 1** If \( R_{h,s} = R_{h,t} \), (e.g. \( v(s, h) = -\psi_s h^{1+1/\gamma} \)) then the Ramsey allocations can be implemented by constant taxes and day care expenses being deductible, i.e. \( \tau_s = \theta_s = \tau \).

An instructive way of expressing this result is that it amounts to equalizing, over the life-cycle, the consumption/leisure wedge, i.e. the ratio of marginal private to marginal social returns from working.\(^9\)

It is worth noting that the optimality of constant tax rates and subsidies hinges on the Frisch elasticity being constant. If it is not, taxes and subsidies should typically be age-dependent, as emphasized in a different context by Erosa and Gervais (2002).

### 2.3 A calibrated version of the stylized model

In this section we examine, in the context of this stylized environment, the quantitative significance of optimal day care subsidies. The aim here is not to provide a serious quantitative assessment of the likely effects of reform in Germany or any other country (we leave that for the next section), but to explore the significance of various features of the parameterization and get a rough sense of what the numbers might be under different assumptions.

Throughout this exercise, we assume that

\[ u(c) = \frac{c^{1-\sigma}}{1 - \sigma} \]

and that

\[ v(s, h) = \psi_s h^{1+1/\varepsilon}. \]

We now describe the baseline calibration, which and is designed to provide a reference point when comparing the results. Since in most countries there are six preschool years (ages 0-5), we think of a period as lasting for six years. Since a working life lasts about 45 years, we assume that the agent lives for seven periods so that \( T = 7 \). The subjective discount factor is assumed to equal \( \beta = 0.97^6 \approx 0.833 \) and the interest rate is \( r = 1/\beta - 1 \).

The intertemporal elasticity of substitution of consumption, \( \sigma \) is set to 2 and the Frisch elasticity of labor supply, \( \varepsilon \), is assigned the value 1/2. The age profile of wages is assumed to be quadratic with a coefficient of 0.04 on age (in years) and \(-0.0005\) on age squared. This is in line with our own estimates on Germany as well as results for the United States from, for example, Blau and Kahn (1997).
The parameters \( \psi_s \) are set to a common number \( \psi \); we set \( \psi = 0.7089 \) so that average hours worked over the life cycle is equal to one.\(^{10}\)

The number of children \( b_s \) are set so as to generate a total of two children over the life cycle. In the benchmark calibration, both children arrive when the parent is of age \( s = 2 \), corresponding to 26-30 years.

Unit day care costs are assumed to be 20 percent of the wage of a person of age \( s = 1 \) (20-25 years). This is line with evidence from U.S. Census Bureau (2005).

The labor tax rate in the absence of day care subsidies is set to 43 percent, which is the OECD average of the combined average effective labor and consumption taxes 1991-97 as reported in Carey and Tchilinguirian (2000).

With this benchmark calibration, the welfare gain from optimal day care subsidies (from a baseline of zero subsidies) corresponds to an increase in consumption of 0.89 percent. Meanwhile, the effect of the optimal policy on labor supply is to increase it by 30 percent during parenthood, decrease it at all other ages, and increase the average by a small amount. For details, see Table 1.

\(^{10}\) The precise value of \( \psi \) has to do only with the choice of units for labor supply and doesn’t matter for the welfare calculations.
Table 1: Effects of optimal policy on labor supply

<table>
<thead>
<tr>
<th>Age</th>
<th>Pre-reform</th>
<th>Post-reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0186</td>
<td>0.9947</td>
</tr>
<tr>
<td>2</td>
<td>0.6354</td>
<td>0.8268</td>
</tr>
<tr>
<td>3</td>
<td>1.0914</td>
<td>1.0658</td>
</tr>
<tr>
<td>4</td>
<td>1.0996</td>
<td>1.0738</td>
</tr>
<tr>
<td>5</td>
<td>1.0881</td>
<td>1.0626</td>
</tr>
<tr>
<td>6</td>
<td>1.0575</td>
<td>1.0327</td>
</tr>
<tr>
<td>7</td>
<td>1.0095</td>
<td>0.9858</td>
</tr>
<tr>
<td>Mean</td>
<td>1.0000</td>
<td>1.0060</td>
</tr>
</tbody>
</table>

We now consider the implications of various departures from our benchmark calibration. The results of this section can be understood in light of Proposition 1 by saying that the size of the effects of optimal reform depends on the size of the differences over the life cycle in the ratio of social to private marginal returns to working. When these ratios are smooth, there is not much for day care subsidies to accomplish; when they are very different, optimal day care subsidies has large effects on labor supply and on welfare. Important determinants of the differences between these ratios include the unit day care cost, the spacing of children and the initial level of taxation.

Table 2 details the results. On rows 2 and 3 we can see the importance of timing of children: if children arrive early, both the effect on parental labor supply and the welfare gains are larger. On row 4 we see the effects of spacing of children: if one child arrives at $s = 1$ and another at $s = 2$, then the welfare gains of optimal subsidies are reduced significantly. Indeed, if children are spaced evenly over the entire life-cycle, then the welfare gains are reduced to essentially zero. Row 5 shows that subsidized day care is less important in less tax-distorted economies. Row 6 considers an increase in the per-unit day care cost to 25 percent of a young person’s wage; this leads to a substantial increase in the welfare and employment effects.\(^{11}\)

\(^{11}\) For higher values of the day care costs, we encounter corner solutions. Since our purpose in the present
Apart from the determinants of differences in the ratio of social to private marginal returns to working, there are other aspects that are relevant as well, such as the specification of the period utility function. Rows 7 and 8 show the significance of changing the Frisch labor supply elasticity to a low value of 0.25 or a high value of 1.0. In row 9 we see the results from investigating the implications of making the disutility of labor higher when children are present. Specifically, we let $\psi_2$ be greater than $\psi_s$ for $s \neq 2$ in such a way that, in the absence of day care subsidies, parents with small children work only 45 percent of the average as opposed to about 64 percent when $\psi_s = \psi$ for all $s$. The results indicate, perhaps not surprisingly, that when the presence of children makes it more unpleasant to work, the benefits from subsidizing day care are reduced.

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section is not to provide a serious calibration but to get a sense of how results depend on parameters, there is no need to consider these higher values here. In Section 3 we do allow for corner solutions and hence take into account the possible effects of day care subsidies on the participation choice.

12 The 45 percent figure is close to what we find for German women; see Section 3.2.
Table 2: Sensitivity analysis

<table>
<thead>
<tr>
<th>Departure</th>
<th>Welfare gain</th>
<th>Effect on parental labor supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Benchmark</td>
<td>0.89%</td>
<td>+30%</td>
</tr>
<tr>
<td>2 Children at ( t = 1 )</td>
<td>1.22%</td>
<td>+38%</td>
</tr>
<tr>
<td>3 Children at ( t = 3 )</td>
<td>0.69%</td>
<td>+27%</td>
</tr>
<tr>
<td>4 One child in ( t = 1, 2 )</td>
<td>0.31%</td>
<td>+7%</td>
</tr>
<tr>
<td>5 Tax rate = 20%</td>
<td>0.06%</td>
<td>+7%</td>
</tr>
<tr>
<td>6 High day care cost</td>
<td>1.81%</td>
<td>+62%</td>
</tr>
<tr>
<td>7 Low labor supply elast.</td>
<td>0.56%</td>
<td>+14%</td>
</tr>
<tr>
<td>8 High labor supply elast.</td>
<td>1.17%</td>
<td>+71%</td>
</tr>
<tr>
<td>9 Child-dependent ( \psi )</td>
<td>0.64%</td>
<td>+31%</td>
</tr>
</tbody>
</table>

3 The calibrated model

The previous section taught us that details of the specification are important in determining the quantitative results. This motivates a serious calibration exercise, and that is what we present in this section.

As outlined in the Introduction, several departures from the stylized environment are necessary. First, we introduce heterogeneity with respect to marital status and the number and timing of births. Second, our model is designed to be consistent with the main features of the German tax-transfer system.

As stressed in the Introduction, our guiding principle in designing our model has been that it should replicate some key features of the German data. In particular, we want the model to match average hours worked across individuals of different marital status, age, gender and the presence or absence of young children, as presented in Figure 1. Some striking facts stand out from this Figure. First, the presence or absence of small children has very little impact on the labor supply of men. Second, small children matter a lot for
women’s labor supply. Finally, the age-hours profile is fairly flat in spite of the increasing age-wage profile that we observe in German data.

In order to capture these facts to a reasonable extent, we allow the disutility of labor $\psi$ to depend on age, gender, marital status and the presence of small children. We also assume that total work is the sum of market work and housework, where the required amount of housework is allowed to depend on household characteristics; in particular, the presence of young children. The significance of this latter feature is that it allows the possibly lower market hours of parents to be the result of a stronger aversion to market work at the margin rather than a stronger aversion to every hour of market work; this in turn has implications for the precise welfare effects of day care reform, though quantitatively the difference is not very big.

3.1 Description of the model

Households live for $T$ periods. They work in periods 1, 2, $\ldots$ $R$, where $R < T$. There are three types of households: couples, single men and single women. Let $x = (s, b, q)$ where $s$ denotes age, $b$ is a vector representing the set of children; its $k$th element represents the number of children of age $k$; $k \in \{1, 2, \ldots, K\}$, and $q \in \{\text{single, married}\}$ denotes marital status. Let $g$ denote gender, where $g = m$ for men and $g = w$ for women.

As in the stylized environment, parents in the model are indifferent to the presence of children; time spent with children is like leisure spent without children, no better and no worse. In order to generate heterogeneity with respect to the number and timing of births, we let children arrive at random in the benchmark specification, and the implied supervision cost risk is assumed to be uninsurable.
3.1.1 Singles

A single person of gender $g$ chooses, at each age $s$, labor supply $h_s$ and savings $a_{s+1}$ so as to maximize

$$
E \left[ \sum_{s=1}^{T} \beta^s w^g(x_s, c_s, \ell_s) \right]
$$

subject to

$$a_{s+1} + (1 + \tau^c)c_s = (1 + r)a_s + y_s - \tau(x_s, y_s) - (1 - \theta)db_s^y h_s + \zeta(x_s, y_s, a_s),$$

$$y_s = w^g(x_s) h_s,$$

$$\ell_s = h_s + n_s,$$

$$n_s = \xi^g(b_s)$$

and $a_1 = a_{T+1} = 0$ where

$$w^g(x, c, \ell) = \eta(x)^{\sigma \frac{c^{\varepsilon - 1}}{1 - \sigma} - \psi^g(x) \ell^{1+1/\varepsilon}}.$$ 

Here $\eta(x)$ is the number of consumption equivalents in the household,$^{13}$ $c_s$ is aggregate household consumption, $\ell_s$ is the total amount of work; $h_s$ is market work, $n_s$ is homework,$^{14}$ $b_y^g$ is the number of young children, $\tau^c$ is the consumption tax, $\tau(x, y)$ is the tax function, and $\zeta(x, y, a)$ is the amount of income and asset tested family assistance that the household is entitled to.

The wage depends on age, gender and marital status.

$$\ln w^g(x) = \gamma_0 + \gamma_1 s + \gamma_2 s^2 + \gamma_3 I_{\{g=m\}} + \gamma_4 I_{\{q=married\}} + \gamma_5 I_{\{g=m \& q=married\}}.$$ 

$^{13}$ This utility specification has the feature that households want to smooth consumption per consumption equivalent which means that, in the absence of uncertainty, they would consume in proportion to the size of the household over the lifecycle.

$^{14}$ We have also considered a specification where $n_s$ is not allowed to depend on gender or the number of children, and found similar welfare effects to the ones presented in the paper.
The probability of newborns arriving depends on three things: your age, your gender and the number of children you already have. Thus the vector $b$ evolves according to

$$P(b_{1,s+1} = j) = p_{s,b,s}^g$$

and

$$b_{i,s+1} = b_{i-1,s}.$$

### 3.1.2 Couples

Couples choose the labor supply of the man, the labor supply of the woman, and savings so as to maximize

$$E \left[ \sum_{s=1}^{T} \beta^s u(x_s, c_s, \ell^m_s, \ell^f_s) \right]$$

subject to

$$a_{s+1} + (1 + \tau^c)c_s = (1 + r)a_t + y_s - \tau(x_s, y_s) - (1 - \theta)db_s^y \min\{h^m_s, h^f_s\} + \zeta(x_s, y_s, a_s),$$

$$y_s = w^m_s h^m_s + w^f_s h^f_s$$

and

$$u(x, c, \ell^m, \ell^f) = \eta(x)^\sigma \frac{\sigma^{1-\sigma} - \psi^m(x)(\ell^m)^{1+1/\varepsilon}}{1-\sigma} - \psi^f(x)(\ell^f)^{1+1/\varepsilon} \frac{1}{1+1/\varepsilon}.$$  

The idea behind the $\min\{h^m_s, h^f_s\}$ expression in the budget constraint is that day care has to be purchased for every hour that both parents work in the market and that shift work is not allowed. Total male work equals

$$\ell^m_s = h^m_s + n^m_s$$

and total female work equals

$$\ell^f_s = h^f_s + n^f_s.$$  

The homework constraint is

$$n^m + n^f = \xi(b_s).$$
The vector $b$ evolves according to

$$P(b_{1,s+1} = j) = p_{s,b,s,j}$$

and

$$b_{j,s+1} = b_{j-1,s}.$$

### 3.2 Calibration

The length of a time period is six years and people are assumed to live for $T = 10$ periods, the last three periods being spent in retirement so that $R = 7$. We think of the first period of adult life as 20-25 years of age. Children remain children for three periods; $K = 3$, implying that “young” children are in the age range 0-5 years.

The parameter $\beta$ is set to 0.97$^6$ and the interest rate $r$ is set so that the subjective and market discount rates are equal; $r = 1/\beta - 1$.

The parameter $\varepsilon$, representing the Frisch elasticity of labor supply, is set to 0.5; see Domeij and Flodén (2006) and Pistaferri (2003). The reciprocal of the intertemporal elasticity of substitution for consumption, $\sigma$, is set to 2.

The consumption equivalents, $\eta$, are calculated using the OECD consumption equivalence scale. According to this scale, the first adult counts as one unit, the second adult as 0.7 and each child as 0.5.

For all the German data, we use the database (G)SOEP, see http://www.diw.de. A household is defined as a set of people living at the same address. Two adults living in the same household are counted as a couple if (1) they are the two eldest in the household and (2) they are of the opposite sex. The birth probabilities are based on the distribution of young children across households of various characteristics. These probabilities imply that 18.7 percent of the model population is between 0 and 17 years old; according to Eurostat, the corresponding number in Germany was 18.2 percent in 2004. They also

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\(^{15}\) For a more detailed description, see Appendix A.
imply that 4.7 percent of single men, 8.2 percent of single women, and 16.3 percent of couples have small (0-5 years old) children at any given point in time.

The home production constraint functions $\xi^g$ and $\xi$ are based on the number of hours of housework per week reported in 2003 by single males, single females and couples, respectively as a function of the number of children in the household. More specifically, we regress number of hours worked at home on a constant ($\mu_1$), the number of children less than 6 years of age ($\mu_2$) and the number of children above six years of age ($\mu_3$). To focus on necessary household chores we only include individuals who work at least 1200 hours per year in the market. For couples we find $\mu = [19.079 \ 0.557 \ 3.445]$. For single men and women we find $[6.592 \ 0.086 \ -0.772]$ and $[10.478 \ 2.230 \ 4.323]$, respectively.\(^\text{16}\) (It is noticeable that single men spend less time on housework than single women.)

The vector $\gamma$ is based on a regression of the log wage on time dummies, age, gender and marital status. The wage is defined as the ratio of labor earnings to hours worked. Specifically, the regression coefficients are $\gamma_1 = 0.0468$ (the age premium), $\gamma_2 = -0.0005$ (the coefficient on age squared), $\gamma_3 = 0.2048$ (the male wage premium), $\gamma_4 = -0.0406$ (the gender-independent cohabitation effect) and, finally , $\gamma_5 = 0.0095$ (the male-specific cohabitation effect).

The tax function $\tau$ is modelled following the description of the German tax system in OECD (2005). For example, the average and marginal tax rates for a single individual with one child are displayed in Figure 2. The consumption tax is set to $\tau^c = 0.158$ based on Carey and Tchilinguirian (2000).\(^\text{17}\) The family assistance function $\zeta$ is modelled following the description of the German family assistance policies in Adema et al (2003). For example, a single woman with one small child receives €884 per month and a married couple with one child 7 years or younger and one child older than 7 receives €1263 per month. See Appendix B for details.

The parameter $d$ determining the real cost of day care is set to €2 per hour. This

\(^{16}\) We obtain very similar regression coefficients if we constrain the sample to those working at least 2000 hours per year.

\(^{17}\) The German sales tax on food is 7% and 19% on other consumption goods.
corresponds to 20\% of the wage of a young woman (age 20-25). We also consider a high-cost scenario where \( d \) is set so as to correspond to €4 per hour. Ideally this number should be set equal to the average actual cost of day care in Germany. However, data on this cost are hard to come by because of the small size of the daycare sector in Germany.\(^{18}\) On the other hand, both the United States and Sweden do provide good data on the costs of day care. According to U.S. Census Bureau (2005), the average ratio of day care costs to earnings for full-time working mothers is about 20 percent in the United States. In Sweden, the cost of a day care spot varies quite a bit across municipalities. In the municipality of Täby, where day care costs are the lowest in the nation, the costs correspond to about 38 percent of the wage of a young woman (age 18-24). Meanwhile, the Swedish national average cost of day care is about 53 percent of young women’s wages.\(^{19}\) The two values of \( d \) that we consider, then, correspond to costs that are in the same order of magnitude as the ones we observe in Sweden and the United States.

Note that if labor is the only input into the day care production technology, then our \( d \) implies that each day care worker can take care of 5 children. This is consistent with evidence from most OECD countries, where there are five to seven children per day care worker when children are in the age-range 0-3.\(^{20}\) It is also worth noting that in the steady-state equilibrium, about 0.56 percent of GNP is spent on day care; the corresponding number in Germany is 0.59 percent.\(^{21}\)

The disutility of labor functions \( \psi^g(x) \) are set so as to match labor supply; by marital status (cohabiting or single), over the life-cycle, and for those with and without small children. Specifically, we match average hours worked by gender and marital status for people with and without small children in the age range 26-43 and for people in the age range 44-61.\(^{22}\) Parameter values are given in Table 3 and the results of the calibration

\(^{18}\) Wrohlich (2005) documents the extent to which demand for subsidized day care falls short of the rather small supply of it.

\(^{19}\) Source: SCB (2007).

\(^{20}\) Source: OECD (2007), chart 6.3.

\(^{21}\) Source: OECD (2006).

\(^{22}\) We exclude the hours worked by 18-25-year-olds from the data since many people in this age group
are shown in Table 4.

are students; this fact has a big impact on labor supply in this age range (see Figure 1) in a way that our model is not designed to capture.
<table>
<thead>
<tr>
<th></th>
<th>Cohabiting</th>
<th></th>
<th>Single</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Females</td>
<td>Males</td>
<td>Females</td>
<td>Males</td>
</tr>
<tr>
<td>Age 20-43, (b^y = 0)</td>
<td>20.5</td>
<td>40.0</td>
<td>44.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Age 20-43, (b^y &gt; 0)</td>
<td>33.5</td>
<td>43.0</td>
<td>29.0</td>
<td>24.0</td>
</tr>
<tr>
<td>Age 44-61</td>
<td>23.5</td>
<td>42.0</td>
<td>50.0</td>
<td>32.0</td>
</tr>
</tbody>
</table>
### Table 4: Hours per year

<table>
<thead>
<tr>
<th></th>
<th>Data 1 26-43</th>
<th>Data 2 44-61</th>
<th>Model 1 26-43</th>
<th>Model 2 44-61</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ages</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohabiting men with children</td>
<td>2199</td>
<td>–</td>
<td>2219</td>
<td>–</td>
</tr>
<tr>
<td>Cohabiting men without children</td>
<td>2238 2265</td>
<td>2212 2244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohabiting women with children</td>
<td>709</td>
<td>–</td>
<td>745</td>
<td>–</td>
</tr>
<tr>
<td>Cohabiting women without children</td>
<td>1616 1650</td>
<td>1612 1654</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single men with children</td>
<td>2171</td>
<td>–</td>
<td>2192</td>
<td>–</td>
</tr>
<tr>
<td>Single women with children</td>
<td>798</td>
<td>–</td>
<td>752</td>
<td>–</td>
</tr>
<tr>
<td>Single women without children</td>
<td>1719 1794</td>
<td>1698 1826</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: A child in this context is one below the age of 6.

Data source: GSOEP

Figures 3 and 4 shows that the model also captures the complete life-cycle profile of hours worked for all individuals except young single women with children. The life-cycle profile of average hours worked for single women with children turns out to be rather more difficult to capture, however. The problem is that the profile is quite flat. Meanwhile, the German social assistance system is such as to discourage part-time work. For low positive hours, earnings are clawed back by reducing social assistance payments, so that the effective marginal income tax rate is very close to 100 percent. This means that small wage differences are associated with large differences in labor supply as single mothers move in and out of the zero corner solution. Meanwhile, the age-wage profile is increasing, so that the only way to match average hours while assuming a constant work aversion parameter $\psi$ is to make the youngest women stay out of the labor force and the slightly older women work more than in the data. Obviously, this problem is not impossible to solve. One way is to allow for more age-specificity of the work-aversion parameter. In order for this to work, the youngest women have to be assumed to be much less averse to work than their slightly older selves; this seems quite artificial. Another way to solve the problem is to add further dimensions of heterogeneity, e.g. permanent
or transitory productivity differences. We have experimented with this approach, and it shows much promise when it comes to matching the data, but on the other hand it makes the model highly time-consuming to solve. In particular, the maximization problem of couples quickly becomes intractable, since we are forced to consider not just all possible levels of productivity, but all possible pairs of productivity levels.

3.3 Reforms

We now consider various reforms, all involving day care subsidies at various rates; we also consider a policy of making day care expenses tax deductible. The reforms are financed by a vertical shift of the marginal tax schedule for those income levels where taxes are paid. The new tax schedule is constant over time. The reforms are designed in such a way that those already born at the moment of reform are not affected; they pay the taxes under the old system and get no subsidies. We adopt this approach in order to avoid any issues of intergenerational redistribution. In particular, we want to avoid the result that the initial old and middle-aged lose from the reform simply because they pay for it but get nothing in return. Such a result would not be particularly interesting. We solve for the transition; this is necessary in order to check government budget balance, but in terms of allocations this transition is trivial because of the grandfather clause in our reforms.

In Figure 5-6 we plot the effects on labor supply, the change in the marginal tax rate and the welfare effects of various subsidy rates and of deductibility. Figure 5 show substantial increases in hours worked for cohabiting women, single men and especially for single women. Labor supply of cohabiting men is almost constant.

Figure 6 shows the welfare gains expressed as equivalent percentage consumption increases for individuals and couples of age 1 (20-25 years old). Notice that deductibility, in contrast to what we found in the stylized model, is not the optimal policy in the current framework. Deductibility does deliver a sizeable welfare gain for all groups, but we can do better than that. Figure 6 shows that welfare increases for everyone as a function of the subsidy rate up to a rate of about 90 percent, which implies a much greater extent of subsidization
than deductibility does. At that rate, the welfare gain for couples corresponds to a 1.8 percent increase in consumption; for single men the number is 1.6 percent, and for single women it is 1.9 percent. Meanwhile, aggregate labor supply increases by 1.3 percent, GNP increases by 1.5 percent and resources allocated to day care goes up from 0.56 percent of GNP to 1.04 percent. At the same time, means-tested family assistance falls from 0.45 percent of GNP to 0.12 percent. This implies that the reform can be financed by an increase in the marginal tax rate of just 0.14 percentage points; this means that the tax/GNP ratio falls from 26.3 percent to 26.2 percent.

A striking aspect of the welfare results is that the gains are so similar across groups. A priori one might expect day care subsidies to benefit single women much more than the other groups. After all, day care is more of a necessity for single women than it is for single men, who on average have fewer children, or for couples, who have the option of letting one spouse work and the other take care of the children. However, there are factors working in the opposite direction. The most important one is that Germany’s family assistance system tends to favour single women with children who do not work. An increase in day care subsidies decreases the extent of redistribution from couples and single men to single women with children as the labor supply of single women with children goes up. A related factor is that the fall in family assistance payments resulting from an increase in day care subsidies makes it possible to finance the reform with a small or even negative tax increase, as shown in Figure 6. This means that the reform heavily favors those households that supply many hours in the absence of day care subsidies even when they have small children, i.e. couples and single men.

While the result that the welfare gains are similar for all three groups seems to depend on the day care subsidies being nearly self-financing, the result that all groups gain substantially from large subsidies does not hinge on this feature. We see this if we perform some sensitivity analysis with respect to the cost of day care \(d\). Consider an economy where the cost of day care is doubled to \(\€4\) per hour and child. Figure 7 shows that all household types still experience sizable gains from high subsidies. Meanwhile, these high subsidies are associated with significant increases in marginal tax rates. As a results couples and
single men prefer a lower subsidy rate than in the benchmark calibration; their preferred subsidy is around 60 percent in this case. Single women on the other hand prefer a 100 percent subsidy in this case as well, and they experience much larger gains than in the benchmark.

3.4 Decomposing the welfare gains

There are several motives for subsidizing day care in the present environment. As in the stylized environment, there is a tax smoothing argument. In addition, there is an insurance motive due to the risk associated with the random number and timing of births, as well as a redistributive motive due to the co-existence of three distinct groups (single men, single women, and couples).

In order to analyze how the welfare gains depend on each of these motives, we consider two alternative economies in addition to the benchmark. In the first economy, we remove the insurance motive by making the timing and number of births deterministic (though still heterogeneous). In the second, we also remove the between-group redistributive motive. We retain the assumption of perfect foresight with respect to the number and timing of births, and impose a separate government budget constraint for each household type so that redistribution across types is ruled out. In both economies we impose the same distribution of characteristics (age, gender, marital status, number of children of various ages) as in the benchmark economy, except that for numerical reasons we constrain the analysis to the 95 percent most common child-paths over the life-cycle for each household type. In practice this means we drop households that give birth to 3 or more children in any six-year period.

Figure 8 shows the welfare gains for each group and the tax changes associated with each reform in the three economies. This figure calls for a few remarks. First, removing the insurance motive for day care subsidy does reduce the welfare gains, but the reduction is small and the welfare gain from a 90 percent subsidy is still in the order of 1.5-2.0 percent. Second, the redistributive motive does indeed matter, but perhaps in a somewhat
unexpected way. If each group has to finance the subsidy by itself, then the welfare gains are still positive for all groups, but for couples they are somewhat smaller, for single men they are tiny and for single women they increase by a factor of two. How can this be? Well, single women with children do not work very much in the initial steady state. Instead they rely to a large extent on welfare benefits; the family assistance benefits are 9 percent of GNP in the initial steady state of this economy compared to 0.45 percent in the benchmark economy. When day care is subsidized, single women with children increase their labor supply dramatically and dependence on welfare benefits is drastically reduced. This leads to a fall in the marginal tax rate by several percentage points and the welfare gains are therefore huge.

Single men with children, on the other hand, already work a lot in the initial steady state, and hence do not receive family assistance. When the cost of subsidized day care is shared with other groups, a subsidy is essentially a windfall gain since the subsidy is basically self-financing. When single men have to finance their own daycare subsidy, the marginal tax rate must increase because there is no reduction in the already non-existent family assistance payments. The welfare gain is therefore much smaller, though still positive because of the tax smoothing argument. The reason why the gains from smoothing are so small in this case is that only a rather small fraction of single men have any children so that there isn’t much to smooth.

The results for couples are conceptually similar to those for single men. In the initial steady state, no couples are entitled to family assistance because their joint income is too large. Subsidized day care increases labor supply of cohabiting women by a sizeable amount, but this comes at a cost of a higher marginal tax rate, which is a disincentive for non-parents to work. Nevertheless, the benefits outweigh the costs. The welfare gains of a 90 percent subsidy is above 1 percent even though the marginal tax rate is increased by almost 1 percent. This is again the tax smoothing argument at work, but it is much more significant for couples than it is for single men, not only because couples have more children, but because of the high degree of heterogeneity across ages and households of the number of children. This is consistent with what we found in Section 2.3, namely
that the welfare gains from day care subsidies are larger the more heterogeneity there is in this respect.

4 Concluding remarks

In this paper we have found that a subsidy on daycare raises welfare by improving the trade-off between consumption and leisure in a life-cycle setting with distortionary labor taxes and exogenous government expenditure. In a stylized environment, we proved that the optimal policy is to make daycare expenses tax deductible.

Applying our analysis to the case of Germany, we found that the welfare gains associated with subsidizing day care would be considerable, and that the maximum gains would be realized if day care were to be subsidized to an even greater extent than what tax deductibility implies.

In our analysis we have not considered the possible effects of day care on child welfare and development. It is an open question whether taking these effects into account would weaken or strengthen our results. There is some evidence that day care has a positive effect on child development and parental welfare; see OECD (2006). If we trust this evidence, then our assessment of the benefits of day care subsidies are conservative. On the other hand, Baker et al. (2005) find some contrary evidence from the province of Québec, so that issue remains unsettled. Either way, there is a strong efficiency case to be made for day care subsidies that must be weighed against any other possible effects.

Appendix A Data

Our main source of data is the German Socio-Economic Panel (SOEP). More information about SOEP can be found at http://www.diw.de/english/soep/29012.html.
A.1 Measurement of marital status

Conceptually, we are not interested in whether anyone is legally married, only whether they are living with one other person in a relationship that is economically similar to marriage. The approach we adopt is an imperfect attempt to capture that notion. What we do is to group people who belong to the same household and try to pick out among the members of a given household a pair of individuals who appear to be in a marriage-like relationship. If the household has just one member, the situation is clear: we then consider the sole household member to be single. On the other hand, if the household has more than one member, then we order the household members by age and consider the two eldest. Occasionally it happens that there are more than two eldest members; we then randomly choose two of them. If these two individuals turn out to be of the opposite sex and if the age gap is strictly less than 20 years, we consider them to be married. The exclusion of same-sex couples is there because we are interested in using marital status information to draw inferences about the probability of the arrival of children, and the exclusion of couples 20 years apart or more in age is there to exclude single parents living with their children from being assigned as married.

Evidently our approach excludes some couples that, conceptually speaking, are married in the economic sense of that term. Nevertheless, we take the view that our approach yields an acceptable approximation.

A.2 Probabilities of having children

The probabilities of having children for singles $p^g_{s,n,s,j}$ and couples $p_{s,n,s,j}$ are set so as to match the number of young children that various categories of adults have in the household. For this purpose, we use GSOEP data from 1984 to 2004. In the files Xkind (where X is a letter representing the year), there is an entry corresponding to each child in the sample with information on birthyear and a number identifying the household. We then merge this data with the information on marital status, age, and, if unmarried, the
gender of each apparent parent (any adult in the same household as the child), and remove those adult-children pairs that are such that the child is less than 18 years younger than the parent. We then consider those children that are between 0 and 5 years old; these are considered “newborn” for the purpose of the calibration. The probabilities of having 0, 1, 2 and 3 new children for potential parents categorized by marital status, age, gender and number of children aged 6-17 are then simply given by the corresponding fractions in the data, e.g. the fraction of 26-30-year-old single women with two children aged 6-17 who have exactly one child between 0 and 5.

A.3 Housework

Data on hours of housework are only available in any significant sample size in 2003. For that year, we use the variables TP1007 (hours of housework weekdays), TP1008 (hours of housework Saturdays), and TP1009 (hours of housework Sundays). (We exclude observations on individuals below the age of 17.) We then use data on household membership to draw conclusions about marital status, and then merge these data with data on the presence of children of various ages. Finally, we run separate regressions for couples, single men and single women, with housework per week on the left hand side and, on the right hand side, a constant and three variables representing the number of children in three age groups: 0-5, 6-11 and 12-17 years.

A.4 Life-Cycle hours profiles

The data on hours are based on the GSOEP variable “average hours worked per week”, called, for example, BP41 in 1985 and NP47 in 1997. What we would like to do is to match the life-cycle profile of hours worked for the first cohort to be affected in a major way by day care reform, i.e. those who are young today. For obvious reasons, there is no data on the entire life-cycle profile of hours for this cohort. Therefore, we use the entire GSOEP panel from 1984 to 2004 and regress hours on age and cohort dummies. The data
presented in Figure 1 are the predicted values for the cohort born between 1970 and 1984.

A.5 Wages

Wages are defined as individual annual labor earnings divided by annual hours worked. The names of these GSOEP variables are e11101XX and i11110XX, where the XX stands for the year.

Appendix B Tax and transfer system

B.1 Tax system

The German tax system is modelled following the description in OECD (2005). Spouses are assessed jointly using the income splitting method. We define taxable income, $x$, as earnings less a basic allowance which consists of three parts. First, there is a allowance of €1308 for single parents. Second, there is a work-related allowance of €920 per employed person. Third, there is lump-sum allowance of €36 for singles and €72 for couples.\footnote{Legally, the child benefit is treated as a tax credit, but in cases where the tax liability is less than the tax credit, the difference is paid out as a cash transfer. The child benefit is equivalent to a direct cash transfer, and we have therefore chosen to model it as such.}

The tax liability, $T$, is then calculated as follows. Let $y = (x - 7664)/1000$ and $z = (x - 12739)/10000$.

$$T = \begin{cases} 0 & \text{if } x \leq 7664, \\ (793.10y + 1600) y & \text{if } 7664 < x \leq 12739, \\ (265.78z + 2045) z + 1016 & \text{if } 12739 < x \leq 52152, \\ (0.48x - 10410) & \text{if } 52152 < x. \end{cases}$$

These formulae are used directly to calculate the tax liability for a single individual. For couples, we apply these formulae on half the taxable income and then double the resulting
amount to arrive at the tax liability. A “solidarity surcharge” (Solidaritätszuschlag) is then levied at 5.5 percent of the tax liability subject to an exemption limit of €972 for singles and €1944 for couples. Above the exemption limits the solidarity surcharge is phased on at a higher rate of 20 percent until it equals 5.5 percent of the tax liability. Total tax payments are equal to the tax liability plus the solidarity surcharge. Note finally that we do not model social security contributions or benefits. The reason is that a large part of benefits are tied to contributions in a more or less actuarially fair way which means that the system of social insurance contributions and benefits is not distortive in the way that income taxes and public purchases are.

B.2 Family assistance

German family assistance policies are modelled following the description in Adema and Kahl (2003). First, there is a universal child benefit (kindergeld) of €154 per child and month. Second, there is an gross income tested child rearing benefit of €307 per month for children below 2 years of age. Claimants are allowed to work up to 30 hours per week. Third, households with little earnings or assets are entitled to several additional benefits. For example, a single parent receives a standard monthly payment of €286 plus €395 for housing, €67 for heating, €121 for larger purchases such as clothing and furniture, and child payments of €169 per child not yet 7 years of age and €232 for per child 7 years and older. The sum of these family assistance payments are reduced by 85% of earnings net of taxes. There are also asset limits above which no family benefits are paid out. Table A1 summarizes the German family assistance benefits.

24 In order to be consistent with the examples of fictitious taxpayers in OECD (2005), the solidarity surcharge is only levied on households without children and on working couples with children.
Table B1: German family assistance policies per month in euros

<table>
<thead>
<tr>
<th></th>
<th>Single without children</th>
<th>Single with children</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Child benefit per child</strong></td>
<td></td>
<td>154</td>
</tr>
<tr>
<td><strong>Child rearing per child not yet 2 years of age</strong></td>
<td></td>
<td>307</td>
</tr>
<tr>
<td><strong>Gross annual income limits</strong></td>
<td></td>
<td>19713</td>
</tr>
<tr>
<td><strong>Hours limit (hours/week)</strong></td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

Net-earnings and asset based benefits

<table>
<thead>
<tr>
<th></th>
<th>Single</th>
<th>Couple</th>
<th>Single</th>
<th>Couple</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard payment</strong></td>
<td>286</td>
<td>515</td>
<td>286</td>
<td>515</td>
</tr>
<tr>
<td><strong>Average housing</strong></td>
<td>357</td>
<td>335</td>
<td>395</td>
<td>444</td>
</tr>
<tr>
<td><strong>Average heating</strong></td>
<td>44</td>
<td>60</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td><strong>Large purchases</strong></td>
<td>46</td>
<td>85</td>
<td>121</td>
<td>159</td>
</tr>
<tr>
<td><strong>Children not yet 7 years of age (per child)</strong></td>
<td>15</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Children 7 years and older (per child)</strong></td>
<td>78</td>
<td>78</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Marginal tax rate on net earnings (%)</strong></td>
<td>85</td>
<td>85</td>
<td>85</td>
<td>85</td>
</tr>
<tr>
<td><strong>Assets limits</strong></td>
<td>1256</td>
<td>1870</td>
<td>1256</td>
<td>1870</td>
</tr>
</tbody>
</table>

\(a\) These income limits are weighted averages of the gross annual income limits during the first 6 months (€51130 for couples and €38350 for singles) and the income limits after the 7th month (€16470 for couples and €13500 for singles). The income limits are raised by €2454 per additional child.

\(b\) These assets limits are raised by €256 per child.

References


Figure 1: Average hours worked over the life-cycle in Germany. Solid lines refer to people without small children, dashed lines to people with small children. For details, see Appendix A.
Figure 2: Average and marginal tax rates for a single individual with one child.
Figure 3: Average hours worked over the life-cycle in the data and in the model for cohabiting men and women. Solid lines refer to the data, dashed lines to model predictions.
Figure 4: Average hours worked over the life-cycle in the data and in the model for single men and women. Solid lines refer to the data, dashed lines to model predictions.
Figure 5: Average hours worked for day care subsidies between -10 and 100 percent. Solid lines refer to people aged 20-43 without small children, dashed lines to people aged 44-61, and dotted lines refer to people aged 20-43 with small children.
Figure 6: Welfare gains and changes in the marginal tax rate for day care subsidies between -10 and 100 percent, depicted by solid lines. The dotted lines represent welfare gains and the change in the tax rate associated with making day care subsidies tax deductible.
Figure 7: High and low day care costs. Solid lines represent the benchmark calibration where the day care cost is €2 per hour. Dashed lines represent the case where the day care cost is €4 per hour.
Figure 8: Decomposing the welfare gains. Solid lines refer to our benchmark specification. Dashed lines refer to the case where households have perfect foresight. Dotted lines depict results from a specification where households have perfect foresight and where the three types of household are isolated in three separate economies.