ABSTRACT
This is the second tech note in Spectra Quest’s signal processing series. This tech note focuses on two techniques associated with rotating machinery fault diagnosis: envelope and cepstrum analyses. Rotating machinery faults usually cause strong harmonics and sidebands. Both the envelope and cepstrum analyses are useful tools to identify the fault frequencies and distinguish them from other frequency contents. Examples of bearing faults, broken rotor bar, and gearbox faults have been presented to demonstrate how to use the software package and how to interpret results. All the experimental data were acquired from Spectra Quest’s Machinery Faults Simulator.

1. INTRODUCTION
Rotating/reciprocating machinery produces vibration signatures depending upon the mechanism involved. Faults may occur at motor, rolling element bearings, gearboxes, belts, fans and other electrical/mechanical components. It is strongly necessary to detect these problems at an early stage and to avoid serious damage and catastrophic failure. The purpose of analysis is to identify the fault frequencies so that root cause can be addressed and corrective action can be taken.

Rotating machinery faults usually associate with strong harmonics and sidebands. Therefore, the fault frequencies can be distinguished from the other frequency contents by identifying the harmonics or sideband components. Envelope analysis is a useful tool to extract the sidebands caused by amplitude modulation, while cepstrum analysis is to separate harmonic families.

In this tech note, the fundamentals of envelope and cepstrum analyses are briefly introduced with examples of rolling element, broken rotor bar, and gearbox faults. The physics associated with the faults are also discussed. The readers do not need to worry about the theories behind the algorithms.

2. ENVELOPE ANALYSIS
For rolling element bearings, when the rolling elements strike a local fault on the inner or outer race, or a fault on a rolling element strikes the inner or outer race, an impact is produced. The bearing frequencies can be categorized as BPFO (ball passing frequency outer race), BPFI (ball passing frequency inner race), BFF (ball fault frequency), and FTF (fundamental train frequency) [1]. Please see the last page of this tech note for the calculation of these fault frequencies. Like rolling element bearings, a faulted gear tooth also generates impact once per revolution when meshing with the other gear [2].

The impacts generated by gearbox and rolling element faults superimpose upon the vibration
signal, resulting in amplitude modulation as shown in Fig. 1, and thus cause sidebands in the spectrum around the frequency bins associated with the vibration signal. The sidebands mingle with the frequency components of the vibration signal so that it is hard to distinguish them in the spectrum. Impacts in time domain generate many harmonics extending to very high frequency in frequency domain. Often some of these harmonics excite resonance in structure, bearings or sensors. Exact location of the resonance is usually not known and cannot be determine easily. However, the resonance amplifies the modulating and carrier signals. Envelope analysis when applied in this region is a useful tool for amplitude demodulation. It should be noted that it is not easy to relate the amplitude of the signal to the fault severity. Envelope analysis is a useful tool for amplitude demodulation. The envelope analysis function in VibraQuest is based on an improved Hilbert transform method. This improved demodulation method attenuates the influences from high frequency contents and makes the envelope frequency easier to identify.

Figure 1 illustrates a simulated amplitude modulated sinusoidal signal. The vibration signal is called the carrier. The red curve indicates the envelope which is directly caused by the impacts mentioned above. The envelope analysis is to extract the frequency of the envelope so that the faults caused by rolling element bearing or gearbox can be identified.

Figure 2 shows a real acceleration signal acquired on a bearing with outer race fault. The running speed of the shaft is 1800 RPM. The sampling rate is 102.4 kHz. Please refer to [1] for detailed experimental setup and discussions. In Fig. 2 (a) we can see the spikes caused by balls passing the local fault of the outer race. The time interval between two adjacent spikes is roughly 0.0095 second. In the spectrum, Fig. 2 (b), it is clearly seen that sidebands around \( f_c \) at 948 Hz. The difference between the carrier frequency and the sideband is about 105 Hz. The spectrum shows
more than one sideband on both sides of the carrier frequency. In Fig. 2 (c) the envelope frequency at 105.45 Hz (= 1/0.0095) and its second harmonic at 210.9 Hz are correctly extracted in the envelop spectrum.

Figure 2. (a) Acceleration signal acquired on a bearing with severely faulted outer race. (b) The carrier frequency and its sidebands. (c) Envelope frequency shows the BPFO.

The envelope analysis panel presents the amplitude spectrum or the power spectrum for the user to simply select the frequency range in which the sidebands occur. The software will automatically calculate the envelope frequency and display the envelope spectrum.
The envelope analysis algorithm used VibraQuest is based on an improved Hilbert transform in two senses. First, it involves a frequency band shift to lower frequency. Such an action leads to narrower frequency range of the envelope spectrum. Since the envelope frequency is at a much lower frequency than the carrier frequency, it is not necessary to keep the wide frequency band for the envelope spectrum. From the engineering point of view, the envelope frequency is thus easier to be identified. Second, before taking the FFT to the extracted envelope, the envelope is squared, which can effectively reduce the harmonics of the envelope frequency at high frequencies.

The envelope analysis is also available in VibraQuest’s MCSA module. Induction motors are commonly used in industrial applications. Motor current signature analysis (MCSA) is a useful analysis and condition monitoring technique for the health of induction motors, since many motor faults cannot be detected from the vibration signals. Air-gap eccentricity, broken rotor bars, bearing damages, and time-varying load all cause sidebands in the current spectrum. Figure 3. (a) Motor current signal obtained from a motor with 6 broken rotor bars. (b) Its spectrum. (c) Envelope spectrum.
3 (a) shows a typical time waveform of the current signal obtained from a motor with 6 broken rotor bars. Its spectrum displayed in the dB scale show the line frequency (33.5 Hz) and its harmonics. Strong sidebands, caused by the slip frequency, can be seen around all the harmonics. Please refer to [3] and [4] for detailed experimental setup by using Spectra Quest’s Machinery Fault Simulator and theoretical background of MCSA. By using the same envelope analysis procedure, the envelope frequency 3.2 Hz is extracted for the slip frequency detection.

3. CEPSTRUM ANALYSIS

Cepstrum, which is an anagram of spectrum, is a nonlinear signal processing technique used to identify and separate harmonic families in the spectra of gearbox and bearing signals. Cepstrum also finds it application in echo cancellation and speech signal processing. Table I compares the terms used in the spectral and cepstral analyses. The calculation of cepstrum involves the inverse Fourier transform of the natural logarithm of a kind of spectrum. Exact definitions vary across the literature. Given a real signal $x[n]$, different cepstrum forms can be found:

complex cepstrum: $C_{\text{cplx}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log|X(e^{i\omega})| e^{i\omega} d\omega$  

real cepstrum: $C_{\text{real}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log|X(e^{i\omega})| e^{i\omega} d\omega$  

power cepstrum: $C_{\text{power}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log|XX^*| e^{i\omega} d\omega$

Since both the Fourier transform and the inverse Fourier transform are complex-domain processes, the cepstrum is complex if the phase information of the original time waveform is preserved. The complex cepstrum has the corresponding inverse complex cepstrum. In this case the time waveform can be reconstructed from a modified cepstrum. Therefore the complex cepstrum can be used for noise reduction and signal separation, such as echo cancellation.

On the other hand, if the input of the inverse Fourier transform is real (no phase information), for example, the power spectrum, or the magnitude of the Fourier transform of the signal, the cepstrum is real-valued. Even though the real-valued cepstrum cannot be reconstructed back to the time domain, we still can “lifter” a harmonic family in the quefrency domain and obtain a liftered spectrum.

<table>
<thead>
<tr>
<th>Spectral analysis</th>
<th>Cepstral analysis</th>
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<tbody>
<tr>
<td>spectrum</td>
<td>cepstrum</td>
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<tr>
<td>frequency (unit: Hz)</td>
<td>quefrency (unit: second)</td>
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<tr>
<td>harmonic</td>
<td>rahmonic</td>
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<tr>
<td>filter</td>
<td>lifter</td>
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Table I. Comparisons of terms used in spectral and cepstral analyses.
Figure 4. Procedures of performing cepstrum analysis.

Figure 5. (a) A typical gearbox signal containing two harmonic families, (b) its power spectrum, and (c) power cepstrum.
Flowchart in Fig. 4 demonstrates the procedures of three different cepstrum forms. This figure can be used as a guideline for performing the cepstrum analysis.

Figure 5 (a) illustrates a typical gearbox signal. The two gears have 18 and 27 teeth, respectively. The input shaft was rotating at a speed of 10 Hz (600 RPM). Therefore in it power spectrum it can be clearly seen that there are two strong components at the 18th and 27th orders, or more precisely, at 180 and 270 Hz. These two components and their harmonics generate two harmonic families which are mingled in the power spectrum such that they are not easily distinguished, as shown in Fig. 5(b). Although we can use the “harmonic cursor” in the VibraQuest system to identify these harmonics manually, it is very time-consuming and it cannot separate the two harmonic families.

Figure 5(c) is the power cepstrum calculated from the power spectrum shown in Fig. 5(b). Please notice that the quefreny axis is in second because it is calculated using the inverse transform of a signal in the frequency domain. As a result, the cepstrum has an appearance to a time waveform. For example, a peak in the spectrum at a particular frequency implies a periodic component (with this frequency) in the time waveform. Similarly, we also expect to see periodic-like behavior in the cepstrum. In Fig. 5 (c) we can see two families of spikes labeled as A and B, respectively. They are called rahmonics. If we look closely, we can see that Rahmonics A occur at 0.0037, 0.0074, 0.0111, 0.0148, 0.0185, 0.0222, 0.0259, etc. second. It is clear that they correspond to the frequency component at 270 Hz. Rahmonics B occur at 0.0055, 0.0111, 0.0167, 0.0222, 0.0278, etc. second. They correspond to the frequency component at 180 Hz. Since the least common multiple of 180 and 270 is 540 which is three times of 180 and two times of 270, in the cepstrum the two rahmonic families coincide with each other every three A’s and every two B’s. When they coincide, that particular rahmonic is strengthened, because the energy of two rahmonics is added together. That is why the rahmonics at 0.0111 and 0.0222 second in the cepstrum, labeled as “A, B”, is higher than the other rahmonics.

After the rahmonic families (or the harmonic families in the power spectrum) are identified, we will show how to “lifter” the cepstrum using the schemes shown in Fig. 4. If the frequency components are required merely to identify, one can simply lifter the power cepstrum. However, the complex cepstrum should be used if the corresponding time waveform should be reconstructed from the liftered spectrum for further processing.

Figure 6 shows how to lifter the power cepstrum. In VibraQuest’s Rotating Machinery Module, you can easily drag two cursors (blue and red cursors shown in Fig. 6(a)) to select the fundamental rahmonic of family A in the power cepstrum. All the associated rahmonics will be captured by the software automatically, and be distinguished by yellow bars accordingly. These distinguished rahmonics are then liftered out to obtain a liftered power spectrum. In Fig. 6 (b) it can be clearly seen that the harmonics associated with the 270 Hz component are removed. Please notice that since the coincident rahmonics are also liftered out, the 3rd, 6th, 9th, … harmonics of the 180 Hz component are also removed in the liftered power spectrum.
Similarly we can **lifter** rahmonic family B, as shown in Fig. 7. Again, **liftering** out coincident rahmonics leads to only odd harmonics of the 270 Hz to be present in the **liftered** power spectrum.

Figure 6. (a) Liftering rahmonic family A. (b) Liftered power spectrum.

Figure 7. (a) Liftering rahmonic family B. (b) Liftered power spectrum.
Figure 8. (a) reconstructed time signal using inverse complex cepstrum by lifting family A. (b) the power spectrum of the reconstructed time signal.

Figure 9. (a) reconstructed time signal using inverse complex cepstrum by lifting family B. (b) the power spectrum of the reconstructed time signal.
By applying the same yellow shaded masks in Fig 6 (a) to the complex cepstrum instead, the rahmonic A family, or actually the harmonics associated with 270 Hz component will be removed. Then using the inverse complex cepstrum we can reconstruct the time waveform so that it only contains the 180 Hz component. Figure 8 illustrates the reconstructed time waveform and power spectrum of the reconstructed signal. Similarly, we can lifter rahmonic B from the complex cepstrum and reconstruct the time waveform which only contains the 270 Hz component, as shown in Fig. 9.

References


Note: all the references are available in pdf file format at www.spectraquest.com/tech/index.html
Fault Frequencies of Rolling Element Bearing

The bearing fault frequencies are given by the following formulae.

\[
BPFO = \frac{nf_r}{2} \left\{ 1 - \frac{d}{D} \cos \phi \right\},
\]

\[
BPFI = \frac{nf_r}{2} \left\{ 1 + \frac{d}{D} \cos \phi \right\},
\]

\[
FTF = \frac{f_r}{2} \left\{ 1 - \frac{d}{D} \cos \phi \right\},
\]

\[
BSF = \frac{Df_r}{2d} \left\{ 1 - \left( \frac{d}{D} \cos \phi \right)^2 \right\},
\]

where \( f_r \) is the shaft running speed, \( n \) is the number of rolling elements, \( \phi \) is the angle of the load from the radial plane, \( d \) and \( D \) are the ball and pitch diameters shown in the figure below.