Lecture Notes for Theory of Finance (PhD):
Consumption-Based Asset Pricing

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Chapter 5

Consumption-Based Asset Pricing

Reference: Bossaert (2002); Campbell (2003); Cochrane (2005); Smith and Wickens (2002)

5.1 Consumption-Based Asset Pricing

5.1.1 Utility Maximization

The investor chooses consumption ($C_t$), and the investments ($v_t$) to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}), \quad \text{subject to} \quad (5.1)$$

$$C_{t+j} + v'_{t+j} P_{t+j} = v'_{t+j-1} X_{t+j}, \quad (5.2)$$

where there are $N$ assets and $P$ and $X$ and denote the price and payoff of the assets.

The Lagrangian is

$$E_t \sum_{j=0}^{\infty} \beta^j [u(C_{t+j}) + \lambda_{t+j} (-C_{t+j} - v'_{t+j} P_{t+j} + v'_{t+j-1} X_{t+j})] \quad \text{or} \quad (5.3)$$

$$u(C_t) + \lambda_t (-C_t - v'_t P_t + v'_{t-1} X_t) + \beta E_t [u(C_{t+1}) + \lambda_{t+1} (-C_{t+1} - v'_{t+1} P_{t+1} + v'_{t} X_{t+1})] + \ldots \quad (5.4)$$

The first order conditions wrt to $(C_t, v_t)$ are

$$u'(C_t) - \lambda_t = 0 \quad (5.5)$$

$$-\lambda_t P_t + \beta E_t \lambda_{t+1} X_{t+1} = 0_{N \times 1} \quad (5.6)$$
These equations must also hold for all other time periods. We can therefore notice that

$$\lambda_t = u'(C_t).$$

Use this in the foc for $v_t$ to get

$$-u'(C_t) P_t + \beta E_t u'(C_{t+1}) X_{t+1} = 0_{N \times 1},$$

or

$$P_t = \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} X_{t+1},$$

(5.7)

and

$$E_t M_{t+1} R_{t+1} = 1, \text{ with } M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}.$$  

(5.8)

which is the classical asset pricing equation. Notice that this expression is just an equilibrium condition. In particular, it does not tie down how consumption depends on wealth, beliefs and other things.

Let the payoff be a dividend and an ex-dividend price

$$X_{t+1} = D_{t+1} + P_{t+1}.$$  

(5.9)

By using this in (5.7) and solving recursively forward (substitute for $P_{t+1}$, then for $P_{t+2}$ and so on) we get

$$P_t = \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} D_{t+1} + \beta^2 E_t \frac{u'(C_{t+1}) u'(C_{t+2})}{u'(C_t) u'(C_{t+1})} D_{t+2} + \ldots$$

$$= \lim_{T \to \infty} \beta^T E_t \frac{u'(C_{t+T})}{u'(C_t)} P_{t+T} + \lim_{T \to \infty} \sum_{j=1}^{T} \beta^j E_t \frac{u'(C_{t+j})}{u'(C_t)} D_{t+j}$$

$$= \sum_{j=1}^{\infty} \beta^j E_t \frac{u'(C_{t+j})}{u'(C_t)} D_{t+j},$$

(5.10)

since the utility discounted future price must go to zero (or else we don’t have an equilibrium). The last equation is the standard expression for an asset price: the risk-adjusted present value of future dividends.

**Example 5.1** (Present value) Suppose $z_{t+j} = \frac{u'(C_{t+j})}{u'(C_t)} D_{t+j}$ in (5.10) is an AR(1)

$$z_{t+1} = (1 - \rho) \mu + \rho z_t + u_{t+1}.$$
We then have

\[ P_t = \frac{\beta \mu}{1 - \beta} + \frac{\beta \rho}{1 - \beta \rho} z_t, \]

which is increasing in \( \rho \) if \( z_t > 0 \).

### 5.1.2 Utility Maximization with Logarithmic Utility

It is possible to derive an explicit consumption function (consumption expressed as a function of wealth and other things) when the utility function is logarithmic. This also provides some additional insights about the portfolio choice.

With \( u(C_t) = \ln C_t \), we have

\[ M_{t+1} = \beta C_t / C_{t+1}. \] (5.11)

The asset pricing equation (5.8) is then

\[ \frac{1}{C_t} = \beta \mathbb{E}_t \frac{1}{C_{t+1}} R_{t+1}. \] (5.12)

Let \( W_t \) be the wealth in period \( t \) \( (v_{t-1} X_t) \) and write the budget restriction as

\[ W_{t+1} = R^W_{t+1} (W_t - C_t), \text{ where } R^W_{t+1} = v'_t X_{t+1} / v'_t P_t \] (5.13)

is the return on the portfolio. It is then straightforward to show that the consumption choice is

\[ C_t = (1 - \beta) W_t \] (5.14)

so the consumption–wealth ratio is constant. In particular, it does not depend on beliefs about the future asset returns.

**Proof.** (of (5.14)) Define \( \alpha_t \) to be the wealth–consumption ratio in period \( t \): \( C_t = W_t / \alpha_t \). Write (5.12), applied to the portfolio return, as

\[
\frac{\alpha_t}{W_t} = \beta \mathbb{E}_t \frac{\alpha_{t+1}}{R^W_{t+1} W_{t+1}} R^W_{t+1} \\
\frac{\alpha_t}{W_t} = \beta \mathbb{E}_t \frac{\alpha_{t+1}}{R^W_{t+1} (W_t - C_t)} R^W_{t+1} \\
\frac{\alpha_t}{W_t} = \beta \mathbb{E}_t \frac{\alpha_{t+1}}{R^W_{t+1} W_t (1 - 1/\alpha_t)} R^W_{t+1} \\
\alpha_t = \beta \mathbb{E}_t \alpha_{t+1} + 1.
\]
Solve recursively forward (substitute for $\alpha_{t+1}$ and then for $\alpha_{t+2}$ and so forth) to get

$$\alpha_t = \lim_{T \to \infty} \beta^T E_t \alpha_{t+T} + \lim_{T \to \infty} \sum_{j=0}^{T-1} \beta^j$$

$$= \frac{1}{1 - \beta},$$

since $\beta < 1$ and $\alpha_{t+T}$ is non-explosive. \[\Box\]

By using this in the budget restriction (5.13) we have $W_{t+1} = R^W_{t+1} W_t \beta$, so the asset pricing equation (5.8) can be written

$$1_{N\times 1} = \beta E_t \frac{W_t}{W_{t+1}} R_{t+1}$$

$$= E_t \frac{1}{R^W_{t+1}} R_{t+1},$$

(5.15)

which shows that the SDF is $1/R^W_{t+1}$. This expression is interesting since it implicitly defines the portfolio choice (since $R^W$ depends on the portfolio weights): if you know the conditional (in $t$) distribution of the asset returns in $t + 1$ then we can solve for the portfolio weights chosen. This means that the portfolio weights do not depend (directly) on consumption or wealth—or on the distribution of returns in future periods: the portfolio choice is myopic. This is different from ICAPM (to be discussed later).

**Example 5.2** (of (5.15) with 2 assets) Let $w_i$ be the weight on asset $i$ in the portfolio. Since the weights must sum to unity (5.15) becomes

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} E_t w_1 R_{t+1} + (1-w_1) R_{2t+1} \\ E_t w_1 R_{t+1} + (1-w_1) R_{2t+1} \end{bmatrix}. $$

For given beliefs about the returns, this determines $w_1$.

In addition, linearizing (5.15) gives

$$M_{t+1} \approx a + b R^W_{t+1},$$

(5.16)

so the log utility case is approximately the same as CAPM.
5.1.3 A Lucas Model

The Lucas model is a very simple general equilibrium model—since it defines the output side of the economy by an exogenous “endowment” process.

Let asset $i$ have the dividend $D_{it}$ and assume that all dividends are in terms of the (only) consumption good so market equilibrium requires

$$C_t = D_t, \text{ where } D_t = \sum_{i=1}^{N} D_{it}. \quad (5.17)$$

(This assumes that the mass of investors equal one—or alternatively, that the market equilibrium is expressed in terms of per capita figures.) Using the present value expression (5.10) gives that the price of a claim on aggregate dividends (that is, a portfolio with one unit of each of the $N$ assets) is

$$P_t = \sum_{j=1}^{\infty} \beta^j \mathbb{E}_t \frac{D_t}{D_{t+j}} D_{t+j} = D_t \frac{\beta}{1 - \beta}, \quad (5.18)$$

so the dividend–price ratio is constant. In particular, it does not depend on beliefs about future dividends—which is yet another aspect of the myopic asset pricing under log utility.

**Remark 5.3** (CRRA utility) If the utility function is of CRRA type, then $u(C) = C^{-\gamma}$, so (5.10) becomes

$$P_t = D_t \gamma \sum_{j=1}^{\infty} \beta^j \mathbb{E}_t D_{t+j}^{1-\gamma},$$

which depends on beliefs about future dividends (unless $\gamma = 1$).

5.1.4 The Basic Asset Pricing Equation

The basic asset pricing equation says

$$\mathbb{E}_{t-1}(R_t M_t) = 1. \quad (5.19)$$

where $R_t$ is the gross return of holding an asset from period $t - 1$ to $t$, $M_t$ is a stochastic discount factor (SDF). $\mathbb{E}_{t-1}$ denotes the expectations conditional on the information in period $t - 1$, that is, when the investment decision is made. This equation holds for any assets that are freely traded without transaction costs (or taxes), even if markets are incomplete.
In a consumption-based model, (5.19) is the Euler equation for optimal saving in \( t - 1 \) where \( M_t \) is the ratio of marginal utilities in \( t \) and \( t - 1 \), \( M_t = \beta \frac{u'(C_t)}{u'(C_{t-1})} \). I will focus on the case where the marginal utility of consumption is a function of consumption only, which is by far the most common formulation. This allows for other terms in the utility function, for instance, leisure and real money balances, but they have to be additively separable from the consumption term. With constant relative risk aversion (CRRA) \( \gamma \), the stochastic discount factor is

\[
M_t = \beta (C_t / C_{t-1})^{-\gamma}, \quad \text{so} \quad \ln(M_t) = \ln(\beta) - \gamma \Delta c_t, \quad \text{where} \quad \Delta c_t = \ln(C_t / C_{t-1}).
\]

The second line is only there to introduce the convenient notation \( \Delta c_t \) for the consumption growth rate.

The next few sections study if the pricing model consisting of (5.19) and (5.20) can fit historical data. To be clear about what this entails, note the following. First, general equilibrium considerations will not play any role in the analysis: the production side will not be even mentioned. Instead, the focus is on one of the building blocks of an otherwise unspecified model. Second, complete markets are not assumed. The key assumption is rather that the basic asset pricing equation (5.19) holds for the assets I analyse. This means that the representative investor can trade in these assets without transaction costs and taxes (clearly an approximation). Third, the properties of historical (ex post) data are assumed to be good approximations of what investors expected. In practice, this assumes both rational expectations and that the sample is large enough for the estimators (of various moments) to be precise.

To highlight the basic problem with the consumption-based model and to simplify the exposition, I assume that the excess return, \( R^e_t \), and consumption growth, \( \Delta c_t \), have a bivariate normal distribution. By using Stein’s lemma, we can write the the risk premium as

\[
E_{t-1}(R^e_t) = \text{Cov}_{t-1}(R^e_t, \Delta c_t) \gamma.
\]

The intuition for this expressions is that an asset that has a high payoff when consumption is high, that is, when marginal utility is low, is considered risky and will require a risk premium. This expression also holds in terms of unconditional moments. (To derive that, start by taking unconditional expectations of (5.19).)
We can relax the assumption that the excess return is normally distributed: (5.22) holds also if $R^e_t$ and $\Delta c_t$ have a bivariate mixture normal distribution—provided $\Delta c_t$ has the same mean and variance in all the mixture components (see Section 5.1.4 below). This restricts consumption growth to have a normal distribution, but allows the excess return to have a distribution with fat tails and skewness.

**Remark 5.4** (Stein’s lemma) If $x$ and $y$ have a bivariate normal distribution and $h(y)$ is a differentiable function such that $E[|h'(y)|] < \infty$, then $\text{Cov}[x, h(y)] = \text{Cov}(x, y) E[h'(y)]$.

**Proof.** (of (5.22)) For an excess return $R^e$, (5.19) says $E(R^e M) = 0$, so

$$E(R^e) = -\text{Cov}(R^e, M)/E(M).$$

Stein’s lemma gives $\text{Cov}[R^e, \exp(\ln M)] = \text{Cov}(R^e, \ln M) E M$. (In terms of Stein’s lemma, $x = R^e$, $y = \ln M$ and $h() = \exp()$.) Finally, notice that $\text{Cov}(R^e, \ln M) = -\gamma \text{Cov}(R^e, \Delta c)$. ■

**The Gains and Losses from Using Stein’s Lemma**

The gain from using (the extended) Stein’s lemma is that the unknown relative risk aversion, $\gamma$, does not enter the covariances. This facilitates the empirical analysis considerably. Otherwise, the relevant covariance would be between $R^e_t$ and $(C_t/C_{t-1})^{-\gamma}$.

The price of using (the extended) Stein’s lemma is that we have to assume that consumption growth is normally distributed and that the excess return have a mixture normal distribution. The latter is not much of a price, since a mixture normal can take many shapes and have both skewness and excess kurtosis.

In any case, Figure 5.1 suggests that these assumptions might be reasonable. The upper panel shows unconditional distributions of the growth of US real consumption per capita of nondurable goods and services and of the real excess return on a broad US equity index. The non-parametric kernel density estimate of consumption growth is quite similar to a normal distribution, but this is not the case for the US market excess return which has a lot more skewness.
Figure 5.1: Density functions of consumption growth and equity market excess returns. The kernel density function of a variable $x$ is estimated by using a $N(0, \sigma)$ kernel with $\sigma = 1.06 \text{Std}(x) T^{-1/5}$. The normal distribution is calculated from the estimated mean and variance of the same variable.

An Extended Stein’s Lemma for Asset Pricing*

To allow for a non-normal distribution of the asset return, an extension of Stein’s lemma is necessary. The following proposition shows that this is possible—if we restrict the distribution of the log SDF to be gaussian.

Figure 5.2 gives an illustration.

**Proposition 5.5** Assume (a) the joint distribution of $x$ and $y$ is a mixture of $n$ bivariate normal distributions; (b) the mean and variance of $y$ is the same in each of the $n$ components; (c) $h(y)$ is a differentiable function such that $E[|h'(y)|] < \infty$. Then $\text{Cov}[x, h(y)] = E[h'(y)] \text{Cov}(x, y)$. (See Söderlind (2009) for a proof.)

5.2 Asset Pricing Puzzles

5.2.1 The Equity Premium Puzzle

This section studies if the consumption-based asset pricing model can explain the historical risk premium on the US stock market.

To discuss the historical average excess returns, it is convenient to work with the
Figure 5.2: **Example of a bivariate mixed-normal distribution** The marginal distributions are drawn at the back.

unconditional version of the pricing expression (5.22)

\[
E(R^c_t) = \text{Cov}(R^c_t, \Delta c_t) y.
\] (5.23)

*Table 5.1* shows the key statistics for quarterly US real returns and consumption growth.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>Autocorr</th>
<th>Corr with ( \Delta c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta c )</td>
<td>1.984</td>
<td>0.944</td>
<td>0.362</td>
<td>1.000</td>
</tr>
<tr>
<td>( R^c_m )</td>
<td>5.369</td>
<td>16.899</td>
<td>0.061</td>
<td>0.211</td>
</tr>
<tr>
<td>Riskfree</td>
<td>1.213</td>
<td>2.429</td>
<td>0.642</td>
<td>0.196</td>
</tr>
</tbody>
</table>

Table 5.1: US quarterly data, 1957Q1-2008Q4, (annualized, in %, in real terms)

We see, among other things, that consumption has a standard deviation of only 1% (annualized), the stock market has had an average excess return (over a T-bill) of 6–8%
(annualized), and that returns are only weakly correlated with consumption growth. These figures will be important in the following sections. Two correlations with consumption growth are shown, since it is unclear if returns should be related to what is recorded as consumption this quarter or the next. The reason is that consumption is measured as a flow during the quarter, while returns are measured at the end of the quarter.

Table 5.1 shows that we can write (5.23) as

\[
E(R_t^e) = \text{Corr}(R_t^e, \Delta c_t) \times \text{Std}(R_t^e) \times \text{Std}(\Delta c_t) \gamma
\]

\[
0.05 \approx 0.21 \times 0.17 \times 0.01 \gamma
\]

which requires a value of \( \gamma \approx 140 \) for the equation to fit.

The basic problem with the consumption-based asset pricing model is that investors enjoy a fairly stable consumption series (either because income is smooth or because it is easy/inexpensive to smooth consumption by changing savings), so only an extreme risk aversion can motivate why investors require such a high equity premium. This is the equity premium puzzle stressed by Mehra and Prescott (1985) (although they approach the issue from another angle). Indeed, even if the correlation was one, (5.25) would require \( \gamma \approx 29 \).

**Remark 5.6** (Relation to the Hansen and Jagannathan (1991) approach) For an excess return \( R^e \), (5.19) says \( E(R^e M) = 0 \), so

\[
E(R^e) = -\text{Cov}(R^e, M) / E(M), \text{ so } \ E(R^e) / \text{Std}(R^e) = -\text{Corr}(R^e, M) \text{ Std}(M) / E(M).
\]

Since a correlation is between \(-1\) and \(1\), the following "volatility bound" must hold: \( \text{Std}(M) / E(M) \geq |E(R^e)| / \text{Std}(R^e) \). The approach in Hansen and Jagannathan (1991) is essentially to search in a set of returns for the portfolio with the largest Sharpe ratio and then study if a given model of the SDF satisfies the volatility bound. In (5.24)–(5.25), we only study one asset (the broad stock market) and ask how we can tweak the SDF to make the bound hold.
5.2.2 Assumptions about \((C, R)\) or about \((C, D)\)?

The Mehra and Prescott (1985) approach to analyzing the consumption-based asset pricing model is somewhat less direct than what we did above. However, it has had a large impact on the research community, so it is worth summarising. The starting point is a Lucas (1978) economy where (aggregate) consumption equals the exogenous (aggregate) endowment. To see how this works, write the gross return as

\[ R_t = \frac{P_t + D_t}{P_{t-1}}, \]

where \(P_t\) is the asset price and \(D_t\) the dividend (per asset) in period \(t\). We can then write (5.19) as

\[ P_{t-1} = E_{t-1}[M_t(P_t + D_t)]. \]

Iterating forward, applying the law of iterated expectations, ruling out bubbles, and using the CRRA SDF in (5.20) give

\[ P_{t-1} = E_{t-1}\left[ \beta(C_t/C_{t-1})^{-\gamma} D_t + \beta^2(C_{t+1}/C_{t-1})^{-\gamma} D_{t+1} + \ldots \right]. \]

The price of a real one-period bond is found by setting \(D_t = 1\) and all future dividends to zero. The price of a claim to the stream of future non-storable endowments—which is interpreted as a proxy of a broad stock market index—is found by setting \(D_t\) equal to \(C_t\). With a time series process for consumption, and values of \(\beta\) and \(\gamma\), it is straightforward to calculate asset returns. Mehra and Prescott (1985) construct a time series process by postulating that consumption growth follows a two-state Markov process: \(C_t/C_{t-1}\) is either low or high and the probability of which depends on whether \(C_{t-1}/C_{t-2}\) was low or high. This makes it very simple to calculate the expectations: there is only one state variable and it can only take two values. In spite of this, the process allows both autocorrelation and time-varying volatility. The model parameters are calibrated to fit the mean, standard deviation, and autocorrelation in consumption growth. With this machinery, Mehra and Prescott (1985) find that a very high value of \(\gamma\) is needed to explain the historical equity premium—basically because consumption is smooth. This approach is interesting, but it is a fairly indirect method for studying the equity premium, and it adds a few very strong assumptions, in particular, about the consumption process and that the stockmarket corresponds to a claim on future consumption. In any case, several extensions have been made, for instance to investigate if more extreme consumption growth processes (fat tails or “crash states”) can rescue the model—but so far with limited success (see, for instance, Salyer (1998) and Bidarkota and McCulloch (2003)).

The general insight is that making assumptions about the processes for consumption and dividendens map into implications for returns—and vice versa. Sometimes one ap-
approach makes more sense than the other.

5.2.3 The Equity Premium Puzzle over Time

In contrast to the traditional interpretation of “efficient markets,” it has been found that excess returns might be somewhat predictable—at least in the long run (a couple of years). In particular, Fama and French (1988a) and Fama and French (1988b) have argued that future long-run returns can be predicted by the current dividend-price ratio and/or current returns.

Figure 5.3 illustrates this by showing results of the regressions

\[ R_{t+k}^e = a_0 + a_1 x_t + u_{t+k}, \text{ where } x_t = E_t/P_t \text{ or } R_t^e(k), \]  

(5.26)

where \( R_t^e(k) \) is the annualized \( k \)-quarter excess return of the aggregate US stock market and \( E_t/P_t \) is the earnings-price ratio.

It seems as if the earnings-price ratio has some explanatory power for future returns—at least for long horizons. In contrast, the lagged return is a fairly weak predictor.

This evidence suggests that excess returns may perhaps have a predictable component, that is, that (ex ante) risk premia are changing over time. To see how that fits with the consumption-based model, (5.22) says that the conditional expected excess return should equal the conditional covariance times the risk aversion.

Figure 5.4.a shows recursive estimates of the mean return of the aggregate US stock market and the covariance with consumption growth (dated \( t+1 \)). The recursive estimation means that the results for (say) 1965Q2 use data for 1955Q2–1965Q2, the results for 1965Q3 add one data point, etc. The second subfigure shows the same statistics, but estimated on a moving data window of 10 years. For instance, the results for 1980Q2 are for the sample 1971Q3–1980Q2. Finally, the third subfigure uses a moving data window of 5 years.

Together these figures give the impression that there are fairly long swings in the data. This fundamental uncertainty should serve as a warning against focusing on the fine details of the data. It could also be used as an argument for using longer data series—provided we are willing to assume that the economy has not undergone important regime changes.

It is clear from the earlier Figure 5.4 that the consumption-based model probably can-
not generate plausible movements in risk premia. In that figure, the conditional moments are approximated by estimates on different data windows (that is, different subsamples). Although this is a crude approximation, the results are revealing: the actual average excess return and the covariance move in different directions on all frequencies.

5.2.4 The Riskfree Rate Puzzle

The CRRA utility function has the special feature that the intertemporal elasticity of substitution is the inverse of the risk aversion, that is, $1/\gamma$. Choosing the risk aversion parameter, for instance, to fit the equity premium, will therefore have direct effects on the riskfree rate.

A key feature of any consumption-based asset pricing model, or any consumption/saving model for that matter, is that the riskfree rate governs the time slope of the consumption
Recursive estimation

\[ E(R^e_t) \]
\[ \text{Cov}(R^e_t, \Delta c_t) \]

Figure 5.4: The equity premium puzzle for different samples.

From the asset pricing equation for a riskfree asset (5.19) we have \( E_{t-1}(R_{ft})E_{t-1}(M_t) = 1 \). Note that we must use the conditional asset pricing equation—at least as long as we believe that the riskfree asset is a random variable. A riskfree asset is defined by having a zero conditional covariance with the SDF, which means that it is regarded as riskfree at the time of investment \( t-1 \). In practice, this means a real interest rate (perhaps approximated by the real return on a T-bill since the innovations in inflation are small), which may well have a nonzero unconditional covariance with the SDF.\(^1\) Indeed, in Table 5.1 the real return on a T-bill is as correlated with consumption growth as the aggregate US stockmarket.

When the log SDF is normally distributed (the same assumption as before), then the

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\( ^1\) As a very simple example, let \( x_t = z_{t-1} + \varepsilon_t \) and \( y_t = z_{t-1} + u_t \) where \( \varepsilon_t \) are \( u_t \) uncorrelated with each other and with \( z_{t-1} \). If \( z_{t-1} \) is observable in \( t-1 \), then \( \text{Cov}_{t-1}(x_t, y_t) = 0 \), but \( \text{Cov}(x_t, y_t) = \sigma^2(z_{t-1}) \).
log expected riskfree rate is

$$\ln E_{t-1}(R_{ft}) = -\ln(\beta) + \gamma E_{t-1}(\Delta c_t) - \gamma^2 \text{Var}_{t-1}(\Delta c_t)/2. \quad (5.27)$$

To relate this equation to historical data, we take unconditional expectations to get

$$E \ln E_{t-1}(R_{ft}) = -\ln(\beta) + \gamma E(\Delta c_t) - \gamma^2 E \text{Var}_{t-1}(\Delta c_t)/2. \quad (5.28)$$

Before we try to compare (5.28) with data, several things should be noted. First, the log gross rate is very close to a traditional net rate ($\ln(1 + z)$), so it makes sense to compare with the data in Table 5.1. Second, we can safely disregard the variance term since it is very small, at least as long as we are considering reasonable values of $\gamma$. Although the average conditional variance is not directly observable, we know that it must be smaller than the unconditional variance\(^2\), which is very small in Table 5.1. In fact, the variance is around 0.0001 whereas the mean is around 0.02.

**Proof.** (of (5.27)) For a riskfree gross return $R_f$, (5.19) with the SDF (5.20) says

$$E_{t-1}(R_{ft}) E_{t-1}[\beta(C_t/C_{t-1})^{-\gamma}] = 1.$$ Recall that if $x \sim N(\mu, \sigma^2)$ and $y = \exp(x)$ then $E(y) = \exp(\mu + \sigma^2/2)$. When $\Delta c_t$ is conditionally normally distributed, the log of $E_{t-1}[\beta(C_t/C_{t-1})^{-\gamma}]$ equals $\ln \beta - \gamma E_{t-1}(\Delta c_t) + \gamma^2 \text{Var}_{t-1}(\Delta c_t)/2)$. \(\blacksquare\)

According to (5.28) there are two ways to reconcile a positive consumption growth rate with a low real interest rate (around 1% in Table 5.1): investors may prefer to consume later rather than sooner ($\beta > 1$) or they are willing to substitute intertemporally without too much compensation ($1/\gamma$ is high, that is, $\gamma$ is low). However, fitting the equity premium requires a high value of $\gamma$, so investors must be implausibly patient if (5.28) is to hold. For instance, with $\gamma = 25$ (which is a very conservative guess of what we need to fit the equity premium) equation (5.28) says

$$0.01 = -\ln(\beta) + 25 \times 0.02 \quad (5.29)$$

(ignoring the variance terms), which requires $\beta \approx 1.6$. This is the *riskfree rate puzzle* stressed by Weil (1989). The basic intuition for this result is that it is hard to reconcile a steep slope of the consumption profile and a low compensation for postponing consumption if people are insensitive to intertemporal prices—unless they are extremely patient.

\(^2\)Let $E(y|x)$ and $\text{Var}(y|x)$ be the expectation and variance of $y$ conditional on $x$. The unconditional variance is then $\text{Var}(y) = \text{Var}[E(y|x)] + E[\text{Var}(y|x)]$. 

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(actually, unless they prefer to consume later rather than sooner).

Another implication of a high risk aversion is that the real interest rate should be very volatile, which it is not. According to Table 5.1 the standard deviation of the real interest rate is perhaps twice the standard deviation of consumption growth. From (5.27) the volatility of the (expected) riskfree rate should be

$$\text{Std}[\ln E_{t-1}(R_{ft})] = \gamma \text{Std}[E_{t-1}(\Delta c_t)].$$

(5.30)

if the conditional variance of consumption growth is constant. This expression says that the standard deviation of expected real interest rate is $\gamma$ times the standard deviation of expected consumption growth. We cannot observe the conditional expectations directly, and therefore not estimate their volatility. However, a simple example is enough to demonstrate that high values of $\gamma$ are likely to imply counterfactually high volatility of the real interest rate.

As an approximation, suppose both the riskfree rate and consumption growth are AR(1) processes. Then (5.30) can be written

$$\text{Corr}[\ln E_{t-1}(R_{ft}), \ln E_{t-1}(R_{ft})] \times \text{Std}[\ln E_{t-1}(R_{ft})] = \gamma \times \text{Corr}(\Delta c_t, \Delta c_{t+1}) \times \text{Std}(\Delta c_t)$$

(5.31)

$$0.75 \times 0.02 \approx \gamma \times 0.3 \times 0.01$$

(5.32)

where the second line uses the results in Table 5.1. With $\gamma = 25$, (5.32) implies that the RHS is much too volatile. This shows that an intertemporal elasticity of substitution of 1/25 is not compatible with the relatively stable real return on T-bills.

**Proof.** (of (5.31)) If $x_t = \alpha x_{t-1} + \epsilon_t$, where $\epsilon_t$ is iid, then $E_{t-1}(x_t) = \alpha x_{t-1}$, so $\sigma(E_{t-1} x_t) = \alpha \sigma(x_{t-1})$. ■

### 5.3 The Cross-Section of Returns

The previous section demonstrated that the consumption-based model has a hard time explaining the risk premium on a broad equity portfolio—essentially because consumption growth is too smooth to make stocks look particularly risky. However, the model does predict a positive equity premium, even if it is not large enough. This suggests that the model may be able to explain the relative risk premia across assets, even if the scale is
wrong. In that case, the model would still be useful for some issues. This section takes a closer look at that possibility by focusing on the relation between the average return and the covariance with consumption growth in a cross-section of asset returns.

The key equation is (5.23), which I repeat here for ease of reading

\[ E(R^e_t) = \text{Cov}(R^e_t, \Delta c_t) \gamma. \]

This can be tested with a GMM framework or a to the traditional cross-sectional regressions of returns on factors with unknown factor risk premia (see, for instance, Cochrane (2005) chap 12 or Campbell, Lo, and MacKinlay (1997) chap 6).

Figure 5.5 shows the results of both C-CAPM and the standard CAPM—for the 25 Fama and French (1993) portfolios. It is clear that both models work badly, but CAPM actually worse.

Figure 5.6 takes a careful look at how the C-CAPM works in different smaller cross-sections—and Figure 5.7 does the same for CAPM. A common feature of both models is that growth firms (low book-to-market ratios) have large pricing errors (in the figures with lines connecting the same B/M categories, they are the lowest lines for both models). See also Table 5.2–5.4)

In contrast, a major difference between the models is that CAPM shows a very strange pattern when we compare across B/M categories (lines connecting the same size category): mean excess returns are decreasing in the covariance with the market—the wrong sign compared to the CAPM prediction. This is not the case for C-CAPM.

The conclusion is that the consumption-based model is not good at explaining the cross-section of returns, but it is no worse than CAPM—if it is any comfort.

### 5.4 Refinements of the Consumption-Based Model

There are many suggestions for how the problems with the consumption-based can be solved. One major strand proposes changes to the utility function; another to how we measure the volatility of returns and consumption.

If we focus on the utility function, we need a high risk aversion (to get a high equity premium), a high elasticity of intertemporal elasticity of substitution (to get a low and stable real interest rate), and maybe also time-variation of the risk aversion (to fit the time-variation in expected returns). This could be produced by using a functional form
that explicitly separates risk aversion from the intertemporal elasticity of substitution as in Epstein and Zin (1989a) or by certain types of habit persistence models as in Campbell and Cochrane (1999).

If we instead focus on the measurement of volatility, we need a high consumption volatility (to get a high equity premium) and time-variation in the volatility of returns and/or consumption (to get time-variation in expected returns). The first could be achieved
by taking into account the effects of uninsurable idiosyncratic shocks as in Mankiw (1986); the second by a model of time-variation of second moments (typically a GARCH or regime switching model) as in, for instance, Bansal and Lundblad (2002).

There are of course many unresolved issues. First, some of the proposed “fixes” have serious side effects. For instance, some of the habit persistence models create implausible
implications for consumption smoothing and optimal fiscal policy (see, Lettau and Uhlig (2000), Lettau and Uhlig (2002), and Ljunqvist and Uhlig (2000)).

Second, it is unclear if these modifications of the SDF can improve the consumption-based model’s ability to account for the cross-section of expected returns discussed in Section 5.3, since the cross-sectional variation often depends on the covariance with con-
Table 5.2: **Historical minus fitted risk premia (annualised %) from the unconditional model.**
Results are shown for the 25 equally-weighted Fama-French portfolios, formed according to size and book-to-market ratios (B/M). Sample: 1957Q1-2008Q4

<table>
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<tr>
<th>B/M</th>
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<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<tr>
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<tr>
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</tr>
<tr>
<td>5</td>
<td>-3.1</td>
<td>-0.5</td>
<td>-0.7</td>
<td>-1.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 5.3: **Historical risk premia (annualised %).** Results are shown for the 25 equally-weighted Fama-French portfolios, formed according to size and book-to-market ratios (B/M) Sample: 1957Q1-2008Q4

<table>
<thead>
<tr>
<th>B/M</th>
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<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<td>6.1</td>
<td>6.3</td>
<td>6.1</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Table 5.4: **Relative errors of risk premia (in %) of the unconditional model.** The relative errors are defined as historical minus fitted risk premia, divided by historical risk premia. Results are shown for the 25 equally-weighted Fama-French portfolios, formed according to size and book-to-market ratios (B/M). Sample: 1957Q1-2008Q4

<table>
<thead>
<tr>
<th>B/M</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
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<td>-11.0</td>
<td>-22.1</td>
<td>4.2</td>
</tr>
</tbody>
</table>
sumption growth only—just as in the CRRA model. The next few sections demonstrate this for some well-known models of time nonseparable utility (Epstein and Zin (1989a)), habit persistence (Campbell and Cochrane (1999)), and idiosyncratic shocks (Constantinides and Duffie (1996)). See Söderlind (2006) for details on the derivations.

5.4.1 The CRRA Model as a Special Case of Epstein-Zin Preferences

The recursive utility function in Epstein and Zin (1989b) and Epstein and Zin (1991) has different parameters for the risk aversion and the elasticity of intertemporal substitution. This generally gives a complicated pricing expression, but it coincides with the CRRA model in special cases.

It can be shown that if all wealth is marketable, then the Euler equation for the excess return is

$$ E_{t-1}[(C_t/C_{t-1})^{\theta/\psi} R_{mt}^{\theta-1} R_t^e] = 0, \text{ where } \theta = (1 - \gamma)/(1 - 1/\psi), \tag{5.33} $$

where $R_{mt}$ is the market gross return, $\gamma$ the risk aversion, and $\psi$ the elasticity of intertemporal substitution.

Let $C_t/W_t = 1/\alpha_t$ be the consumption–wealth ratio. The Euler equation can then be written

$$ E_{t-1}[(C_t/C_{t-1})^{-\gamma \alpha_t^{\theta-1} R_t^e}] = 0. \tag{5.34} $$

If there are no one-period innovations in the consumption-wealth ratio, then $\alpha_t$ can be cancelled. To get this result we need either iid market returns or an elasticity of intertemporal substitution equal to one. In these cases, testing the pricing performance of the CRRA model is the same as testing the Epstein-Zin model.

**Proof.** Let $C_t/W_t = 1/\alpha_t$ and substitute for wealth in the budget restriction to get $C_t \alpha_t = R_{mt}(\alpha_{t-1} C_{t-1} - C_{t-1})$, or $C_t/C_{t-1} = R_{mt}(\alpha_{t-1} - 1)/\alpha_t$ which in turn gives $(C_t/C_{t-1}) \alpha_t/(\alpha_{t-1} - 1) = R_{mt}$. Using in the Euler equation gives $E_{t-1}[(C_t/C_{t-1})^{-\gamma \alpha_t^{\theta-1} R_t^e}] = 0$. Campbell (1993) shows that there are no innovations in $\alpha_t$ if $\psi = 1$ and that $\alpha$ is a constant if the market returns are iid or if $\psi = 1$ and all innovations are homoskedastic.

$\blacksquare$
5.4.2 Habit Persistence

The habit persistence model of Campbell and Cochrane (1999) has a CRRA utility function, but the argument is the difference between consumption and a habit level, \( C_t - X_t \), instead of just consumption. The habit is parameterized in terms of the “surplus ratio” \( S_t = (C_t - X_t)/C_t \), which measures how much aggregate consumption exceeds the habit. Since this ratio is external to the investor, marginal utility becomes \( (C_t - X_t)^{-\gamma} = (C_t S_t)^{-\gamma} \). The log SDF is therefore

\[
\ln M_t = -\gamma (\Delta s_t + \Delta c_t), \tag{5.35}
\]

where \( s_t \) is the log surplus ratio. The process for \( s_t \) is assumed to be a non-linear AR(1) (constant suppressed)

\[
s_t = \phi s_{t-1} + \lambda(s_{t-1}) \Delta c_t, \tag{5.36}
\]

where \( \lambda(s_{t-1}) \geq 0 \) is a decreasing function of \( s_{t-1} \).

It is straightforward to show that the conditional pricing expression is the same as (5.22), but with

\[
\kappa_t = \gamma [1 + \lambda(s_{t-1})] \tag{5.37}
\]

instead of just \( \gamma \). The model therefore has the same cross-sectional implication as the conditional CRRA model. The only difference is that the slope coefficient will be time-varying since it involves \( 1 + \lambda(s_{t-1}) \).

Since the log surplus ratio, \( s_t \), is unobservable, the unconditional pricing expression is not well suited for empirical testing—unless we make some further assumptions. In particular, it can be shown that if we assume that \( \lambda(s_{t-1}) \) is a constant \( \lambda \) and that the excess return is unpredictable (at least by \( s_{t-1} \)) then the unconditional pricing expression is the same as (5.23), but with

\[
\kappa = \gamma (1 + \lambda) \tag{5.38}
\]

instead of \( \gamma \). The only difference to the standard CRRA model is the extra \( 1 + \lambda \) term. This special case clearly ruins several of the interesting features of the model in Campbell and Cochrane (1999), so using (5.38) should probably be interpreted as focusing on a more standard habit persistence model. In contrast, the conditional expression (5.37) is a direct implication of the Campbell and Cochrane (1999) model.

**Proof.** (of (5.37) and (5.38)) Use (5.36) in (5.35) to get \( \ln M_t = -\gamma (\phi - 1)s_{t-1} - [1 +
\( \lambda(s_{t-1}) \gamma \Delta c_t \). Since \( s_{t-1} \) is known in \( t-1 \), the conditional covariance is \( \text{Cov}_{t-1}(R_t^e, \ln M_t) = \text{Cov}_{t-1}(R_t^e, \Delta c_t)[1+\lambda(s_{t-1})]\gamma \). Use in (5.22) to get (5.37). If \( \lambda(s_{t-1}) \) is a constant \( \lambda \), then unconditional asset pricing equation becomes \( E(R_t^e) = \text{Cov}(R_t^e, \gamma(\phi - 1)s_{t-1} + (1 + \lambda)\gamma \Delta c_t) \). The \( s_{t-1} \) term cancels if it cannot predict \( R_t^e \) which gives \( E(R_t^e) = \text{Cov}(R_t^e, \Delta c_t) \gamma(1 + \lambda) \).

5.4.3 Idiosyncratic Risk

The volatility of aggregate consumption may underestimate the risk faced by investors if there are uninsurable individual shocks. Such shocks mean that the consumption growth of investor \( j \) is the aggregate consumption growth plus an idiosyncratic component. His/her log SDF, assuming CRRA utility, is therefore

\[
\ln M_{jt} = -\gamma \Delta c_t - \gamma u_{jt+\delta} ,
\]  
(5.39)

where \( u_{jt+\delta} \) is the idiosyncratic shock. For analytical convenience, I let the shock be realized a split second (\( \delta \)) after the asset return and aggregate consumption. Using the law of iterated expectations, the Euler equation of investor \( j \), \( E_{t-1}(R_t^e M_{jt}) = 0 \), can now be written

\[
E_{t-1}[R_t^e \exp(-\gamma \Delta c_t) E_t \exp(-\gamma u_{jt+\delta})] = 0 .
\]  
(5.40)

To simplify, assume that the distribution of the idiosyncratic shock, conditional on the information set in \( t \), is normal. It is important that the mean of this distribution does not depend on the return (or else the shock is insurable) or consumption growth (or else the shock is non-idiosyncratic). I therefore assume that the mean is always zero. In contrast, the variance is assumed to be \( 2\lambda(\varepsilon_t) \) where \( \lambda(\varepsilon_t) \) is some function of the aggregate shock, \( \varepsilon_t = \Delta c_t - E_{t-1} \Delta c \). To simplify, I approximate \( \lambda(\varepsilon_t) \) by a time-varying linear function. In short, I assume

\[
u_{jt+\delta}| \text{info}_t \sim N(0, 2\lambda(\varepsilon_t)] , \text{with} \]

\[
\lambda(\varepsilon_t) \approx a + b_{t-1} \varepsilon_t .
\]  
(5.41)

where \( b_{t-1} \) is known in \( t-1 \).\(^3\)

With these assumptions, it can be shown that the conditional pricing expression is the

\(^3\)This is similar to Lettau (2002) who assumes that \( \varepsilon_t \) and \( \lambda(\varepsilon_t) \) have a bivariate normal distribution.
same as (5.22), but with
\[ \kappa_t = \gamma(1 - \gamma b_{t-1}) \quad (5.42) \]
instead of \( \gamma \). If \( b = 0 \), so idiosyncratic risk has a constant variance, then it drops out of the pricing expression and we are back in the standard CRRA model: idiosyncratic shocks have no effect on risk premia unless their variance depends on the aggregate shock (see Mankiw (1986) and Constantinides and Duffie (1996)).

Instead, if \( b_{t-1} < 0 \), so bad times (negative consumption surprise) are also risky times, then the idiosyncratic shocks add to the expected return. However, the implication for the cross-sectional pattern of expected returns is the same as in the conditional CRRA model without idiosyncratic shocks.

In contrast, the unconditional pricing expression is not that easily tested. However, if we assume that consumption growth is unpredictable and \( b_{t-1} \) is constant, then it can be shown that the unconditional pricing expression is the same as (5.23), but with
\[ \kappa = \gamma(1 - \gamma b) \quad (5.43) \]
instead of \( \gamma \).

An additional aspect of models with idiosyncratic risk is that they allow us to study the effects of transaction costs in a serious way: heterogenous agents with idiosyncratic costs trade, representative agents do not. The first effect of transaction costs is to transform the first order condition of an investor to a set of inequalities
\[ 1/(1 + \tau) \leq E_{t-1}(R_t M_{jt}) \leq 1 + \tau \] where \( \tau \) is the (proportional) cost of a round-trip (the upper limit is for buying the asset in \( t - 1 \) and selling in \( t \); the lower limit is for the opposite). This has been used to modify the Hansen and Jagannathan (1991) bounds (see He and Modest (1995) and Luttmer (1996)) with mixed results. One of the key assumptions in that analysis seems to be the assumption length of the period: a 0.5% transaction cost is quite substantive compared to a 1% monthly (expected) return, but small compared to a 12% annual return. We therefore need to find the equilibrium (including the endogenous trading frequency) in order to analyse the importance of transaction costs. So far, the findings indicate that the effect on prices is relatively small (unless borrowing/lending is very costly) but that the effect on turnover is large (see Heaton and Lucas (1996) and Vayanos (1998)).

**Proof.** (of (5.42) and (5.43)) We have that \( E_t \exp(-\gamma u_{jt+\delta}) = \exp[\gamma^2 \lambda(\varepsilon_t)] \), and \( \Delta c_t = E_{t-1} \Delta c_{t-1} + \varepsilon_t \). Equation (5.40) can therefore be written \( E_{t-1}\{R_t^e \exp[-\gamma E_{t-1} \Delta c_t - \Delta u_{jt+\delta}] \exp(-\gamma u_{jt+\delta}) \} = \gamma(1 - \gamma b_{t-1}) \exp\left[\gamma^2 \lambda(\varepsilon_t) - E_{t-1}(\Delta c_{t-1})\right] \).
\( \gamma \varepsilon_t + \gamma^2 \lambda(\varepsilon_t) \) = 0. Letting \( \lambda(\varepsilon_t) = a + b_{t-1} \varepsilon_t \) and cancelling the non-random term \( (\gamma^2 a) \) gives

\[
E_{t-1} \{ R_t^e \exp[-\gamma E_{t-1} \Delta c_t - \varepsilon_t \gamma (1 - b_{t-1} \gamma)] \} = 0.
\]

Clearly, the \( \exp[\ ] \) term then corresponds to \( M_t \). The conditional asset pricing expression becomes

\[
E_{t-1}(R_t^e) = -\text{Cov}_{t-1} [R_t^e, -\gamma E_{t-1} \Delta c_t - \gamma \varepsilon_t (1 - b_{t-1} \gamma)]
\]

\[
= \text{Cov}_{t-1}(R_t^e, \varepsilon_t) \gamma (1 - b_{t-1} \gamma),
\]

since \( E_{t-1} \Delta c_t \) is known already in \( t - 1 \). We can replace \( \varepsilon_t \) in this expression by \( \Delta c_t \) if we want to. The unconditional asset pricing expression becomes

\[
E(R_t^e) = \text{Cov} [R_t^e, \gamma E_{t-1} \Delta c_t + \varepsilon_t \gamma (1 - b_{t-1} \gamma)].
\]

If \( E_{t-1} \Delta c_t \) cannot predict \( R_t^e \), then \( \gamma E_{t-1} \Delta c_t \) cancels. If, in addition, \( E_{t-1} \Delta c_t \) and \( b_{t-1} \) are constants (which certainly implies that they cannot forecast \( R_t^e \)), then we get

\[
E(R_t^e) = \text{Cov} (Z_{it}, \Delta c_t) \gamma (1 - b \gamma).
\]
Bibliography


