Traffic Accident Reconstruction

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Overview

Why?
The purpose of these training programs is to prepare participants for the process of thoroughly investigating serious traffic collisions.

Learning Objectives

- Define the term yaw as it applies to a vehicle.
- List the characteristics of yaw marks.
- Demonstrate the ability to plot actual yaw marks.
- List four types of information necessary to estimate speed from critical speed scuff marks.
- Demonstrate one method for calculating radius.
- Demonstrate one method for calculating critical speed.
- Identify five instances when the critical speed equation should not be used to estimate speed.
- Define, in mathematical terms, the Law of Pythagoras.
- Demonstrate methods for calculating the sine, cosine, and tangent of a right angle.
- List the three Newtonian Laws of Motion.
- Define mass, acceleration and force.
- List the mathematical expression that defines uniform circular motion.
- Copy and analyze the radius and critical speed derivation
- Define and describe falls, flips and vaults
- List three kinds of information needed to calculate the speed of objects that fall, flip or vault
- Demonstrate an ability to estimate speed from falls, flips and vaults
- Define and describe uniform projectile motion
- Explain when a critical speed yaw becomes a spin.
- Define a spin angle.
• Define velocity vector.
• Define heading vector.
• Define rolling resistance
• Demonstrate one method for adjusting the drag factor of a vehicle having less than 100% slippage which has left spin marks.
• Demonstrate one method for calculating speed loss from vehicles which leave spin marks.
• Demonstrate one method for combining speeds.
• Analyze percentage of braking
• List five kinds of information needed to evaluate how vehicles react to a collision sequence
• Analyze damage profiles to determine rotation and the amount of collapse
• Define vector
• Demonstrate two methods for adding vectors
• List the sign of X and Y in each quadrant of the left hand coordinate system
• Define Newton’s Second Law of Motion
• List four types of information needed for a momentum analysis
• Demonstrate two methods for calculating impact speeds
• Demonstrate the ability to apply three acceleration equations to traffic accident reconstruction problems
• Demonstrate the ability to apply three initial velocity equations to traffic accident reconstruction problems
• Demonstrate the ability to apply two end velocity equations to traffic accident reconstruction problems
• Demonstrate the ability to use work, force and distance to calculate energy.
• Demonstrate one method for calculating elastic potential energy when force constant and distance are known.
• Define the Law of Conservation of Energy.
• Demonstrate one method for calculating energy of a vehicle that skids over numerous surfaces, when weight, drag factor and distance are known.
• Demonstrate two methods for calculating vehicle speed when crush and after impact vehicle dynamics are known.
• Define the terms dynamic collapse and restitution as they apply to vehicle damage.
Traffic Accident Reconstruction Level I

This course introduces students to the proper techniques for making accurate speed estimates when certain scuff mark evidence is available. Critical issues such as scuff mark characteristics, measuring techniques, equation application, and the equation derivation are addressed.

**Learning Objectives**

- Define the term yaw as it applies to a vehicle.
- List the characteristics of yaw marks.
- Demonstrate the ability to plot actual yaw marks.
- List four types of information necessary to estimate speed from critical speed scuff marks.
- Demonstrate one method for calculating radius.
- Demonstrate one method for calculating critical speed.
- Identify five instances when the critical speed equation should not be used to estimate speed.
- Define, in mathematical terms, the Law of Pythagoras.
- Demonstrate methods for calculating the sine, cosine, and tangent of a right angle.
- List the three Newtonian Laws of Motion.
- Define mass, acceleration and force.
- List the mathematical expression that defines uniform circular motion.
- Copy and analyze the radius and critical speed derivation
Yaw Mark Definition and Documentation

Terminal Performance Objective

Given a simulated traffic accident, students will demonstrate the ability to accurately recognize and evaluate yaw mark evidence in accordance with the information presented in class.

Enabling Objectives

1. Define the term yaw as it applies to a vehicle.
2. List the characteristics of yaw marks.
3. Demonstrate the ability to plot actual yaw marks.

Definitions

Yaw is a sideways movement of a vehicle in turning; movement of a vehicle in another direction than that in which it is headed; sideways motion produced when centrifugal force exceeds traction force. Often this is the result of overreaction or exceeding the critical speed. Sometimes it is revealed by tire marks on the highway.

Critical Speed is a speed above which a particular highway curve, or a curve demanded by the driver, could not be negotiated by a motor vehicle without all the wheels side slipping. It is the speed at which the centrifugal force of a vehicle following a specific curve exceeds the traction force of the tires on the surface.

Yaw Mark is a scuff mark made while a vehicle is yawing; the mark on the road is made by a rotating tire which is slipping in a direction parallel to the axle of the wheel.

Critical Speed Scuff Mark is a scuff mark made while a vehicle is yawing; the mark on the road is made by a rotating tire which is slipping in a direction parallel to the axle of wheel. In critical speed scuff marks the rear wheels will track to the outside of the front wheels, striations should be present, and the marks will always be
curved. All critical speed scuff marks are yaw marks but not all yaw marks are critical speed scuff marks.

Identifying and Plotting Yaw marks

Yaw mark evidence is often mistaken for skid mark evidence. It is important that accident investigators be able to distinguish the difference between the two types of marks. Accident investigators must also be able to measure the marks so they can be located on a scale drawing. When plotted properly these marks can assist the investigator in determining the speed of the vehicle, as well as the position of the vehicle at the time the marks were being made.
Estimating Critical Speed

Terminal Performance Objective

Given the need, students will correctly estimate the speed of vehicles by using mathematical formulae and analyzing yaw mark evidence in accordance with the information presented in class.

Enabling Objectives

1. List four types of information necessary to estimate speed from critical speed scuff marks.
2. Demonstrate one method for calculating radius.
3. Demonstrate one method for calculating critical speed.
4. Identify five instances when the critical speed equation should not be used to estimate speed.

A wheel that is rolling straight down the road surface is not slipping provided the brakes are not being applied. At the contact patch (tire-road interface), there is basically no movement relative to the road surface. This means the velocity at the tire contact patch is zero relative to the road.

When brakes are applied a retarding force is generated by the road. The velocity at the tire contact patch is no longer zero and begins to increase. The tire is locked by the brakes and begins to slide completely, leaving skid marks. When there is 100% slippage (wheels completely locked) the speed of the wheel at the contact patch is equal to the speed of the vehicle.

Braking is not the only thing that can cause a tire to slip. Steering can also cause slippage. With braking, the slippage occurs longitudinally because the tire is rotating slower than the angular velocity required for it to roll on the surface without slippage.

The slippage that occurs during steering is lateral rather than longitudinal and, for most steering inputs, minimal. During extreme cornering maneuvers at higher
speeds, the tires may exceed friction limits and begin to slip excessively. At this point the vehicle will begin to yaw and leave scuff marks. In these situations, the required force at the tires exceeds the available force (coefficient of friction).

Marks left by a vehicle in a critical speed scuff (yaw) can be used to estimate the speed of the vehicle similarly to the way braking skid marks are used. A critical speed scuff mark on the road surface is an indication the tire was side slipping and exceeded its frictional limit. The speed at which a vehicle will begin to sideslip on a given surface along the radius indicated by the mark can be calculated if certain data are known. The following information is needed to calculate speed from critical speed scuff marks.

- Confirmation the marks are critical speed scuff marks
- The grade in the direction of slippage.
- The coefficient of friction of the road surface.
- The radius of the curve followed by the center mass of the vehicle.

Grade/Super elevation

Grade and super elevation are the same except for the direction they are measured in. The grade in the direction of slippage is the combination of any cross-grade (super elevation) and grade (slope) that is present. The percent grade is measured parallel to the direction the vehicle is slipping or parallel to the striation marks. Grade is positive (+) if the surface rises in the direction of the slippage and negative (-) if it falls in that direction.

- \( G = \frac{V}{H} \)

Drag Factor

Drag factor can be thought of as the force needed to produce acceleration (deceleration) in the direction of the acceleration divided by the weight of the object. Test skids should be done on level ground. If they are not, the drag factor must be adjusted to level ground before it is substituted into the critical speed equation.
Radius

The following are components of a circle:

- Circumference is the perimeter of a circle.
- Diameter is a straight line drawn across a circle and through its center.
- Radius is a line extending from the center of a circle to any point on the circumference. The radius is always one-half the diameter.

A chord and middle ordinate are the two measurements needed from the critical speed scuff marks. A chord is the distance of a tape measure stretched from one edge of the scuff mark to the other. A distance of 30 to 50 feet is usually sufficient. A middle ordinate is a measurement taken from the mid-point of the chord and perpendicular to it, to the edge of the scuff mark. Once a radius has been measured it can be calculated by using the following equation:

\[ R = \frac{C^2}{8M + M/2} \]

- \( R \) = Radius
- \( C \) = Chord
- \( M \) = Middle Ordinate

Critical Speed Equation

The speed at which the vehicle will just begin to sideslip is called the critical speed. A vehicle reaches its critical speed when centrifugal force equals the maximum attainable traction (centripetal) force. This can be shown mathematically by using the following equation:

\[ S = 3.86 \sqrt{\frac{R (f+G)}{1-fG}} \]

- \( S \) = Speed
- \( R \) = Radius in feet adjusted for the path of travel of center mass of the vehicle (Subtract 1/2 Wheel Track Width)
- \( f \) = Coefficient of friction (level ground)
- G = Grade/Super elevation expressed as a decimal measured parallel to the slippage (striations) is positive (+) if the surface rises in the direction of slippage, and negative (-) if it falls in that direction.

Critical speeds cannot be combined with any speed which occurs after the measured scuff mark. This is not a minimum speed equation. Critical speed is the average speed the vehicle traveled throughout the chord and middle ordinate measured.

Critical Curve Speed

The critical speed equation can also be used to calculate the critical speed of a curve. The radius of the center of the lane of travel is used for this calculation. This radius is determined by measuring the chord and middle ordinate on either the inside or outside edge of the road. Then add or subtract the distance from the center of the lane to the edge measured.

Critical curve speed may be used to indicate vehicle speed when there are no scuff marks or if the scuff marks have been inadequately reported. There must be reliable information that the vehicle side slipped off the curve, as opposed to being driven off the curve. A fatigued driver might drive the vehicle off the curve at speed limit speed or less, with no attempt to steer the vehicle around the curve. The use of the critical speed equation to estimate the speed of the vehicle would be inappropriate in this case.

For the purpose of estimating curve speed, be careful not to assume, without justification, that the vehicle was moving normally or attempting to move normally in its lane when it slid off the curve. Often a vehicle is turned more or less sharply than the road. This takes place when drivers try to "straighten out" the curve by using the entire roadway. If this is the case, the radius of the curved path the vehicle followed will not be the same as that of its normal path of travel.

Misapplication of the Critical Speed Equation

There are certain instances when the critical speed equation should not be applied.
The critical speed equation should never be used to estimate after collision speeds. The equation is based on balancing centrifugal and centripetal forces and not collision forces.

The critical speed equation should not be used to estimate the speed of an articulated vehicle. In theory the equation applies to articulated vehicles. Yet, practical application is difficult. At high speeds in a turn a towed vehicle could "swing out" due to a change in the articulation angle between the towing and towed vehicles. This can cause a jackknife situation which causes the towing vehicle and the towed vehicle to travel in different paths. Speed calculations made in this situation would be suspect.

The critical speed equation should not be used to estimate the speed of a motorcycle. Motorcycles will begin to slide if the speed and radius (of the curved path followed by center mass) cause an imbalance between the centrifugal force and the frictional force. Yet, motorcycle scuff marks are generally quite short as the motorcycle slides out from under the rider due to the rider's lean angle. Measuring such marks is difficult because they are generally very short.

The critical speed equation should not be used to estimate speed anytime there is a strong indication of load shift.

The critical speed equation should not be applied if the radius begins to change rapidly or if the offset between the front and rear tires becomes more than half the track width over the length of the chord. A radius that changes rapidly may indicate acceleration, or locked brakes during the yaw. The equation does not account for the longitudinal acceleration caused by engine power or brakes. When the offset is more than half the track width the vehicle may be in a side slide. Determining the speed in this instance would require a different approach.

The critical speed equation should not be applied if the marks on the roadway do not exhibit the characteristics of critical speed scuff marks (curved, rear wheels outside the front, striations).
Basic Trigonometry

Terminal Performance Objective

Given the need, students will correctly apply basic trigonometric functions as a part of the speed estimate process in accordance with the information presented in class.

Enabling Objectives

1. Define, in mathematical terms, the Law of Pythagoras.
2. Demonstrate methods for calculating the sine, cosine, and tangent of a right angle.

A right triangle is a triangle where only one of the angles is 90 degrees. The law of Pythagoras (or the Pythagorean theorem) states that the square of the hypotenuse of a right triangle equals the sum of the squares of the legs. The hypotenuse is defined as the side opposite the 90 degree (right) angle of the triangle. The hypotenuse will be designated as side c. The legs of the right triangle are the sides that are not the hypotenuse. They are designated by a and b.

The law of Pythagoras stated mathematically is as follows:

\[ c^2 = a^2 + b^2 \]

To solve for the hypotenuse simply take the square root of both sides of the equation.

\[ c = \sqrt{a^2 + b^2} \]

Sides a and b can also be solved:

\[ a = \sqrt{c^2 - b^2} \]
\[ b = \sqrt{c^2 - a^2} \]
Sine

The sine of θ, written as \( \sin \theta \), is defined as the length of the opposite side over the length of the hypotenuse. As an equation, it is written:

- \( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \) or \( \sin \theta = \frac{a}{c} \)

Cosine

The cosine of the angle θ, written as \( \cos \theta \), is defined as the length of the adjacent side over the hypotenuse. The equation would be:

- \( \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \) or \( \cos \theta = \frac{b}{c} \)

Tangent

The tangent of the angle θ, written as \( \tan \theta \), is defined as the length of the opposite side over the length of the adjacent side. The equation would be:

- \( \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \) or \( \tan \theta = \frac{a}{b} \)

This is the same equation you learned in On-Scene Level II for calculating grade.
Introduction to Newton’s Law’s of Motion

Terminal Performance Objective

Given the need, students will correctly use basic principles of physics to analyze vehicle dynamics in accordance with the information presented in class.

Enabling Objectives

1. List the three Newtonian Laws of Motion.
2. Define mass, acceleration and force.
3. List the mathematical expression that defines uniform circular motion.

Newton’s Laws of Motion

Newton's First Law may be stated like this: Every body at rest tends to remain at rest, while every body in motion tends to remain in motion, unless it is acted upon by an unbalanced external force.

This law describes the concept of inertia. Inertia is that resistance to stopping you feel when you are riding in a car that has to stop suddenly. The inertia of our bodies tends to resist the stopping, and we feel ourselves slide forward against our seat belts. The definition of force can also be seen in this law. An unbalanced force must exist to change the motion of a body. Force can be defined as an action which tends to change the motion of a body.

Newton's Second Law may be stated like this: The acceleration of any body is directly proportional to the force acting on the body, while it is inversely proportional to the mass of the body.

Mass is different from weight. Mass is the amount of matter that an object contains. Weight is the measure of how strongly the earth attracts an object of a given mass. A block of steel would weigh less on the moon than it does on earth because the gravitational attraction is less. However, the mass of the block of steel on the moon would be the same as it is on earth as it would contain the same amount of matter.
Acceleration is the rate change of velocity with respect to time. In other words, acceleration is the measurement of how fast the velocity of an object changes. This change can be either positive or negative. If the acceleration is positive, the object will speed up.

If the acceleration is negative, the object will slow down. Acceleration will also result any time the direction component of velocity changes. Example: A vehicle traveling at a constant velocity is suddenly struck by another vehicle. This will cause a change in direction and consequently a change in the vehicle's velocity. The change in velocity is acceleration.

Newton's second law of motion states a body that is acted upon by a force will be accelerated in the direction of the force. This means that the magnitude of the acceleration (change in motion) will be directly proportional to the force.

The second part of the law states that the acceleration of an object is inversely proportional to the mass of the object. If the mass of the body is doubled and the force remains the same, the acceleration will be only half what it originally was.

In the equation $a = F/m$, (a) is acceleration, F is force, and M (w/g) is mass. This equation can be rewritten in a more common form by solving it for force, F:

- $F = Ma$

With this equation we can see that if (F) gets bigger, then so does (a), if (M) remains the same. When (M) gets bigger, then (a) gets smaller, as long as (F) remains the same.
Newton's Third Law has to do with the relationship between acting and reacting forces. It may be stated as follows: For every force exerted on a body by another body, there is an equal but opposite force reacting on the first body by the second.

Another way to say this: For every action, there is an equal but opposite reaction. Let's say there is a five pound weight sitting on a table. The weight pushes down on the table with a force of five pounds. At the same time the table pushes up on the weight with a force of five pounds. Or if we pull with a given force on a rope tied to a tree, the rope in turn pulls back on us with the same force.

In the field of accident investigation, acting and reacting forces are found when a vehicle skids to a stop. The tires exert a force on the road while the road, in turn, exerts a force on the tires that is equal in magnitude but opposite in direction. This law indicates that forces exist in pairs not singularly, and are equal in magnitude but opposite in direction.

Speed and Velocity

In accident investigation there is a difference between speed and velocity. Mathematically they are both defined the same.
- S = D/T
- V = D/T

However, in accident investigation velocity is a vector quantity, while speed is a scalar quantity. A vector quantity is any quantity that must be described by both a magnitude and a direction. A scalar quantity may be described by magnitude alone.

In the case of a vehicle, we might say that its speed is 55 mph with respect to the road surface. However, to describe the velocity of the vehicle, we would also have to specify the direction. Example: The vehicle is traveling 55 mph due north. To effectively deal with some traffic accident problems, we must know the velocity of the vehicle rather than just its speed.

Uniform Circular Motion

Uniform circular motion is nothing more than the movement of a body in a circular (constant radius) path at a constant speed. In traffic accident investigation, uniform circular motion is important to us when a vehicle leaves the road in a critical speed scuff situation.

Recalling the difference between velocity and speed we know that velocity is specified by both a magnitude and a direction. Speed is specified by magnitude only. Reason tells us that the velocity of a body moving in a circular path at a constant speed is constantly changing, since the direction of the body is constantly changing.

We will recall that a change in velocity results in an acceleration. We will also recall, from Newton's first law, that a mass undergoing a change in velocity has force acting upon it. Thus, we may conclude that an external unbalanced force is acting on a body to cause it to move in a circular path.

In the case of a vehicle on a road surface, the force of friction between the tires and the road keeps the vehicle moving in a circular path. The external unbalanced force may have several sources. When we begin the derivation of the critical speed equation we will have to express uniform circular motion mathematically.
Consider what happens when an object, such as an auto, is not moving in a straight line, but is following a curve that is an arc of a circle. Assume that it has a constant velocity of \( V \) on a circular path that is at a distance \( R \) (radius) from the center point \( O \). At point 1 the direction of motion, that is the velocity \( V_1 \), will be tangent to the circular path and at a right angle to the radius, \( R \) at that point.

![Diagram of circular motion](image)

A short time later, say \( t \) sec later, the object has moved to point 2. The velocity is the same as it was at point 1. Remember the object is moving at a constant velocity. However, the direction has changed and that is indicated by \( V_2 \).

What we are really interested in is the change occurring from \( V_1 \) to \( V_2 \), since it is this change of velocity with respect to time that results in acceleration. We will call this change in velocity \( V_3 \), as it is the difference in motion from \( V_1 \) to \( V_2 \) in time \( T \).
The angle between the radius at 1 and 2 is the same as the angle between V1 and V2. The triangle between the radius at 1 and that at 2 in Figure 1 is a similar triangle (ratios are the same) to the triangle formed by V1, V2, and V3 in Figure 2. These both form Isosceles triangles. Isosceles triangles have two equal sides. Therefore, the corresponding sides of similar triangles are proportional.

\[ a = \frac{V^2}{R} \]

This is the form of the equation we are interested in. It can be described as centripetal acceleration and is directed in toward the center of the circular path followed by the body. The force associated with this acceleration, is called centripetal force. This holds the body in its path.

Remember Newton's third law. Forces come in pairs and are equal and opposite. Along with the centripetal acceleration and centripetal force, there is another acceleration and force, called centrifugal force. Centrifugal force has the same magnitude as centripetal force but has opposite direction. This is the force that makes the car slide off the road.

If the centrifugal force causes the vehicle to slide off the road then it has overcome the centripetal force. You might ask how this can happen if forces are equal. The forces are equal only as long as there is no change in the motion. When the centrifugal force gets out of balance with the centripetal force there is new unintended motion. It is that new unintended motion that causes the vehicle to leave the desired radius or path of travel.

When traveling at normal speeds in a curve the driver is controlling the forward acceleration. When the vehicle is traveling too fast for the desired radius then the centrifugal force is greater than the centripetal traction force between the tires and the road. It is at this point that the undesired lateral acceleration comes into play. Remember where there is acceleration there is force acting on mass.

If the vehicle is leaving critical speed scuff marks, we know that its brakes are not being applied, since the tires are rotating as well as sliding. The force trying to retard the vehicle's forward motion is very slight. The speed of the vehicle is reduced very little in the first hundred or so feet of the scuff mark.
The equation we have just developed deals with a constant speed. Since the vehicle can no longer be steered and since there are no other forces acting to change the direction of the vehicle, it must, then, follow a constant radius path.
Derivation of the Radius and Critical Speed Equations

Terminal Performance Objective

Given a set of variables consistent with Newtonian Physics, students will correctly copy and analyze the derivation of the radius and critical speed equations in accordance with the information presented in class.

Enabling Objective

1. Copy and analyze the radius and critical speed derivation

Radius Equation

The radius equation was derived to give us the radius of a circle if we know the chord and middle ordinate. The radius of a circle is equal to one-half the diameter, or the distance between any points on the circle to its exact center.

Critical Speed Equation

From our previous blocks of instruction we learned that velocity has both magnitude and direction. A change in velocity can be brought about by either a change in magnitude or direction and will always result in acceleration.

Consider a vehicle traveling in a circular path at a constant speed. In this case, the magnitude of the velocity remains unchanged. The direction of the velocity, however, is undergoing change and this will result in a lateral acceleration. Accurately reflecting the value of this lateral acceleration is a major part of the derivation of the Critical Speed Equation.
The relationship between the sides of a right triangle is expressed as $c^2 = a^2 + b^2$.
\[
\begin{align*}
\text{Fn} &= W \cos \theta + Ma \sin \theta \\
\text{Ft} + W \sin \theta &= Ma \cos \theta
\end{align*}
\]
Traffic Accident Reconstruction Level II

Traffic Accident Reconstruction Level II, introduces students to the proper techniques for collecting and evaluating evidence associated with accident vehicles that fall, flip, or vault. Students who successfully complete this course learn the procedures necessary to accurately estimate the speed of accident vehicles involved in this type of accident.

Learning Objectives

- Define and describe falls, flips and vaults
- List three kinds of information needed to
- Calculate the speed of objects that fall, flip or vault
- Demonstrate an ability to estimate speed from falls, flips and vaults
- Define and describe uniform projectile motion
- Explain when a critical speed yaw becomes a spin.
- Define slip angle.
- Define velocity vector.
- Define heading vector.
- Define rolling resistance.
- Demonstrate one method for adjusting the drag factor of a vehicle having less than 100% slippage which has left spin marks.
- Demonstrate one method for calculating speed loss from vehicles which leave spin marks.
- Demonstrate one method for combining speeds.
- Analyze percentage of braking.
Falls, Flips, and Vaults

Terminal Performance Objective

Given a simulated traffic accident, students will accurately evaluate falls, flips and vaults for the purpose of estimating the speed of objects that travel through the air in accordance with the information presented in class.

Enabling Objectives

1. Define and describe falls, flips and vaults
2. List three kinds of information needed to
3. Calculate the speed of objects that fall, flip or vault
4. Demonstrate an ability to estimate speed from falls, flips and vaults
5. Define and describe uniform projectile motion

The definition of a fall, flip, or vault must be understood by the accident investigator.

A fall occurs when a vehicle is traveling forward and is no longer supported by the surface it is moving over. At the beginning of the fall the vehicle can be going uphill, downhill, or on a level surface. A side slipping vehicle could experience a fall also. In fall situations the vehicle will generally land lower than the takeoff point. In some situations the landing point could be higher than the takeoff point.
Flips occur when vehicles are moving sideways and the resistance at the tires is sufficient to cause the vehicle to rise and move through the air. This can be caused by the wheels hitting a curb or furrowing in loose material. Vehicles "tripped" in this manner generally travel through the air horizontally for a considerable distance.

In these situations it is difficult to determine the takeoff angle. Consequently, a takeoff angle that will give a minimum speed is assumed. A rollover is sometimes used to describe a flip. However, rollovers may occur without the wheels striking a curb or furrowing in loose material. Therefore a rollover and a flip are not the same.

Vaults are similar to flips, except they are "end-over-end" flips. A vehicle which vaults must be moving forward instead of sideways. The term vault is often interchanged with the term fall. They are not the same.
The most accurate estimates of speed in accident reconstruction are those made from reliable observations and measurements in a situation where the vehicle left the ground and fell through the air before landing. In fall situations the vehicle nearly always lands right side up, although it may roll or flip after it lands.

A car which is moving through the air and is no longer supported by the ground, tends to keep moving in a straight line in the direction it was headed when it left the ground. The weight of the car (force of gravity) makes it fall toward the ground with increasing vertical velocity until it lands (what goes up, must come down). If grade is present at the takeoff point (either up or down) the car will have a vertical component of velocity.

In the vertical direction, gravity is the only force acting on the car. It causes the car to be accelerated downward at a rate of 32.2 fps². The car's horizontal component of velocity will remain essentially the same at landing as it was at takeoff. The only force acting on the car as it travels through the air is air resistance. Since the distance traveled by cars through the air is relatively short, the effect of air resistance is negligible and does not have to be considered.

If we can develop equations to determine the time it takes a car to travel both the vertical and horizontal distances in a fall, then we can also calculate the velocity of the car at the beginning of the fall.
Information Needed

To make speed calculations from vehicles which have traveled through the air, the following information is needed:

- The horizontal distance traveled by the vehicle's center mass from where it left the ground to where it landed
- The vertical distance traveled by the vehicle's center mass from where it left the ground to where it landed
- The angle of take off of the path the vehicle was traveling before it left the ground

Normally, a vehicle does not come to rest where it strikes the ground. It rolls or slides on the surface until it stops. Measurements must be made from the point where the vehicle left the ground to its position when it first landed. The investigator must be certain that he measures to the first point of ground contact. The takeoff angle must be measured along the path the vehicle actually traveled before it left the ground. This path is often shown by tire marks of one kind or another.
Measuring and calculating the take off angle \((G = V/H)\) then \((\text{Inv. Tan})\)

Measuring the distance a vehicle falls when it takes off into the air may be quite simple or a bit complicated. The spot where the vehicle took off is located by coordinates or triangulation. Measuring the vertical fall is more difficult when the vehicle goes off the top of a bank into a deep ditch.

You will usually have little difficulty determining where the takeoff was. Landing is not so simple. It is usually marked by scars in the bank or ground or tire marks on hard surfaces. Remember the distance traveled by the vehicle's center mass must be calculated. Match the marks at the landing point to the object on the vehicle which made the marks and adjust for distance traveled by center mass.

Some special equipment may be needed to measure accurately a large or long vertical drop. A surveyor, who has special equipment, may have to help in some cases. However, the simplest equipment such as string and line level, measuring tapes, and a carpenter's level will work for most situations.

When the vehicle falls a considerable distance the investigator may have to measure the fall using the "step" method or another method which involves using trigonometry.
The equation to calculate the speed of a vehicle at the point of takeoff is:

\[ S = 2.73 \frac{d}{\sqrt{d}} \sin \theta \cos \theta - h \cos^2 \theta \]

- \( S \) = Speed at takeoff
- \( d \) = Distance traveled horizontally by the vehicle's center of mass from takeoff to landing.
- \( h \) = Distance traveled vertically by the vehicle's center of mass from takeoff to landing. The value of \( h \) is positive if the landing point is above the takeoff point, and negative if the landing point is below the takeoff point.
- \( \theta \) = The angle of takeoff as measured relative to a horizontal plane. If the takeoff angle is uphill the sine and cosine value of that angle will be calculated. If the takeoff angle is downhill the angle will be subtracted from 360 and the sine and cosine value calculated.

You can say the speed calculated is how fast a vehicle would have had to be going to takeoff on the slope where it left the ground and traveled the distance horizontally while dropping the vertical distance to its landing spot. This is neither a maximum nor a minimum speed. If the measurements are accurate, you could not say that the vehicle would have had to be going faster or slower than the speed calculated. Therefore, the calculated speed is a very good estimate.

A vehicle's position when it first landed can be a difficult to locate. For example, if the landing place is a steep slope, water surface, or is covered with rocks or trees. Establishing the landing position may involve considering three circumstances:

- Dimensions of the vehicle, to determine the location of its center of mass;
- Scars on the ground showing where the vehicle first landed;
- Damage to the vehicle showing which part of it struck the ground first.

Speeds from falls, flips, and vaults can not be combined with any speed after the fall, flip, or vault.

Unknown Takeoff Angles (Flips)
Previously we have dealt with examples of falls. The equation to determine the speed of vehicles which flip is the same as that for vehicles which fall. Flips are quite different in that the determination of the takeoff angle can be very difficult.

A flip will occur when the vehicle strikes an object that stops forward movement of part of the vehicle at or near the ground. The rest of the vehicle tends to keep going in the direction the vehicle was initially heading. It can only do this by pivoting on the part that is stopped, usually a wheel. This pivoting requires that the vehicle's center of mass rise or lift. While in the air the vehicle can rotate rapidly causing it to land upside down. The vehicle will rarely stop where it first lands but will continue to roll until it comes to rest.

Speed estimates from flips require the same information as those from falls. The investigator must document the horizontal distance the vehicle's center mass moved through the air from takeoff to landing. The vertical distance traveled by the vehicle's center mass between take off and landing must also be measured as well as the takeoff angle.

As previously mentioned, determining the takeoff angle for vehicles that flip is difficult and can rarely be determined accurately. An equation can be used to calculate the angle which will give a minimum speed. The result will be an angle of approximately 45 degrees which will give the greatest distance for the lowest speed.

The equation for the angle $\theta$ that allows the minimum speed estimate to be calculated is:

$\theta = \text{Inv Cos } \left[ -h/\sqrt{(d^2 + h^2)} \right] \div 2$

- $\text{Inv Cos} =$ Inverse cosine of everything in the brackets
- $\theta =$ Angle which will give the minimum speed
- $h =$ Height of fall (+h is the vehicle lands higher than takeoff and -h if it lands lower)
- $d =$ Horizontal distance
- $2 =$ Constant
Speeds from flips calculated in this manner are subject to the usual, generally minor errors reflecting inaccuracies of original measurements. Using any angle larger or smaller than the angle given by the equation will result in a higher speed. The calculation assumes that the angle of takeoff is that at which the vehicle would go farthest for its speed.

Additional speed is lost in tearing up the ground or possibly damaging wheels (for example, when curbs are struck). Therefore, it can be said that the speed of the vehicle was actually greater than that estimated, possibly much greater, so the calculated speed is a minimum.

Derivation

The Fall equation is based on a concept in physics called Uniform Projectile Motion. This concept means that motion in the horizontal direction and motion in the vertical direction are independent of one another. The time it takes a vehicle to move through the air vertically is the same time it would take it to move through the air horizontally.
Estimating Speed Loss from Spin Marks

Concepts and Definitions

Terminal Performance Objective

Given the need students will correctly apply principles and define terms related to a vehicle spin analysis in accordance with accepted principles of traffic accident reconstruction.

Enabling Objectives

1. Explain when a critical speed yaw becomes a spin.
2. Define spin angle.
3. Define velocity vector.
4. Define heading vector.

Many times a vehicle will lose directional control and spin about its vertical axis. This loss of control can be caused by a collision or an extreme turning maneuver.

- The marks made by a vehicle which is rotating about its vertical axis are called spin marks.

- Trooper Ward Holton of the Georgia State Patrol states that “in 82% of my cases, at least one of the crash vehicles could have been assessed for the spin analysis”.

The critical speed equation should not be used to calculate speed from spin marks once the distance between the front and rear tracking tires exceeds $\frac{1}{4}$ of the vehicle wheelbase.

$1/4$ of $9.8' = 2.4'$

If all the wheels were locked during the spin, the minimum speed equation can be used to calculate the vehicle’s speed loss.

$$S = 5.47 \sqrt{\mu N + G} \cdot pd + (\mu N + G) \cdot pd + (\mu N + G) \cdot pd + (\mu N + G) \cdot pd$$

When the wheels are rolling, or there is less than 100% slippage then the marks must be analyzed in segments.

The drag factor for each segment will have to be adjusted.

The speed loss for each segment can then be calculated and combined to determine the total speed loss.

Definitions

- Heading vector is the direction that the vehicle headlights are pointing.
- Velocity vector is the direction taken by the vehicle’s center of mass.
- Spin Angle ($\alpha = \text{alpha}$) is the difference between the tangent angle of the spin mark (velocity vector) and longitudinal axis of the vehicle (heading vector).
Adjusting Drag Factor

Terminal Performance Objective

Given the need students will accurately assess drag factors and apply the necessary adjustments for vehicles which leave spin marks in accordance with accepted principles of traffic accident reconstruction.
Enabling Objectives

1. Define rolling resistance.
2. Demonstrate one method for adjusting the drag factor of a vehicle having less than 100% slippage which has left spin marks.

Rolling resistance is the horizontal force required to keep a vehicle in motion on a level surface, with the engine disconnected from the wheels and with no brake application; drag factor produced by friction within the vehicle and deformation of the tires and road surface.

- For a free rolling tire the rolling resistance factor is between 0.01 and 0.02 according to some sources.

- From testing done by Trooper Ward Holton the rolling resistance is between 0.02 and 0.04.

When estimating speed loss from a spin, using the coefficient of friction over the spin distance can lead to a significant error. Instead the drag factor over the spin distance must be determined.

Begin by understanding the difference between the heading vector and the velocity vector.
Derivation

An equation can be developed which will allow us to calculate the drag factor for a spinning vehicle.

From Newton’s Second Law

1) \( F = ma \)

2) \( m = \text{Mass} = \frac{w}{g} \)

Drag factor (f) is a ratio of accelerations that is expressed as:

3) \( f = \frac{a}{g} \)

Equation (3) solved in terms of (a) is expressed as:

4) \( a = fg \)

Equation (1) can be written as:

5) \( F = \frac{w}{g} (fg) \) Cancel the (g) leaving:

6) \( F = wf \)

Coefficient of friction is expressed as:

7) \( \mu = \frac{F}{w} \)

Drag factor can also be expressed as,

8) \( f = \frac{F}{w} \) or \( f = \text{all parallel forces/weight} \)
On level ground with 100% slippage,

9) \( f = \mu \)

If there is grade and the vehicle has 100% slippage then:

10) \( f = \mu + G \)

Equation (10) would also apply if there was grade and the vehicle was sliding laterally or 90 degrees to its heading.

If there is no braking and the wheels are free rolling the vehicle has a rolling resistance factor between 0.01-0.02. In this situation equation (10) would be written as:

11) \( f = fr + G \)

Once a vehicle begins to spin across a surface, the drag factor on the vehicle is changing if any of its wheels are free to rotate.

The amount of resistance from the tires, which is opposite to the direction the vehicle is moving (true in a straight skid also) is equal to the rolling resistance plus the sideways component of the tire forces from the vehicle.

The difference between the angle the vehicle’s center mass is traveling (velocity vector) and the direction the vehicle is pointing (heading) is known as the spin angle, \( \alpha \).

This is an absolute value and can never be a negative term as would be the case if the vehicle spun in a complete 360 degree circle. Therefore equation (11) is written as:

12) \( f = fr + (\mu - fr) \sin \alpha + G \)
The sine component is the constant ratio between 0 degrees and 90 degrees which is directly proportional to 0 to 1 (or 0 percent to 100 percent).

The Sine of 0 degrees is 0. The Sine of 90 degrees is 1. This means a vehicle sliding 90 degrees to its heading has 100% slippage as previously stated in equation (10).

If the Sine value were 90 degrees then the fr term would go away because there would be 100% slippage.

The fr term must be subtracted or else there would be a chance to have a higher than possible drag factor when the vehicle is in a side slip.

The terms fr and µ are inversely proportional. As one goes up the other must go down.

\[ f = fr + \frac{\mu - fr}{\sin \alpha} + G \]

- \( f \) = adjusted drag factor
- \( fr \) = rolling resistance factor
- \( \mu \) = coefficient of friction
- \( \sin \alpha \) = sine of the spin angle
- \( G \) = grade

Speed Estimates

Terminal Performance Objective

Given the need students will correctly estimate the speed of vehicles by using mathematical formulae and analyzing spin mark evidence in accordance with accepted principles of traffic accident reconstruction.
Enabling Objectives

1. Demonstrate one method for calculating speed loss from vehicles which leave spin marks.
2. Demonstrate one method for combining speeds.
3. Analyze percentage of braking.

Two things are a must before a spin analysis can begin.

1. Scale Diagram
2. Accurate evidence interpretation

Plot the marks and place the vehicle tires on each respective tire mark. Show the path of travel of center mass. The curved blue line is the vehicle’s center mass travel.

Draw a straight line from the center mass of the vehicle at the beginning of the spin marks. Draw offset lines at intervals along the straight line. These do not have to be equal. Consider placing them where there are surface or grade changes.
Measure the distance traveled between each offset. These measurements should be along the path of travel of the vehicle's center mass.
Draw a heading and velocity vector for the vehicle at each offset. The velocity vector is tangent to the blue arced line that is drawn between the vehicle’s center mass.
Measure the spin angle at each offset. This is done by taking the angular difference between the vehicle’s velocity vector and the vehicle’s heading vector. In this example the angle is approximately 104 degrees.
Once each angle is measured the average is taken as shown.
If the average of angle $\alpha$ were taken over the entire distance, it is possible that the angle would be inaccurate. For example, if the vehicle spun 180 degrees then the average would be 90 degrees. Since the sine of 90 degrees is 1, that would be equal to the full drag factor or 100% slippage over the spin distance.

By taking smaller increments of the vehicle’s trajectory an average spin angle is obtained for each segment. That average is close to the starting and ending spin
angles for each segment. The more angles used to figure the average, the more accurate the speed estimate.

Create a chart.

The following chart was created in Excel. The variables for this example are:

\[ \mu = .70 \]
\[ fr = .02 \]
\[ G = .01 \]
\[ d1 = 22 \text{ ft.} \]
\[ d2 = 21 \text{ ft.} \]
\[ d3 = 20 \text{ ft.} \]

<table>
<thead>
<tr>
<th>Interval</th>
<th>Distance (ft)</th>
<th>( \alpha ) Avg (degrees)</th>
<th>( \mu )</th>
<th>( fr )</th>
<th>( \sin \alpha )</th>
<th>Grade</th>
<th>f adj</th>
<th>Speed</th>
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Combined Speed 28.76 mph
In the previous example the combined speed is 29.10 mph, the drag factor is .70 and the spin distance is 64 feet. The adjusted drag factor can be calculated as follows:

- \( f = \frac{S^2}{30d} \)
- \( f = \frac{29.10^2}{30(64)} \)
- \( f = \frac{846.81}{1920} \)
- \( f = .44 \)

The drag factor ration is equal to:

- \( \eta = \frac{\text{Adjusted Drag Factor}}{\text{Total Possible Drag Factor}} \)
- \( \eta = .44 / .71 \)
- \( \eta = .61 \)
Traffic Accident Reconstruction Level III

Traffic Accident Reconstruction Level III, introduces students to this procedure. The conservation of linear momentum is another useful speed calculation method. It also provides a starting point for many of the time/distance calculations that are common to traffic accident investigation. Students who successfully complete this course will have another necessary ingredient to add to their accident investigation skills.

Learning Objectives

- List five kinds of information needed to evaluate how vehicles react to a collision sequence
- Analyze damage profiles to determine rotation and the amount of collapse
- Define vector
- Demonstrate two methods for adding vectors
- List the sign of X and Y in each quadrant of the left hand coordinate system
- Define Newton’s Second Law of Motion
- List four types of information needed for a momentum analysis
- Demonstrate two methods for calculating impact speeds
Collision Analysis

Terminal Performance Objective

Given the need, students will accurately analyze vehicle dynamics by evaluating the results of the collision on the accident vehicle in accordance with the information presented in class.

Enabling Objectives

1. List five kinds of information needed to evaluate how vehicles react to a collision sequence
2. Analyze damage profiles to determine rotation and the amount of collapse

When examining accident vehicles the following information is needed.

- Personal inspection of vehicles, including measurements of collapse
- Reports by others of vehicle examination, preferably in writing, but possibly verbal
- Photographs
- Collision Analysis
- Dimensions of the vehicle from actual measurements or published data
- Sometimes vehicle parts such as wheels, bumpers, lamps, and paint chips are useful in identifying contact-damage areas, especially when more than two vehicles are involved

Reconstructing a traffic accident depends greatly on the quality and quantity of data which have been collected.

Elements Of A Collision

During collision between two vehicles, a whole series of events takes place while the vehicles are in contact. Three of these events are especially noteworthy.
First Contact:

First contact is the beginning of a collision. At that instant, force begins to develop between the objects.

Maximum engagement:

From first contact, penetration and force increase to maximum engagement. For an instant at maximum engagement with a fixed object, the vehicle, and particularly the part of it in contact with the object struck, stops. Then, for that instant, the vehicle and object are at the same speed, zero. Thereafter, due to elasticity of the material of which the vehicle is made, the vehicle moves backward.

As the penetration and force decrease the rearward velocity increases until last contact. Then the objects in the collision disengage and separate. In general terms, the greatest penetration or collapse determines the greatest force; and the amount of force determines the change in speed. Motor-vehicle bodies have very little elasticity so the amount of rebound (restitution) after maximum engagement is practically zero.

Separation or stopping if the vehicles or other objects remain engaged occurs when force between the colliding objects becomes zero. Virtually all of the deformation at maximum engagement remains as vehicle damage.

Last Contact

Last contact is the end of the collision. Decreasing force and penetration cause rearward motion and increasing rearward speed. Obviously, the force becomes zero.
Many people tend to think of impact as only the first contact or, sometimes as maximum engagement. Impact and collision mean essentially the same thing just like speed and velocity signify practically the same idea.

Impact may be divided into two classes:

A full impact means some part of the colliding surfaces attain the same speed during impact. If the colliding bodies remain in contact when motion ceases, the impact is full because the parts in contact are at the same speed. Motion between parts in contact will cease momentarily. Full impact does not mean that either of the objects in collision necessarily stopped on the ground (relative to each other not relative to the ground).

A partial impact means no substantial parts of colliding surfaces attain the same speed during collision. All partial impacts involve disengagement of colliding surfaces.

In opposite direction collisions, a full impact is often loosely referred to as head-on, whereas a partial impact is spoken of as a sideswipe. In motor vehicle accidents, one of the colliding objects is a motor vehicle. The other may be another motor vehicle, a pedestrian, cyclist, fixed object, or the roadway surface.
The force of impact modifies the speed, direction, and rotation of the accident vehicle. In braking and turning, friction forces change the speed and direction of the vehicle. Collision forces are ordinarily much more violent. Friction forces often leave signs on the road surface of what happened; collision forces also leave signs: damage to vehicles, injury to pedestrians, and scars in the road or roadside. Consider the effects of collision forces on a vehicle.

**Velocity Change**

Thrust between a vehicle and other object results in collapse of vehicle parts. The amount of collapse depends on 1) the amount of force; and 2) the strength of the vehicle’s structure. If a vehicle runs head-on into something, the vehicle is slowed (decelerated); but if another vehicle hits it from behind, the vehicle is speeded up (accelerated).

**Rotation**

Thrust can make an object rotate or spin as well as change its speed. Rotation depends on three things:

- Strength of the thrust
- Direction of the thrust
- Point of application

If the force is directly toward the center of mass of the vehicle, the vehicle is slowed down or speeded up in line with that force, but it does not rotate. This is a centered force. Centered forces are rare, and they do not have to be applied to the center of the vehicle. They may be applied anywhere on the vehicle, as long as they are directed toward the center of mass. Usually, forces produced by collision are more or less eccentric, that is, the force is not directed toward the center of mass. An eccentric force results in rotation.
Direction Change

If the force is only slightly eccentric and there is a full impact, the vehicle cannot continue in the same direction. When one side of the vehicle stops and the other keeps going, the vehicle must pivot on the part which is stopped. The ensuing rotation forces the vehicle to take a new direction, before disengagement.

This is a common kind of collision; the vehicle departs from impact obliquely to the right of its approach if the rotation is counterclockwise. In this kind of collision, the vehicle is not moved to one side as a vehicle may be if it strikes a guardrail at a small angle; that would be a partial impact. Changing direction after collision is an important effect of eccentric impact.

Collisions With Fixed Objects

A number of conclusions can be made when a moving vehicle collides with a fixed object.

- The moving vehicle struck the stationary object.
- The direction of force developed in the collision is in the direction of movement of the moving vehicle.
- All of the damage and all of the motion after collision is due to the energy of the moving vehicle. This total energy is an indication of the speed of the striking vehicle.
Collisions Between A Moving Vehicle And A Stopped Vehicle

A number of conclusions can be made when a moving vehicle collides with a stopped vehicle.

- The moving vehicle strikes the stopped vehicle
- The initial force against the stopped vehicle is in the direction of motion of the moving vehicle.
- The moving vehicle will be slowed, and if the force is sufficient, the stopped vehicle will be set in motion in the general direction of movement of the moving vehicle.
- Rotation of the vehicles after first contact depends on the first contact point on each, the angle between the vehicles, and the angle of the moving vehicle to its direction of motion (angle related to its heading).
- Damage to both vehicles can be attributed to the energy of the moving vehicle; this total damage is an indication of that vehicle’s velocity if there is no post impact movement. If they move the movement must be accounted for in the overall speed calculation.

Collisions between Vehicles In Motion

A number of conclusions can be made when two moving vehicles collide.

- It is useless to discuss which vehicle hit which; they simply struck each other.
- The direction of thrust on neither vehicle is the same as its direction of approach except in exact same direction or opposite direction collisions.
- Damage to the vehicle is not an indicator of the speed of either.
Damage is done by the forces of impact which deform the materials of the vehicles. At all times during impact, the force against one vehicle is exactly the same as the force against the other but in the opposite direction. Then whatever material in the contact area that is weakest will collapse first and most.

Damage And Movement

Three conditions are necessary to have the same damage result from different vehicle velocities in two-vehicle collisions:

The first contact position of the vehicles must be the same.
The vehicles must be approaching each other from the same direction.
The closing velocity must be the same.
Damage reflects only motion of the vehicles with respect to each other, not movement with respect to the road. When vehicles collide you can not tell which vehicle was traveling fastest from damage alone. Other information is needed to determine that.

Damage And Speed

In a two vehicle collision, does the faster or slower vehicle suffer the most damage? The fact is that in a two-vehicle collision, the stronger vehicle suffers the least damage. Since the forces between the impacting vehicles are equal and opposite at all times, the weaker structure of either is crushed and broken most.

In an impact between the channel-iron bumper of a big truck and the ornamental grill on a compact car, the truck bumper remains virtually unscathed whereas the grill is torn and crushed. This is true with the truck stopped and the car moving, with the car stopped and the truck moving or with both moving; the angle of impact makes no difference. It is only the strength of the parts in contact that counts.

Damage And Thrust

When vehicles make initial contact, the force of one vehicle against another vehicle or other object, begins to crush parts of the vehicle in the direction of the thrust. As the impact progresses, the vehicles rotate at different rates or in different directions.

This collision could be 90 degrees at maximum engagement
Vehicles in contact rotating at different rates and directions causes a corresponding change in direction of forces between the vehicles. This also leads to a change in direction of crush of vehicle parts. The direction of crush at maximum engagement, when forces between vehicles are greatest, gives the final direction and penetration of one vehicle into the other.

After that the force lessens, deformation remains because vehicle parts are essentially inelastic; they do not return to their original shape after deformation. Therefore, what you see after a collision is damage which indicates the direction and extent of penetration at maximum engagement, not that at first contact. There may be a slight difference between the maximum deformation in a collision and the final deformation.

Contact Damage

A thorough examination of contact damage areas can reveal how the vehicles behaved during collision. Obviously the contact damage areas of two vehicles reflect that they were against each other during collision. Deciding exactly how they were in contact can be difficult. Matching contact damage can be easy in some cases if there is a recognizable imprint from one vehicle on the other, or if the contact damage areas on the two vehicles have the same dimensions.

If damage to the side of a vehicle is approximately as long as the width of the vehicle that hit it, the damage may suggest a right angle collision.
A long area of contact damage on one vehicle and a short one on the other usually indicates a sideswipe (partial impact). This may not always be the case as one vehicle moving at an angle with a corner against the side of another can leave a long damage area. Remember, penetration increases from first contact to maximum engagement where slippage stops. Then impact is complete.

Carefully made measurements and photographs are necessary to substantiate your observations of damage. As you begin to learn and use momentum to calculate impact speeds, you will realize the significance of evaluating contact damage.

In order to analyze impact the investigator must have data to depend on. That data is the damage to the vehicles and their dimensions. You were taught in On-Scene Level I how to collect that type of data. Now we hope to show you how to use that information to determine impact speeds. The first step in understanding vehicle behavior in a particular collision is to draw an outline of each vehicle approximately to scale. On each diagram, show the collapsed or deformed area, contact-damage area, and direction of principal force.

The principal direction of force constantly changes throughout the collision sequence due to the rotation of the vehicles. The procedure used on the overhead should be used at various locations along the contact damage area. You are trying to determine the principal direction of force which is similar to finding an average.

Once you have the principal direction of force identified for one vehicle you can match it to the other (also drawn to scale). Place the two together as they would be at maximum engagement. The principal direction of force on one will be directly opposite of the principal direction of force of the other. Maximum engagement position of each vehicle with respect to the other can be derived from these diagrams.

Two simple rules govern this operation:

- The greatest collapse in the contact-damage area of each vehicle must be against the greatest collapse in the contact-damage area of the other.
- Thrust directions must be in line, that is, the thrust of one must be directly opposite to the thrust of the other.
The angle of first contact position of the vehicles in the previous slide is less than at maximum engagement because the blue vehicle has rotation while the red one has none. This is because the force line passes through the center mass of the blue vehicle.

All of this is important in determining speed from impact. The angles of approach (first contact) and departure are necessary pieces of information which must be determined. This type of information is also used with other information from the road to develop theories about how vehicles collided.

For example, if a theory of how two vehicles collided is developed from marks on the road, the theory can be tested by considering how well vehicle movements, according to the theory, correspond to those developed by collision analysis.

If the movements correspond closely, the theory is supported (but not necessarily proved); if the movements do not correspond well, the value of the theory must be questioned. Be sure to consider the evidence. Consider the following:

The following is a summary of the processes which have been explained up to this point.

- Draw an outline of each vehicle approximately to scale and preferably to the same scale as the after situation diagram.
- For each vehicle study the damage (or photos of the damage) and show on the vehicle outline its collapsed or deformed shape.
- Indicate on the outline, by a bracket with a pointer at each end, the extent of the contact damage areas.
- Locate the spot of greatest collapse, penetration, or distortion in the contact damage area, and then estimate the direction a vehicle part was moved to force it to that position. Show this on the outline as a thrust (force) arrow. Do this from several viewpoints in order to get the principal direction of force.
- Consider carefully how the direction of thrust is related to the center of mass of the vehicle. This gives you the eccentricity of the maximum engagement force and indicates rotation, if any, of the vehicle.
- On the outline, show direction of rotation by a curved arrow partly encircling the center of mass as indicated in several of the overheads.
- Put the vehicles together at maximum engagement position. Adjust these so that the maximum engagement deformation of each is against that of the other and
the thrust arrows are aligned. Keep the maximum engagement positions in mind, stick the adjusted outlines together or trace them on a single sheet.

- Carefully compare the direction of rotation and its rate (based on the eccentricity of the collision) for the vehicles involved. This is the basis for estimating the change in angle between the vehicles from first contact to maximum engagement. Then adjust positions of original outlines (which do not show damage) to represent first contact positions.

- Do the same to show positions of vehicles at separation.

- Go to the after-accident situation drawing. Look for signs of first contact position mainly irregularities in tire marks but sometimes scars on fixed objects. Look for signs of maximum engagement positions: collision scrubs, gouges, scrapes, spatter. Place your outline of the two vehicles on the drawing as close as you can.

- Put maximum engagement outlines on signs of maximum engagement, and so on. If there are no signs on the drawing of first contact or maximum engagement, place the outline according to the witness statement if any, or according to some theory of what happened.

- Consider how each vehicle would move from its maximum engagement position to its final rest position. Be aware of how it must rotate and move. You may have to change the maximum engagement position to make it conform to rotation predicated on vehicle damage, perhaps several times. You may have to re-evaluate your analysis of thrust direction and rotation.

- Try to account for all signs of the accident: tire marks, gouges, debris and so on reported or shown in photographs. If any seem not to be associated with the accident, be prepared to explain exactly what makes you think so.
Vector Addition

Terminal Performance Objective

Given the need, students will correctly demonstrate the ability to use vector addition for purposes of determining a vehicle’s pre-impact and post-impact momentum in accordance with the information presented in class.

Enabling Objectives

1. Define vector
2. Demonstrate two methods for adding vectors
3. List the sign of X and Y in each quadrant of the left hand coordinate system

Vectors have magnitude and direction
The Magnitude is the length of the vector

Tail ------------------------------- Head

The direction is give by the head of the vector

The most commonly used forms of vectors in accident reconstruction are to add two momentum vectors and to divide a momentum vector into two parts. Momentum is the product of mass time’s velocity and will be discussed in more detail later in the course.

Parallelogram Method of Vector Addition

The resultant (sum) of two vectors can be obtained by drawing the diagonal of a parallelogram with the two vectors as sides. This is done by placing the tails of both vectors at the same origin. Draw a line through the head of each vector parallel to the other vector. These lines intersect is one corner of the parallelogram. The diagonal line extending from the origin to the intersecting dotted lines is the resultant of the two vectors.
Polygon Method of Vector Addition

This method is no more difficult than the parallelogram method. All that is necessary is to place a head to a tail of consecutive vectors. Start at a convenient spot for the origin. Place each vector head to tail. For vector addition only the length and direction must be maintained. Vectors can be moved provided there direction and length is not changed. The resultant of the vector addition is a vector drawn from the origin to the head of the last vector.
Vector Components

A vector component is its effective value in any specified direction. In some momentum problems it is desirable to know the components of a vector that are not horizontal and vertical. This can be done by drawing parallelograms.

Left Hand Coordinate System

The left hand coordinate system is the system most accident reconstruction specialists use to analyze their momentum problems. In the left-hand system, the x-axis is horizontal and the y-axis is vertical. When measuring degrees from the x-axis, we measure counter-clockwise from 0 degrees to 360 degrees.

The numbers to the right along the x-axis from the vertex (where x and y axis cross) will be positive and those to the left along the x-axis from the vertex will be negative. Moving up the y-axis from the vertex, the numbers will be positive and moving down the y-axis from the vertex, they will be negative.
Left Hand Coordinate System

Quadrant II \((X-,Y+)\)  \quad Quadrant I \((X+,Y+)\)

\[ X - \quad X + \]

Quadrant III \((X-,Y-)\)  \quad Quadrant IV \((X+,Y-)\)

\[ Y - \quad Y + \]
Linear Momentum

Terminal Performance Objective

Given the need, students will accurately estimate vehicle impact speeds by using mathematical formulae and analyzing vehicle collision dynamics in accordance with the information presented in class.

Enabling Objectives

1. Define Newton’s Second Law of Motion
2. List four types of information needed for a momentum analysis
3. Demonstrate two methods for calculating impact speeds

Applying linear momentum to accident investigation allows us to calculate vehicle impact speeds. This is a critical element in the investigative process. Knowing impact speeds is the beginning step in determining pre-impact vehicle to roadway and vehicle to vehicle relationships.

Newton’s Laws of Motion

First Law

Every object persists in a state of rest or a state of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.

Newton’s first law is a statement about vectors. A car travels a constant velocity unless acted upon by an unbalanced force. As long as pressure is applied to the accelerator the car keeps moving at a constant velocity. If we try to coast, our car will rapidly slow down (unless it is traveling downhill). The reason it slows is the presence of other forces such as air resistance and friction.
Second Law

If there is an unbalanced force the car will accelerate in the direction of the force. The force is equal to mass times acceleration.

- $F = Ma$
- or
- $a = F/M$

The acceleration of a vehicle’s center mass is governed by $F = Ma$, no matter where the force is applied.

Third Law

For every action, there is an equal but opposite reaction. Or any force acting on a vehicle has a companion force equal and opposite to it acting on something else.
Momentum Equation

\[ W_1v_1 + W_2v_2 = W_1v_3 + W_2v_4 \]

It is important to recognize at this point that momentum and energy are not the same. In the derivation of the minimum speed equation, which comes from energy, we saw that Work = Force times distance = \( \frac{1}{2} MV^2 \).

Momentum is Mass times Velocity or MV. The two look similar, like house and horse. Even though they have similar units they are not the same thing.

Energy produces the damage in a collision. Impulse causes a change in momentum, which redirects the vehicles during collision. Impulse is directly proportional to both force and time.

Momentum is directly proportional to both mass and velocity. It takes a great deal more effort to stop a train than it does to stop a car, even though both are moving with the same velocity. The greater mass of the train gives it more momentum. A bullet fired from a gun has more penetrating power than a bullet that is thrown by hand, even though both bullets may have the same mass. The greater velocity of the fired bullet gives it more momentum.
A car weighing 4000 pounds is traveling 50 feet per second. Calculate its momentum.

Momentum = Mass x Velocity
Momentum = 4000/32.2*(50)
Momentum = 12422*(50)
Momentum = 6211 pounds-seconds

A car weighing 2000 pounds is traveling 100 feet per second. Calculate its momentum.

Momentum = Mass x Velocity
Momentum = 2000/32.2*(100)
Momentum = 6211 (100)
Momentum = 6211 pounds-seconds

The following example may help you visualize how the momentum of two vehicles before a collision is the same as the momentum of the two vehicles after the collision.

Consider two cars heading toward each other. They have equal weight and velocity, but in opposite directions. Assume the cars are aligned head light to head light. Most people would agree that after the vehicles collide, they will immediately slow to a stop because of the collision. This is indeed what they would do. After the collision there is no momentum, because neither vehicle has velocity. If there is not after collision momentum, there is no before collision momentum.

The reason there is no before collision momentum is because we must think of the momentum of the entire system, that is, the two cars together before the collision. Think of the momentum before the collision in terms of positive and negative quantities, because the collision takes place along the same line of travel. If the direction of travel of vehicle 1 is considered positive, then the before collision momentum can be expressed as:

\[ W_1V_1 + W_2V_2 = WV + (-WV) = 0 \]

Remember the left hand coordinate system. Movement to the right is positive and movement to the left is negative. The investigator must know the following in order to do a momentum analysis:
The approach path of each vehicle to its first-contact position;
The departure path of each vehicle from the collision;
The total weight of each vehicle (including its load);
The after-collision velocity of each vehicle

Solving Momentum Graphically

Angles: Always set the drawing up so W1V1 will be headed towards zero prior to impact. 0 degrees will be at 3 o'clock, 90 will be at 12 o'clock, 180 will be at 9 o'clock, and 270 will be at 6 o'clock. W1V1 will always be on the X axis, and W2V2 will always be on the Y axis. The angle is determined by the direction the vehicle is headed either prior to or after collision.

- Angle A will always be the approach angle of W1V1
- Angle B will always be the approach angle of W2V2
- Angle C will always be the departure angle of W1V3 which is W1V1 after impact
- Angle D will always be the departure angle of W2V4 which is W2V2 after impact

The vehicles correct weights and after impact speeds must be known to solve for impact speeds. The best way to obtain vehicle weights is to have the vehicles weighed and then add in passenger weights. After impact speeds are calculated by whatever means possible. Often the after impact skid marks may be used.

- W1 = Weight of vehicle # 1 (X axis)
- W2 = Weight of vehicle # 2 (Y axis)
- V1 = Impact speed (unknown)
- V2 = Impact speed (unknown)
- V3 = After impact speed of V1
- V4 = After impact speed of V2

Remember the law of the conservation of momentum. Momentum is not lost in a collision. The momentum before impact equals the momentum after impact.
Example:

A = 0  W1 = 2875 lbs
B = 90  W2 = 3900 lbs
C = 80  V3 = 32.58 mph (from skid marks)
D = 50  V4 = 31.36 mph (from skid marks)

Step 1

Determine the scale to be used.

First multiply W1V3, 2875 (32.58) = 93667.5 momentum units

Second multiply W2V4, 3900 (31.36) = 122304 momentum units

Total the two products,

122304 + 93667.5 = 215971.5

We want this to fit on our paper so the scale needs to be quite small. Since one-half our paper width is 4 inches we will divide the total by 4.

215971.5/4 = 53992.875 momentum units per inch.

Now we will divide the product of W1V3 and W2V4 by the scale we are using (53992.875). This will give us the length of the after impact vectors.

93667.5/53992.875 = 1.73 inches

122304/53992.875 = 2.26 inches
Step 2

Draw an X axis across the center of your paper and a Y axis to cross the center of the X axis. The origin will be the point where the two axes intersect.

Step 3

Draw a vector representing W1V3 which is 1.73 inches long and at an 80 degrees angle from the origin. Do the same with W2V4 which has a length of 2.26 inches and an angle of 50 degrees.

All departure angles are established from the X axis.

Step 4

Create the parallelogram by striking an arc with the bow compass from the end of W1V3 the length of W2V4. Strike another arc from the end of W2V4 the length of W1V3.

Step 5

Draw in the resultant from the origin to the point where the two arcs intersect. The resultant is the sum of the total after impact momentum.

Step 6

Extend the resultant from the point of origin in the opposite direction. It should be exactly the same length as the resultant you have already drawn. Remember total momentum after impact is equal to total momentum before impact.

Step 7

Draw a line at a 90 degree angle from the Y axis below the X axis to the end of the new resultant. From the end of that resultant draw a line at a 90 degree angle from the line you have just drawn. Make it intersect the X axis.
Step 8

Now measure the distance from where the last line drawn crosses the X axis to the point of origin. This is W1V1 and it should be approximately 1.8 inches long. From the point of origin measure down the Y axis to the line that is 90 degrees from the end of the resultant. This is W2V2 and it should be approximately 3.47 inches long.

Step 9

Multiply the length of W1V1 and W2V2 times the scale we are using (53992.875).

\[
\begin{align*}
W1V1 &= 1.8 \times 53992.875 = 97187.175 \\
W2V2 &= 3.47 \times 53992.875 = 187355.28
\end{align*}
\]

Step 10

Divide the weight of vehicle 1 into the product 97187.175 and divide the weight of vehicle 2 into the product 187355.28. The result is the speed for V1 and V2 in miles per hour.

\[
\begin{align*}
97187.175/2875 &= 33.80 \text{ mph} \\
187355.28/3900 &= 48.03 \text{ mph}
\end{align*}
\]

Circular Momentum

Vehicle impact speeds can also be solved by using the momentum equation which we derived earlier. This method is called circular momentum or 360 degree momentum. Let us look at the X and Y axis of the left-hand coordinate system we mentioned earlier in the course. When we place the vectors representing W1V1, W2V2, W1V3, and W2V4 onto the axis we will need to extend each from their point of origin to determine the direction they are headed.

The approach vector W1V1 will be 0 degrees. The drawing can be turned any way necessary so that one of the vehicles is heading towards 0. That vehicle will always be W1V1. Keep in mind, a momentum diagram must be used to establish respective approach and departure angles.
When using circular momentum we will always solve for V2 (Y direction) first. Recalling the momentum equation:

\[ W_1V_1 + W_2V_2 = W_1V_3 + W_2V_4 \]

Each velocity component in the equation is a vector and has a direction (angular measurement from a fixed reference point) and sense (direction the vector points, the head of the vector gives it the sense of direction) associated with it.

The vectors \( W_1V_3 \) and \( W_2V_4 \) are simply the hypotenuse of a right triangle which represents each vehicle’s after impact momentum. In solving for V2 we are only concerned with \( W_1V_3 \) and \( W_2V_4 \) movement in the Y or sine direction.

\[ V_2 = \frac{(W_1V_3 \sin C + W_2V_4 \sin D)}{W_2 \sin B} \]

In solving for V1 we are only concerned with \( W_1V_3 \) and \( W_2V_4 \) movement in the X or cosine direction.

\[ V_1 = \frac{(W_1V_3 \cos C + W_2V_4 \cos D - W_2V_2 \cos B)}{W_1} \]

These two equations are all that is needed to solve any momentum problem. In the case of in-line collisions the speed of one of the vehicles (V2) must be known. The value of V2 can not be calculated as it has a sine value of 0.

Momentum Limitations

Momentum has its limitations as does all the speed equations you have been introduced to. First of all, a momentum analysis is not possible without the following:

- Scale drawings are a must
- Vehicle damage information from actual inspection as well as photographs
- Straight edge
- Protractor or 360 degree overlay
- Template
Post crash vehicle dynamics information
Drag factor
Angles of Approach/Departure (center mass)
Vehicle weights to include load

Do Not Use Conservation Of Momentum Unless You Can Establish These Points:

The paths of approach for each vehicle must be known. They can be found from damage that shows force lines or skid marks leading up to the point of impact.

The paths of departure for each vehicle must be known. Usually these can be found from marks on the road that lead from the point of impact to separation.

The speed of each vehicle after impact must be known. Use whatever drag factor can be established for metal or rubber or rolling wheels, speeds from cars which fall, flip, or vault, or from cargo being launched into the air from force of impact.

The approximate weight of each vehicle plus its load must be known. This need not be exact, but weighing the car is the ideal situation.

Momentum Will Not Work If:

- Vehicles are steered after impact
- The angle of departure must be the result of the collision
- Acceleration after collision
- A vehicle hits a parked car and then is driven down the street before stopping
- Must be an uncontrolled final rest position
- Momentum does not work with IN-LINE collisions (head-on or rear-end) UNLESS you know the pre-impact speed of one of the vehicles
- There is little or no post impact data
A momentum analysis becomes very sensitive if the following conditions exist:

- If there is a large difference in pre impact momentum or if the approach angles are less than 20 degrees. Either of these situations will result in a momentum diagram that is long and skinny. Those types of momentum analysis are very sensitive. However, the information on the vehicle having the most momentum is usually good. Disregard the momentum of the vehicle having the least momentum.

- Secondary impacts can adversely affect the accuracy of a momentum analysis. If a secondary impact alters the natural redirection of the accident vehicles from the initial impact, then the momentum analysis may be subject to error. Secondary impacts that occur over a short period of time and result from violent collisions should not affect the accuracy of the momentum analysis.

Sensitivity Analysis Example

A 60,000 lb truck collides with a 3000 lb automobile and they stick together. Skid marks after the collision indicates that the post impact speed of the two vehicles after collision was 50 +/- 3 mph.

Just prior to the collision, the truck passed another vehicle, whose driver said that the truck had been traveling 55 mph. How fast was the automobile traveling just prior to the collision?

Sensitivity Analysis: V1 = Car; V2 = Truck

\[ W1V1 + W2V2 = (W1+W2)V_f \]
\[ 3000*V1 + 60000(55) = (3000+60000)50 \]
\[ 3000V1 + 3300000 = 3150000 \]
\[ V1 = 3150000-3300000/3000 \]
\[ V1 = -50 \text{ mph} \]
Sensitivity Analysis  Change Vf to 47 mph

\[ 3000 \times V_1 + 60000(55) = (3000 + 60000) \times 47 \]
\[ 3000V_1 + 3300000 = 29610000 \]
\[ V_1 = \frac{29610000 - 3300000}{3000} \]
\[ V_1 = -113 \text{ mph} \]

Sensitivity Analysis  Change Vf to 53 mph

\[ 3000 \times V_1 + 60000(55) = (3000 + 60000) \times 53 \]
\[ 3000V_1 + 3300000 = 3339000 \]
\[ V_1 = \frac{3339000 - 3300000}{3000} \]
\[ V_1 = 13 \text{ mph} \]

Coefficient of Restitution

Coefficient of restitution relates to a vehicle’s ability to return to its normal shape after a collision. Since cars do not bounce, car to car collisions are generally considered to be inelastic collisions. Once they are deformed they essentially stay deformed. However, in very low speed collisions the coefficient of restitution might need to be considered since the vehicle may retain its original shape.

For typical collision speeds encountered in traffic accident reconstruction cases, the coefficient of restitution can be considered to be zero. This means none of the kinetic energy was lost. A coefficient of restitution of approximately 1 is possible in very low speed collisions. This means all of the kinetic energy was lost.
Traffic Accident Reconstruction Level IV

In serious traffic accidents questions continually arise regarding positions of vehicles and or pedestrians prior to the collision. In order to fully analyze traffic accident, these time/distance relationships must be established. Traffic Accident Reconstruction Level IV, introduces the students to twelve motion equations that will enhance their investigative abilities. Students who successfully complete this course will be able to establish pre-impact vehicle to vehicle or vehicle to pedestrian relationships.

Learning Objectives

- Demonstrate the ability to apply three acceleration equations to traffic accident reconstruction problems
- Demonstrate the ability to apply three initial velocity equations to traffic accident reconstruction problems
- Demonstrate the ability to apply two end velocity equations to traffic accident reconstruction problems
Motion Equations

Terminal Performance Objective

Given the need, students will accurately calculate vehicle to vehicle and vehicle to roadway relationships using motion equations in accordance with the information presented in class.

Enabling Objectives

1. Demonstrate the ability to apply three acceleration equations to traffic accident reconstruction problems
2. Demonstrate the ability to apply three initial velocity equations to traffic accident reconstruction problems
3. Demonstrate the ability to apply two end velocity equations to traffic accident reconstruction problems

Understanding the relationships of the variables acceleration (a), time (t), distance (d), initial velocity (Vi), and end velocity (Ve), is essential when addressing accident reconstruction problems.

- Distance is a linear measurement from some point. For reconstruction problems distance is measured relative to a coordinate system fixed on earth. Distance is a scalar quantity, having magnitude only. In U.S.A. units the dimension for distance is always feet.
- Time is measured in seconds. In some cases it may be useful to know the distance that could be traveled during an hour.
- Velocity Initial and End is a rate of change of distance with respect to time. Thus velocity has the units of distance per time. The values used are ft/sec. If a vehicle is traveling at constant velocity, then \( v = \frac{d}{t} \). By rearranging this equation the other two equations for constant velocity can be seen. They are \( d = vt \) and \( t = \frac{d}{v} \). These equations can only be used if the velocity is constant. If the velocity is not constant the answer will be wrong. When velocity changes from one to another, the first velocity is initial velocity (Vi) and the second velocity is called end velocity (Ve). The change in velocity takes place over a time period, t. By definition, velocity is a vector quantity. In most time-distance analysis only the magnitude of the velocity is of interest.
Acceleration is the rate change of velocity with respect to time. Because velocity is a vector quantity, this rate change of velocity can be either in magnitude or direction. Think of an object speeding up or slowing down as an example of a change of magnitude. An object being deflected from its path by some side force and undergoing a lateral acceleration is an example of a change in direction. The units of acceleration are in feet per second per second (ft/sec²).

Solving Time/Distance Problems

You have been given an equation sheet to use to solve time, distance, acceleration and velocity problems. There are more equations that can be used than those listed. It is recommended that you limit your confusion by using only those equations on the equation sheet.

Drag factor, f, will be given to you in many problems. The first thing you should do is change drag factor to acceleration. From On-Scene Level II we know drag factor is related to acceleration by the following equations:

- \( a = fg \)
- \( f = a/g \)

The quantity, \( g \), is the acceleration of gravity. The acceleration rate of gravity is 32.2 ft/sec/sec. Drag factor does not have a positive or negative sign associated with it. You must remember to add the negative sign in cases where you have deceleration (decreasing velocity). Velocity units will be in ft/sec. In some cases you will have to convert from/to mi/hr to/from ft/sec.

- \( S = 15/22 \times \text{Velocity} \)
- \( V = 22/15 \times \text{Speed} \)

Analysis procedure is made easier if you use the equations provided in the student manual. The most common question asked by investigators is: Which equation do I use? This is not a difficult question to answer. First of all, ask yourself “What do I want to know?” This may be velocity, time, acceleration factor, distance, or whatever else is important.
Look for an equation that isolates this desired quantity on the left side of the equals sign. Next, ask yourself what information is already known. You must be thorough here, especially if there is a change in velocity. You must find an equation that has those things that you know or can get on the right side of the equals sign. You can then solve the equation.

It is always a good idea to write the equation you are going to use on your worksheet. Once you have done the arithmetic to get the answer check to see if your answer makes sense. For example, if you just calculated the time it takes to accelerate from a stop across two lanes of traffic and your answer is 44 seconds, you clearly have made an error. Check your work.
Acceleration

1. \( a = \frac{v_e - v_i}{t} \)
2. \( a = \frac{2d - 2v_i t}{t^2} \)
3. \( a = \frac{v_e^2 - v_i^2}{2d} \)

Initial Velocity

4. \( v_i = v_e - at \)
5. \( v_i = \frac{d}{t} - \frac{at}{2} \)
6. \( v_i = \sqrt{v_e^2 - 2ad} \)

End Velocity

7. \( v_e = v_i + at \)
8. \( v_e = \sqrt{v_i^2 + 2ad} \)

Distance

9. \( d = v_i t + \frac{1}{2} at^2 \)
10. \( d = \frac{v_e^2 - v_i^2}{2a} \)
11. \( d = \frac{t (v_i + v_e)}{2} \)

Time

12. \( t = \frac{v_e - v_i}{a} \)
Traffic Accident Reconstruction Level V

Traffic Accident Reconstruction Level V, introduces students to the proper techniques for making accurate speed estimates from crush damage. Critical issues such as work, energy, the conservation of energy, equation application and the energy equation derivation are addressed. Students who successfully complete this course should have the knowledge and confidence to accurately estimate speed from vehicle crush in some instances.

Learning Objectives

- Demonstrate the ability to use work, force and distance to calculate energy.
- Demonstrate one method for calculating elastic potential energy when force constant and distance are known.
- Demonstrate one method for calculating energy of a vehicle that skids over numerous surfaces, when weight, drag factor and distance are known.
- Demonstrate two methods for calculating vehicle speed when crush and after impact vehicle dynamics are known.
- Define the terms dynamic collapse and restitution as they apply to vehicle damage.
Speed Estimates from Crush Damage

Terminal Performance Objective

Given the need, students will evaluate vehicle crush data to accurately calculate vehicle impact speeds in accordance with the information presented in class.

Enabling Objectives

1. Demonstrate the ability to use work, force and distance to calculate energy.
2. Demonstrate one method for calculating elastic potential energy when force constant and distance are known.
4. Demonstrate one method for calculating energy of a vehicle that skids over numerous surfaces, when weight, drag factor and distance are known.
5. Demonstrate two methods for calculating vehicle speed when the amounts of crush and after impact vehicle dynamics are known.
6. Define the terms dynamic collapse and restitution as they apply to vehicle damage.

Work

Work can be described mathematically as the product of the force (F) and the distance (d) through which it acts, provided the force and distance covered are in the same direction. This may be written in equation form:

- $W = Fd$
- $F = \text{force (lb)}$
- $d = \text{distance (ft)}$
- $W = \text{work (ft-lb)}$
If the force and distance are not in the same direction, then the equation is rewritten:

- \( W = Fd \cos \theta \)

Work is the product of the magnitudes of force and distance. Therefore, work is a scalar quantity. Being a scalar quantity, work has only magnitude and sign (positive or negative), but has no direction associated with it. Work is positive if \( \theta \) is greater than or equal to 0 degrees but less than 90 degrees. Work is negative if \( \theta \) is greater than 90 degrees but less than or equal to 180 degrees.

Energy

When work is done, energy is transferred between different objects. The amount of energy possessed by an object is an indication of its ability to do work. Work and energy are proportional. The more energy an object has the more work it can perform.

Equations can be derived to calculate the amount of energy transferred between objects when work is done under a variety of conditions. These derivations use the positive sign if the object's energy is increased and negative if it is decreased when work is done. This agrees with the previous examples given of the accelerating and decelerating vehicles. The accelerating vehicle is gaining energy and increasing its ability to do work. The decelerating vehicle is losing energy and decreasing its ability to do work.

Energy can be grouped into three general categories:

- Rest energy
- Kinetic energy
- Potential energy

Rest energy is energy an object possesses due to its mass. This form of energy is related to Einstein's theory of relativity.
Kinetic energy (KE) is energy an object possesses due to its motion. A vehicle in motion has more energy than a vehicle at rest. When a vehicle accelerates, it gains velocity and also increases its energy. The energy of a moving body is equal to:

- \( KE = \frac{1}{2}mv^2 \)
- \( KE = \) Kinetic energy (ft-lbs)
- \( M = \) Mass (weight divided by the acceleration of gravity)
- \( v = \) Velocity (fps)

The amount of work required to accelerate the vehicle from rest to a given velocity can be calculated using this equation. The energy calculated is the energy gained by the vehicle and it is kinetic energy. In the equation mass and weight are related by the acceleration due to gravity. Considering this relationship the equation could also be written as follows:

- \( KE = \frac{w v^2}{2g} \)
- \( KE = \) Kinetic energy (ft-lbs)
- \( w = \) Weight (lbs)
- \( v = \) Velocity (fps)
- \( g = \) Gravity (32.2 ft/sec²)

The same amount of work would be required to bring the vehicle to rest. The same amount of energy that was originally gained would have to be dissipated. Energy is dissipated when it is converted into thermal (heat) energy. This happens as a result of the friction between the tires and road surface, tires and brake system, or through rolling resistance in coasting to a stop.

In the derivation of the speed from skid mark equation we learned that the following equation can be used to calculate the amount of kinetic energy that must be converted to another form in order to bring a vehicle to a stop:

- \( W = wfd \)
- \( W = \) Work (ft-lbs)
- \( w = \) weight (lbs)

- \( f \) = drag factor
- \( d \) = distance (ft)

The following two equations can be used to calculate speed estimates from skid marks.

- \( W = wfd \)
- \( KE = wv^2/2g \)

The first equation is used to find the amount of kinetic energy the vehicle dissipated while skidding. The second equation, rearranged to solve for velocity, uses the energy calculated to find the velocity of the vehicle.

Since the quantity of one form of energy is the same as the quantity of any other form of energy, the equation can be written:

- \( v = \sqrt{\frac{(2g)(E)}{w}} \)

The kinetic energy of a given mass depends on the magnitude of its velocity and not on the direction of travel. Any change in kinetic energy depends on the sign of the work done. A positive sign indicates the kinetic energy is increased. A negative sign indicates the kinetic energy is decreased.

Acceleration rate is not a factor in kinetic energy. The mass and velocity of a body govern the amount of kinetic energy the body has. The rate at which the body reaches that velocity does not affect the kinetic energy.

Potential energy (PE) is energy an object possesses due to its position. Two forms of potential energy will be covered in this course.

- Gravitational potential energy (PEh)
- Elastic potential energy (PEk)
Gravitational potential energy is the energy an object possesses due to its position above some reference plane, usually the surface of the earth.

- PE\(_h\) = gravitational potential energy (ft-lbs)
- w = weight (lbs)
- h = vertical distance (ft)

Elastic potential energy is energy an object possesses because of its shape. A spring which is compressed or stretched has elastic potential energy because of its shape. The amount of elastic potential energy it has depends upon the amount of work done to compress or stretch the spring.

A force is required to compress a spring from it free length. The force required is not constant. It must increase as the spring is compressed more and more. The amount of force is directly proportional to the amount the spring is compressed. This relationship is known as Hooke's Law. The proportionality constant for Hooke's Law depends on the stiffness of the spring and is called the force constant (k). A stiff spring will have a larger force constant than a spring that can be easily stretched.

- F = kx
- F = force (lbs)
- k = force constant (lbs/ft)
- x = distance compressed (ft)

The average force would be calculated by using the following equation.

- W = \(\frac{1}{2}kx^2\)
- W = work (ft-lbs)
- k = force constant (lb/ft)
- x = distance compressed/stretched

This is also equal to the amount of elastic potential energy possessed by the spring.

- PE\(_k\) = \(\frac{1}{2}kx^2\)
Remember this equation applies equally for a spring that is stretched and to one that is compressed. The elastic potential energy is equal to one-half the product of the force constant times the square of the distance compressed/stretched from its free length. If the distance is doubled, the energy increases by a factor of 4.

Conservation of Energy

When work is done energy is converted from one form into another. We are able to apply energy to accident situations because it is a conserved quantity. The total energy remains the same, but it may change forms.

The law of conservation of energy states that when work is done and energy is converted from one form into another, no energy is created and no energy is destroyed. As accident investigators, this concept can be another tool which may enable us to determine accident vehicle speed.

The first step is to account for the work done during the accident sequence. Once the total energy dissipated has been calculated it can then be converted into velocity. The total energy must be calculated by using an energy balance equation.

- \( ET = E_1 + E_2 + E_3 \ldots E_i \)
- \( ET = \) initial energy possessed by the vehicle
- \( E_1 = \) first amount of energy dissipated
- \( E_2 = \) second amount of energy dissipated
- \( E_3 = \) third amount of energy dissipated
- \( E_i = \) last amount of energy dissipated or left unchanged

In some instances energy might be dissipated in subtle ways that we are unable to measure. In these cases it may be difficult to account for all the energy in an energy balance equation.

Speed Estimates From Vehicle Damage

After a collision the vehicle is permanently damaged. The crush measurements taken on a damaged vehicle show the resulting damage to the vehicle. In the
collision itself the vehicle will have slightly more damage than is revealed after the collision. This damage is referred to as dynamic collapse.

A vehicle has some restitution in a collision. Restitution means the vehicle tries to return to its original shape. The effect of restitution is insignificant in nearly all severe collisions. In low-velocity impacts, restitution may have to be considered.

The definition of work indicates that the amount of energy dissipated is related to the force and the distance through which it acts. The distance is determined by making crush measurements. Once this has been done, the speed can be calculated provided information exists regarding the amount of force required to produce the damage. This quantity of force is related to the dynamic force deflection characteristics, or crush resistance (stiffness), of the vehicle structure being crushed.

The dynamic force-deflection characteristics of a vehicle are determined by tests, primarily barrier impact tests. In barrier impact tests it is assumed that all the energy is used in doing damage to the vehicle. The velocity at which the vehicle strikes the barrier is an indication of the amount of kinetic energy (crush energy) available to damage the vehicle.

The assumed relationship between a barrier test vehicle and a similar accident vehicle is that an equal quantity of energy must be used to damage the latter the same as the barrier test vehicle. For example, suppose a 1990 Chevrolet Impala is used in a barrier impact test and reveals an average of 15 inches of crush from a 30 mph impact. You investigate an accident in which a 1990 Chevrolet Impala strikes a bridge abutment. You measure the crush damage and find that it averages 15 inches.

In this scenario it would be safe to assume the accident vehicle struck the bridge at a speed of 30 mph. The barrier impact speeds are known as the equivalent barrier speed (EBS) or barrier equivalent velocity (BEV).

Once the EBS for the accident vehicle has been determined, the energy dissipated in doing damage can be calculated. This energy along with energy dissipated in other ways, can be used in an energy balance equation to calculate the vehicle's initial velocity.
Earlier in the course you were given the equation for the elastic potential energy of a compressed spring.

\[ PEK = \frac{1}{2}kx^2 \]

This equation also applies to a linear plastic spring, i.e., one that does not return to its original shape.

\[ \frac{1}{2}mv^2 = \frac{1}{2}kx^2 \]

The actual amount of deformation would depend on the spring's stiffness. This concept can be used to estimate the amount of energy dissipated by damage if the permanent deformation and the vehicle's stiffness are known.

The process of calculating an accident vehicle's speed from damage begins by understanding the variables involved. First of all you have to determine the A and B stiffness coefficients for the accident vehicle. These stiffness coefficients reflect the force deflection characteristics (stiffness) of the vehicle structure.

The A and B coefficients can be estimated from fixed barrier impact tests by plotting permanent crush against the impact speed. Remember, the deceleration force of an automobile is directly proportional to the crush. One of the variables for the A and B coefficients, b1, must be solved first. The following equation will solve for b1.

\[ b1 = \frac{(v - bo)}{C} \]

- \( b1 \) = change in impact speed to change in crush (unknown)
- \( v \) = impact velocity (mph not fps) (crash test vehicle)
- \( bo \) = 3 mph to 8 mph (the maximum barrier impact velocity resulting in no permanent damage. The lower number will give the lower speed in the energy dissipated doing damage equation)
- \( C \) = average crush (inches) (crash test vehicle)

Both \( v \) and \( bo \) must be multiplied by 17.6 (12) (22)/15 = 17.6 (converts to in/sec.).
The A coefficient can be determined from the following equation.

- \( A = \frac{(w)(b_0)(b_1)}{(g)(W)} \)
- \( A \) = A coefficient (unknown)
- \( w \) = vehicle weight (lbs) (crash test vehicle)
- \( b_0 \) = substituted from the b1 equation
- \( b_1 \) = solved from the b1 equation
- \( g \) = 386.4 in/sec²
- \( W \) = crush width (inches) (crash test vehicle)

The B coefficient can be determined from the following equation.

- \( B = \frac{(w)(b_1^2)}{(g)(W)} \)
- \( B \) = B coefficient (unknown)
- \( w \) = vehicle weight (lbs) (crash test vehicle)
- \( b_1 \) = change in impact speed to change in crush, substituted from b1 equation
- \( g \) = 386.4 (in/sec²)
- \( W \) = crush width (inches) (crash test vehicle)

Once the A and B coefficients have been determined, they can be substituted into the equation for energy dissipated in doing damage. There are three equations. They are based on the number of crush measurements ("C" measurements) taken from the accident vehicle.

If only two crush measurements are taken the following equation is used.

- \( E = W\left[\frac{G+A}{2(C1+C2)}+B\right]/6(C1^2+C2^2+C1C2)\{(1+\tan^2\theta)\]}

If four crush measurements are taken the following equation is used.

- \( E = W/6\left[6G+A(C1+2C2+2C3+C4)+B/3(C1^2+2C2^2+2C3^2+C4^2+C1C2+C2C3+C3C4)\right](1+\tan^2\theta)\]
If six crush measurements are taken the following equation is used.

- \( E = \frac{W}{5} [5G + A/2(C_1 + 2C_2 + 2C_3 + 2C_4 + 2C_5 + C_6) + B/6(C_1^2 + 2C_2^2 + 2C_3^2 + 2C_4^2 + 2C_5^2 + C_6^2 + C_1C_2 + C_2C_3 + C_3C_4 + C_4C_5 + C_5C_6)] (1 + \tan^2 \theta) \)

- \( E = \) energy dissipated (in-lbs) (divide end result by 12 to convert to ft-lbs)
- \( G = \frac{A^2}{2B} \) (lbs)
- \( W = \) width of crush region (inches) (accident vehicle)
- \( A = \) the maximum force per inch of damage width which will not cause permanent damage (lb/inch)
- \( B = \) the spring stiffness per inch of damage width (lb/inch²)
- \( \theta = \) the angle between a line perpendicular to the vehicle's damaged surface and the force direction It is recommended that this factor not exceed 2. The tangent squared of any angle above a 45 degree angle added to 1 will exceed 2.
- \( C = \) crush measurements of damage
Equation limitations

Remember that A and B coefficients are established from barrier tests of many vehicles. If the accident vehicle is not listed in the barrier test information, then it must be assigned to a group. That group's A and B coefficients must then be used. This may or may not be accurate.

A different uniform stiffness is assumed for the entire front and rear widths and the entire lengths of the sides. Frame or uni-body structure involvement is an important factor in front and rear collisions. If the uni-body structure is involved, the stiffness

Example:
Assume PDOF angle is 50 degrees

\[ \tan 50 = 1.19 \]

\[ 1 + \tan^2 50 = 1 + 1.19^2 \]

\[ 1 + 1.42 = 2.42 \]

Since the result exceeds 2 it would be an indication that the force came from the side rather than the front.
coefficients may be accurate. If only sheet metal is involved the stiffness coefficient may be too high. This applies to the sides of the vehicle also. There are "hard" spots at the wheel areas and "soft" spots between the wheels in the passenger area.

The frame and frame-type structure provide the major stiffness to the vehicle. It is likely to be more accurate to weigh the crush measurement in favor of the deformation in the frame or uni-body structure area. Be careful when there is under-ride or over-ride.

Damage can be difficult to measure. Refer to the S.A.E. handout on measuring protocol. Do not use the equation for rollover or sideswipe accidents. There is no stiffness data for large trucks.

**Alternative Method For Calculating Speed From Barrier Test Data.**

Some of the NHTSA crush data you have been provided with does not list the width of the damage sustained by the test vehicle. Without knowing the width you would have to refer to the Crash III chart for the A & B coefficients. There is an equation which will give the desired results and is much simpler to use.

EBS = [(test speed-bo)/Cavg](Cavg)+bo
- **EBS = Equivalent Barrier Speed (mph)**
- **Test Speed = Crash Test Vehicle Speed (mph)**
- **bo = Speed without doing permanent damage (mph)**
- **Cavg = Average Crush of the Accident Vehicle**

**Equation limitations**

This equation assumes a straight linear relationship between the amount of vehicle crush and the velocity of that vehicle required to create the designated amount of crush. A limitation of the linear function is its lack of input for exact crush width. This hampers ability to compensate for collisions involving deformation to only a portion of the vehicle end. The linear function assumes damage across the entire end width of the vehicle. This can be compensated for by assigning some of your C1 through C6 measurements the value of zero. These zeros would be used for calculation of Cavg.
Note: In this Example C1, C2, C5, and C6 equal zero inches

Vehicle Crush Profile

\[
C_{avg} = \frac{0+0+C3+C4+0+0}{6}
\]