Chapter 4

Reverse Logistics Network Design

Moritz Fleischmann, Erasmus University Rotterdam
Jacqueline M. Bloemhof-Ruwaard, Erasmus University Rotterdam
Patrick Beullens, University of Leuven
Rommert Dekker, Erasmus University Rotterdam

4.1 Introduction

Throughout this book manifold examples of reverse logistics initiatives are presented, encompassing a variety of products, actors, drivers, and business processes. One of the common elements across all of these cases concerns the need for an appropriate logistics infrastructure. Just as in traditional supply chains, the various business processes need to be embedded in a corresponding logistics network. In conventional supply chains, logistics network design is commonly recognized as a strategic issue of prime importance (see, e.g. Chopra and Meindl, 2001). The location of production facilities, storage concepts, and transportation strategies are major determinants of supply chain performance. Analogously, setting up an appropriate logistics network has a fundamental impact on the economic viability of any reverse logistics program. In order to successfully exploit the opportunities of recovering value from used products, companies need to design a logistics structure that facilitates the arising goods flows in an optimal way. To this end, decisions need to be taken on where to locate the various processes of the reverse supply chain introduced in Chapter 2 and how to design the corresponding transportation links. Specifically, companies need to define how to collect recoverable products from the former user; where to inspect collected products in order to separate recoverable resources from scrap; where to re-process collected products to make them fit for reuse; and how to distribute recovered products to future customers.

In this chapter, we take a detailed look at logistics network design in a reverse logistics context. We start by highlighting key business issues and contrasting them with logistics network design for traditional ‘forward’ supply chains in Section 4.2. The core part of the chapter then discusses a number of alternative modelling approaches that support the design of reverse logistics networks and allow for a quantitative analysis of the underlying tradeoffs. Section 4.3 addresses integer programming based approaches that build upon traditional facility location models. Section 4.4 considers stochastic programming approaches that focus on incorporating the aspect of uncertainty into the network design decisions. Finally, Section 4.5 presents a stream of research based on continuous approximation techniques. Section 4.6 synthesizes the different modelling approaches by exploring them in a common numerical example that highlights the
Chapter 4. Reverse Logistics Network Design

economics of reverse logistics networks. To conclude, Section 4.7 summarizes the key points of this chapter. Before embarking into a systematic analysis of reverse logistics network design we illustrate some of the main issues in a real-life business example.

4.1.1 Illustrative Case: Reverse Logistics Flows at IBM

The electronics industry has been a key sector in the growth of product recovery management. Ever expanding market volumes on the one hand and shorter product lifecycles on the other hand result in huge amounts of used products being disposed of. In this light, it comes as no surprise that electronic waste has been a prime target of environmental regulation, including enacted or pending take-back obligations in several countries (see Chapter 15). At the same time, modular product design and a relatively little extent of mechanical ‘wear and tear’ sustain the reusability of electronic products and components. Together, both developments result in a significant value recovery potential.

Business activities of IBM, as one of the major players in this sector, involve several types of ‘reverse’ product flows, which together cover most of the categories outlined in Chapter 2. From a business perspective, the most important class concerns product returns from expiring lease contracts. To date, leases account for some 35% of IBM’s total hardware sales. Furthermore, IBM has implemented take-back programs in several countries in North America, Europe, and Asia, which allow business customers to return used products for free or for a small fee. For remarketable products customers may even receive a positive contribution. In the consumer market IBM is required to take back end-of-life products in several countries in Europe and Eastern Asia, due to environmental regulation. Besides used products IBM, as most companies, faces a ‘reverse’ stream of new products, which includes e.g., retailer overstocks and cancelled orders. This flow very much depends on contractual agreements along the supply chain (see also Chapter 12). Finally, it is worth mentioning returns of rotable spare parts, as a fairly traditional type of closed-loop flows related to the service business: Defective parts replaced in a customer’s machine are sent back for repair and may then be stocked as spare parts again. (IBM, 2001; Fleischmann, 2001a)

Recognizing the growing importance of reverse logistics flows, IBM set up a dedicated business unit in 1998, which is responsible for managing all product returns worldwide. The main goal of this organization named Global Asset Recovery Services (GARS) is to manage the dispositioning of returned items such as to maximize the total value recovered. To this end, GARS operates some 25 facilities all over the globe where returns are collected, inspected, and assigned to an appropriate recovery option (see IBM, 2001). Equipment that is deemed remarketable may be refurbished and then put into the market again. For this purpose, IBM operates nine refurbishment centers worldwide, each dedicated to a specific product range. Internet auctions, both on IBM’s own Web site and on public sites have become an important sales channel for remanufactured equipment. Equipment that does not yield a sufficient value as a whole is sent to a dismantling center in order to recover valuable components, such as hard-disc assemblies, cards, and boards, which can be fed into IBM’s spare parts network or sold on the open market (for a detailed description of this channel see Fleischmann, 2001a). The remaining returned equipment is broken down into recyclable material fractions, which are sold to external recyclers. In 2000 IBM reports to have processed a total of 51,000 t of used equipment, of which only a residual of 3.2% was landfilled (IBM, 2001).

The above processes concern equipment from the business market. Given the much lower market value, consumer returns follow a different road. To work around inefficiencies of individual collection IBM participates in co-operative industry-wide solutions for this market sector in several countries. In the Netherlands, for example, IBM supports a system organized by the Dutch association of information and communication technology producers, in order to comply
with product take-back legislation. In this case, used machines from different manufacturers are collected by the municipalities and then shipped to recycling subcontractors. Transportation and recycling costs are shared by the member organizations, proportional to their products’ volume contribution (see Nederland ICT, 2002). Yet another system has recently been implemented in the USA. Since November 2000 IBM customers have the option to purchase a recycling service together with any new PC. Once the equipment is no longer needed the customer sends it by UPS to a dedicated recycling center where it is either prepared for donation to charities or broken down into recyclable materials (IBM, 2000).

4.2 Network Design Issues in Reverse Logistics

4.2.1 Delineation of Reverse Logistics Networks

The above example underscores the need for logistics infrastructure that accumulates used products and conveys them to recovery facilities and eventually to another user. In general terms such a structure can be viewed as the logistics link between two market interfaces providing supply of used products and demand for reusable products, respectively. Moreover, this link encompasses the reverse channel functions identified in Chapter 2, namely collection, testing and sorting, and re-processing. Figure 4.1 depicts a general scheme of this setting. It is worth pointing out that the two markets involved may actually coincide, thereby implying a closed-loop network.

From a logistics perspective, one may characterize the general structure illustrated in Figure 4.1 as a many-to-many network. Within this layout one may distinguish a convergent inbound part corresponding to the collection and acquisition function and a divergent outbound part serving a distribution function. The intermediate part of the network hosts the actual transformation processes. Therefore, its structure very much depends on the type of re-processing involved.

One may argue that it is only the inbound part of the network that actually concerns ‘reverse’ logistics processes whereas the remainder very much corresponds with a traditional production-distribution network. However, as discussed in Chapter 2 this segregation may hamper a comprehensive analysis since the different product flows are closely interrelated. In fact, in this light one may wish to extend the scope even further and also include the distribution
of the original new products (see Figure 4.1). It should be clear that this broad scope does not mean that the entire network is managed by a single company. As in a traditional supply chain context, responsibilities may be distributed among several players. However, in line with the supply chain management imperative one should consider the complete picture in order to understand the economics of reverse logistics networks.

Within the above setup examples of reverse logistics networks are far from identical. Significant differences concern e.g. the players involved, their responsibilities, but also the network structure in terms of centralization and the number of layers. In literature several classifications have been proposed for structuring this field.


In a different perspective, Bloemhof–Ruwaard and Salomon (1997) and Fleischmann et al. (2000) attribute differences between reverse logistics networks to the form of the recovery process primarily. In this light the authors distinguish three network types, namely remanufacturing, recycling, and direct reuse networks. Fleischmann (2001b) refines this model by including ownership of the recovery process (original equipment manufacturer (OEM) versus third party) and recovery drivers (economic versus legislation) as additional explanatory variables. Based on this analysis the paper suggests distinguishing the following five network classes: (i) networks for mandated product take–back, (ii) OEM networks for value added recovery, (iii) dedicated remanufacturing networks, (iv) recycling network for material recovery, and (v) networks for refillable containers.

### 4.2.2 Characteristics of Reverse Logistics Networks

Strategic design decisions concerning the logistics networks delineated above include the choice of a collection/acquisition method, the location and capacity of the sorting and re-processing operations and corresponding inventory buffers, and the definition of various transportation links in terms of sourcing, modes, and capacities. When comparing these tasks with the design of a conventional production–distribution network, the network structure may seem the most apparent discriminating factor. As pointed out above reverse logistics implies a many–to–many structure composed of a convergent and a divergent part (see Figure 4.1) whereas production–distribution networks are typically perceived as few–to–many, divergent structures. However, this difference may be a matter of scope rather than an intrinsic element of reverse logistics. By taking a supply chain perspective and hence taking into account the supplier level in the design of a ‘forward’ distribution network one obtains a picture that very much resembles Figure 4.1.

When looking for more fundamental characteristics that are specific of reverse logistics networks three factors are worth observing, namely

- supply uncertainty
- degree of centralization of testing and sorting
- interrelation between forward and reverse flows.

In traditional supply chains demand typically is the main unknown factor. In a reverse logistics setting however, it is the supply side that introduces significant additional uncertainty. Quantity, quality and timing of the used products becoming available are in general not known with certainty and, in addition, may be difficult to influence (see also Chapter 2). Consequently, a logistics network design that is robust with respect to variations in flow volumes and composition
4.3 Mixed Integer Location Models for Reverse Logistics Network Design

4.3.1 Literature Review of Reverse Logistics Location Models

For logistics network design in a more traditional context, facility location models based on mixed integer linear programming (MILP) have become a standard approach. A rich body of literature ranges from simple uncapacitated plant location models to complex capacitated multi-level multi-commodity models. At the same time, various solution algorithms have been proposed relying on combinatorial optimization theory. For a detailed overview of models and solution techniques see e.g. Mirchandani and Francis (1989) and Daskin (1995).

Given this extensive body of research, MILP location models appear to be a natural starting point for quantitative approaches to reverse logistics network design. Several authors have followed this route and have presented MILP location models adapted to a reverse logistics context. Table 4.1 provides an overview of the corresponding literature. We distinguish models that encompass the entire network between the two market interfaces sketched in Figure 4.1 and models with a scope restricted to the 'reverse' network part in a strict sense. Moreover, we indicate whether supply of used products is modelled as a push or a pull process, i.e. whether...
Table 4.1: Reverse Logistics Facility Location Models

<table>
<thead>
<tr>
<th></th>
<th>supply push</th>
<th>supply pull</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spengler et al. (1997)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Barros et al. (1998)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Marín and Pelegrín (1998)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fleischmann et al. (2001)</td>
<td></td>
</tr>
</tbody>
</table>

There is a given collection volume that needs to be processed or whether collection responds to demand primarily.

The summary in Table 4.1 indicates that most of the models published to date address the entire network scope and treat supply as a push process. The model of Kroon and Vrijens (1995), which is applied in the context of reusable packaging, essentially is a conventional uncapacitated warehouse location model with lateral transshipments. Similarly, Marín and Pelegrín (1998) consider a special case of a warehouse location model where each customer’s supply equals a fixed fraction of his demand. Jayaraman et al. (1999) analyze a multi-product variant of this model with general supply and demand volumes. Moreover, the supply process is governed by limited availability rather than by collection obligations.

Thierry (1997) considers a linear programming model that corresponds with the structure outlined in Figure 4.1 with facility locations being fixed. The disposal volume arising at the testing stage is modelled as a fixed fraction of the volume processed. Berger and Debaillie (1998) include location decisions in this model while at the same time limiting its scope to the ‘reverse’ network part. Moreover, they model the disposal volume as a lower bound rather than a fixed fraction. Krikke et al. (1999) apply a similar model in a case study on copier remanufacturing. Fleischmann et al. (2001) analyze a generalization of Thierry’s model including location decisions. This model is discussed in detail in section 4.3.2 below.

The model presented by Barros et al. (1998) captures a more detailed picture in that it explicitly includes an alternative recovery path rather than an external scrap process for material rejected at the testing stage. Spengler et al. (1997) and Realff et al. (1999) take an even broader perspective by modelling multi-commodity flows in general processing networks. While both cases are motivated by applications in the process industry they differ in their view of the supply process. The former considers a supply push in a waste recycling context whereas the latter focuses on the recoverable value of the supply potentially available.

Considering the above contributions one observes much similarity with traditional multi-level location models. From a mathematical perspective, the modifications triggered by the particular characteristics of reverse logistics identified in the previous section appear to be fairly limited. Specific features include additional flow constraints reflecting supply restrictions. Other variations are due to multiple return flow dispositions and to a possible interaction between forward and reverse channel. As a consequence, most of the models have a multi-commodity flow character. In the next section we discuss these aspects in more depth on the basis of a specific model.

4.3.2 A Basic Facility Location Model

To make things specific let us take a look at a concrete MILP formulation of a reverse logistics network design problem. To this end, we discuss a variant of the model we introduced in Fleischmann et al. (2001). The model picks up the general scheme sketched in Figure 4.1. Specifically,
it encompasses the processes between the two market interfaces discussed in Section 4.2. In this setting the model considers three levels of facilities for a single type of product, namely test centers, factories, and distribution warehouses. Moreover, it includes two generic dispositions for the flow of used products, namely recovery and disposal, where recovery is restricted to a certain maximum yield. Figure 4.3.2 displays the general structure of this model. The MILP formulation below uses the following notation.

Index sets

\[ \mathcal{I} = \text{set of potential plant locations}; \]
\[ \mathcal{J} = \text{set of potential warehouse locations}; \]
\[ \mathcal{K} = \text{set of fixed customer locations}; \]
\[ \mathcal{L} = \text{set of potential test center locations}. \]

Variables

\[ Y^p_i = \text{indicator opening plant } i \in \mathcal{I}; \]
\[ Y^w_j = \text{indicator opening warehouse } j \in \mathcal{J}; \]
\[ Y^r_l = \text{indicator opening test center } l \in \mathcal{L}; \]
\[ X^{p}_{ij} = \text{product flow from plant } i \text{ to warehouse } j \text{ (in product units)}; \]
\[ X^{k}_{jk} = \text{product flow from warehouse } j \text{ to customer } k \text{ (in product units)}; \]
\[ X^{c}_{kl} = \text{product flow from customer } k \text{ to test center } l \text{ (in product units)}; \]
\[ X^{r}_{li} = \text{product flow from test center } l \text{ to plant } i \text{ (in product units)}; \]
\[ V_k = \text{unsatisfied demand of customer } k \text{ (in product units)}; \]
\[ U_k = \text{excess supply of used products available at customer } k \text{ (in product units)}. \]

Costs

\[ f^p_i = \text{annualized fixed costs for opening plant } i \in \mathcal{I}; \]
\[ f^w_j = \text{annualized fixed costs for opening warehouse } j \in \mathcal{J}; \]
\[ f^r_l = \text{annualized fixed costs for opening test center } l \in \mathcal{L}; \]
\[ c^p_{ij} = \text{sum of unit production cost at plant } i \text{ and transportation cost from plant } i \text{ to customer } j; \]
\[ c^w_{ij} = \text{sum of unit handling and storage cost at warehouse } j \text{ and } \]
Chapter 4. Reverse Logistics Network Design

Transportation cost from warehouse \( j \) to customer \( k \)

\[ c_{kl}^c = \text{sum of unit transportation cost from customer } k \text{ to test center } l \text{ and test, inspection, and disposal cost; } \]

\[ c_{li}^c = \text{sum of unit transportation cost from test center } l \text{ to plant } i \text{ and reprocessing cost minus production cost; } \]

\[ c_k^b = \text{unit penalty cost for not serving demand of customer } k \]

\[ c_k^o = \text{unit penalty cost for not collecting returns of customer } k. \]

Parameters

\[ d_k = \text{annual demand of customer } k \in K \text{ (in product units); } \]

\[ u_k = \text{annual returns of used products from customer } k \in K \text{ (in product units); } \]

\[ \gamma = \text{average recovery yield; } \]

\[ p_i = \text{annual capacity of plant } i \in I \]

\[ h_j = \text{annual capacity of warehouse } j \in J \]

\[ r_l = \text{annual capacity of test center } l \in L. \]

We then formulate the general reverse logistics network design (RLND) model as

\[
\begin{align*}
\text{min} \quad & \sum_{i \in I} f_p^p Y_p^i + \sum_{j \in J} f_j^h Y_j^h + \sum_{l \in L} f_l^r Y_l^r + \\
& \quad \sum_{i \in I} \sum_{j \in J} c_{ij}^p X_{ij}^p + \sum_{k \in K} \left( c_k^b V_k + \sum_{j \in J} c_{jk}^h X_{jk}^h \right) + \\
& \quad \sum_{k \in K} \left( c_k^o U_k + \sum_{l \in L} c_{lk}^c X_{lk}^c \right) + \sum_{l \in L} \sum_{i \in I} c_{li}^r X_{li}^r \\
\text{subject to} \quad & \sum_{j \in J} X_{jk}^h + V_k = d_k \quad \forall k \in K \qquad (4.2) \\
& \sum_{l \in L} X_{lk}^c + U_k = u_k \quad \forall k \in K \qquad (4.3) \\
& \sum_{i \in I} X_{ij}^p = \sum_{k \in K} X_{jk}^h \quad \forall j \in J \qquad (4.4) \\
& \sum_{i \in I} X_{li}^p \leq \sum_{j \in J} X_{ij}^p \quad \forall i \in I \qquad (4.5) \\
& X_{li}^h \leq \gamma \sum_{k \in K} X_{lk}^c \quad \forall l \in L \qquad (4.6) \\
& \sum_{j \in J} X_{ij}^p \leq \pi_i Y_i^p \quad \forall i \in I \qquad (4.7) \\
& \sum_{i \in I} X_{ij}^p \leq \pi_j Y_j^h \quad \forall j \in J \qquad (4.8) \\
& \sum_{k \in K} X_{lk}^c \leq \pi_l Y_l^r \quad \forall l \in L \qquad (4.9) \\
Y_{ip}^p, Y_{jh}^h, Y_{lr}^r \in \{0, 1\} \quad \forall i \in I, j \in J, l \in L \qquad (4.10) \\
X_{ij}^p, X_{jk}^h, X_{li}^c, X_{lk}^c, U_k, V_k \geq 0 \quad \forall i \in I, j \in J, k \in K, l \in L \qquad (4.11)
\end{align*}
\]

In this formulation Equations 4.2 and 4.3 ensure that all customer demand and returns are taken into account. Equations 4.4 through 4.6 represent balance constraints at the warehouse,
4.3. Mixed Integer Location Models for Reverse Logistics Network Design

plant, and test center level respectively. At the warehouse level inbound and outbound flows need to be equal. At the plant level a potential excess outbound volume corresponds with new production. Similarly, the excess inbound volume at the test center level, which is constrained by the recovery yield corresponds with the disposal volume. Finally, Equations 4.7 through 4.9 are the usual facility opening conditions coupled with capacity constraints. In addition, the above formulation can be strengthened by means of the following valid inequalities in order to speed up the solution process (compare Bloemhof et al., 1996).

\[ X_{hjk} \leq \min(d_k, h_j) \quad Y_{hj}^h \quad \forall j \in J, k \in K \]  
\[ X_{ckl} \leq \min(u_k, r_l) \quad Y_{rl}^r \quad \forall k \in K, l \in L \]

It should be noted that this model is rather general and can capture a large variety of reverse logistics situations. For example, closed–loop and open loop structures both can be represented and are reflected in different settings of the parameters \( d_k \) and \( u_k \). Specifically, a closed–loop situation is characterized by \( d_k \cdot u_k > 0 \) for at least some customer \( k \). Similarly, push and pull drivers for used product collection are reflected in different penalty costs \( c_{ok} \). Furthermore, it is worth emphasizing that the ‘disposal’ route may include any form of recovery that is outsourced to a third party, e.g. material recycling.

Mathematically the above formulation does not differ much from multi–level facility location models in a more traditional production–distribution context. A particular aspect concerns the two sets of exogenous parameters \( d_k \) and \( u_k \), which are linked by the different balance conditions. This reflects the need in reverse logistics for striking a balance between market conditions on the supply and the demand side. Another element which is worth pointing out concerns the additional degree of freedom introduced by the yield condition 4.6. By constraining the disposal volume by a lower bound rather than by a fixed fraction, the recovery strategy and the network design are optimized simultaneously. A relevant question concerns the impact of these features on the performance of specific solution methods. To our knowledge, results on this issue are few to date.

4.3.3 Extensions

The (RLND) model introduced above can be extended in manifold ways. Analogous with traditional facility location models the formulation can be generalized to a dynamic, capacity selection, multi–product setting. We do not elaborate on these features here since they are well known from other contexts. Instead, we indicate a number of additional elements that appear to be specific of a reverse logistics context. For mathematical details we refer to Fleischmann et al. (2001).

- **Integrating forward and reverse channel facilities**
  As discussed in Section 4.2 integration versus separation of different processes is an important issue in reverse logistics. For example, co–locating a warehouse and a test center may allow for sharing fixed assets and therefore result in economies of scale. This effect can be captured by introducing additional indicator variables for combined facilities.

- **Integrating forward and reverse transportation flows**
  Similar synergies may arise from combining transportation routes for forward and reverse goods movements (see also Chapter 5). In the above setup this can be modelled by means of additional flow variables representing simultaneous flows in both directions between two locations.

- **Distinguishing demand for new and recovered products**
  The above formulation includes only one class of demand, which may be fulfilled through
either new production or recovery. Alternatively one may wish to distinguish between markets for new and recovered products. In essence this comes down to explicitly including the leftmost part of the scheme in Figure 4.1. Mathematically this approach results in a multi-commodity network flow formulation.

- **Multiple recovery options** The above formulation uses the most basic representation of a recovery strategy in that it distinguishes two return dispositions, namely internal ‘recover’ versus external ‘disposal’. In order to capture a more refined picture one may wish to distinguish more recovery options. Mathematically this extension again results in a multi-commodity formulation.

### 4.4 Stochastic Location Models for Reverse Logistics Network Design

#### 4.4.1 Stochastic Mixed Integer Modelling Approaches

As discussed in Section 4.2, growing uncertainty in particular on the supply side is frequently named as a major characteristic of reverse logistics networks. In the mixed integer network design approaches presented in the previous section uncertainty is, in general, addressed by means of scenario analyses. Thus a model is solved repeatedly for a set of scenarios and the solution with the best ‘overall performance’, according to some multi-criteria measure is retained. In this section we review modelling approaches that incorporate the aspect of uncertainty more explicitly.

For a general introduction to stochastic programming we refer to Birge and Louveaux (1997). A stochastic mixed–integer linear program seeks to minimize the expected costs over a given set of scenarios with associated probabilities, subject to linear and integrality constraints. In the model definition one needs to specify which decision variables need to be fixed before the realization of a scenario is known and which ones can be adjusted afterwards. Let us denote the vectors of both types of decision variables by $Y$ and $X$, respectively. Moreover, let $\omega \in \Omega$ denote the set of scenarios. Then a stochastic mixed–integer linear program can be written as

$$
\min \ c^T Y + \mathbb{E}_{\omega}[c^*(\omega, Y)] \quad \text{s.t.} \quad Y \geq 0, \ Y_T \in \{0; 1\},
$$

(4.14)

where $c^*$ is the optimal value of a MILP in decision variables $X$, which depends on $\omega$ and $Y$, $c$ is a vector of objective coefficients, and $Y_T$ is some sub–vector of $Y$. If $\Omega$ is finite then (4.14) can be rewritten as an ordinary MILP, though at the expense of an increasing problem size, by introducing scenario dependent decision variables $X_\omega$.

It is important to note that the optimal solution of (4.14) need not be optimal for any single scenario. In this sense stochastic programming is more powerful than a simple scenario analysis. This expansion comes at a cost however, since the problem size of the corresponding MILP formulation increases significantly.

In the context of logistics network design stochastic programming models have been presented to capture the impact of demand uncertainty and price variations (see, e.g. Louveaux, 1986). Typically, these models assume that location decisions are fixed for a longer planning horizon (corresponding to variables $Y$ in our formulation) whereas transportation flows can be adjusted in the short term, according to demand realizations (corresponding to variables $X$).

Stochastic programming models require a probability to be specified for each scenario. Since in practical applications these probabilities often are hard to define some authors have argued that other optimality criteria may be more relevant. Instead of expected costs they suggest to consider some min–max criterion, such as minimizing the maximum cost across all scenarios.
4.4 Stochastic Location Models for Reverse Logistics Network Design

or minimizing the maximum ‘regret’, i.e. the cost deviation from the corresponding scenario-optimal solution. These approaches do not require any probabilities to be given but seek solutions that provide a good performance guarantee in all cases. For a general introduction to these so-called ‘robust’ optimization models we refer to Kouvelis and Yu (1997). It should be noted, however, that despite their name these approaches may be highly sensitive to the set of scenarios considered since extreme scenarios may strongly dominate the solution.

To our knowledge, two groups of authors have presented robust and/or stochastic extensions to network design models in a reverse logistics context. Realff et al. (2002) report on a case study on the design of a carpet recycling network in the USA. The authors extend a corresponding MILP facility location model to a multi-scenario setting, involving different levels of supply volumes and material prices, and seek to minimize the maximum regret across all scenarios. All binary variables, which represent location choices and capacity levels, are fixed at the beginning of the planning horizon whereas the values of all continuous variables are scenario dependent. In a numerical example the authors illustrate that the optimal robust solution is not optimal, in general, for any of the individual scenarios considered. Information on the cost deviation between both approaches is not available, though.

Listes and Dekker (2001) build upon the work of Barros et al. (1998) concerning a case study on the design of a sand recycling network in the Netherlands (see also Section 4.2). The authors extend the original MILP model to a stochastic model that maximizes expected profit under demand and supply uncertainty. In a first approach they consider uncertain demand locations and volume. Location decisions for cleaning and storage facilities are assumed to be fixed at the beginning of the planning period, whereas all transportation, processing, and storage decisions may be adjusted to the demand realization. In a second approach supply volumes are also uncertain. Decisions are now taken in three stages as the scenario realization is revealed successively. In a numerical study the authors document that the optimal stochastic solution need not coincide with the solution for any individual scenario. However, the cost deviation between the stochastic solution and the best solution obtained from a scenario analysis is within a few percentages in each of the cases presented.

4.4.2 A Stochastic Location Model for Reverse Logistics

Let us now apply the above stochastic modeling approaches to the reverse logistics network design model introduced in Section 4.3. To this end, let $\Omega$ denote a finite set of scenarios, and for each scenario $\omega \in \Omega$ let $\pi_\omega$ denote its probability. We assume that scenarios differ in terms of demand and return volumes and recovery yields, which we denote by $d_{k\omega}, u_{k\omega}$ and $\gamma_\omega$, in analogy with Section 4.3. Then a stochastic version of the model in (4.1)-(4.11) can be formulated by adding a scenario index to the continuous variables $X^P, X^h, X^c, X^r, V,$ and $U$, taking the expected value of the objective function across all scenarios, and imposing restrictions (4.2)-(4.9) per scenario.

It is worth noting that the uncertain volume parameters concern the righthand side of the MILP formulation whereas the uncertain recovery yield affects the coefficient matrix. In contrast, all cost parameters are assumed to be fixed. Since all continuous variables depend on $\omega$ whereas the binary variables do not, location decisions are taken under uncertainty whereas transportation and processing flows can be adjusted to individual scenario realizations, in line with the above motivation. Furthermore, note that setting $V_{k\omega} = d_{k\omega}$ and $U_{k\omega} = u_{k\omega}$ for all $k$ and $\omega$ always provides a feasible solution. Hence, each location decision is feasible for all scenarios.

Comparing this formulation with the original deterministic model in Section 4.3 we observe that the number of continuous variables and the number of constraints has increased with a factor of $|\Omega|$. To improve numerical solution procedures the MILP formulation can be strengthened
by means of valid inequalities analogous with Equations (4.12)-(4.13). We illustrate the relation between solutions of the deterministic and the stochastic model in Section 4.6.

4.4.3 Extensions

The above model can be modified in manifold ways, e.g. to allow for different scenario definitions or information evolution. In the remainder of this section let us take a brief look at alternative optimality criteria and multi-stage decision approaches.

If the minimal costs vary largely across scenarios then the expected cost criterion used in Section (4.4.2) may result in a biased solution in the sense that it is dominated by a few high cost scenarios. In this case minimizing the expected ‘regret’ may be a relevant alternative. To this end, the term $-\sum_{\omega \in \Omega} \pi_\omega c^*_\omega$ should be added to the expected cost function, where $c^*_\omega$ denotes the minimum costs for scenario $\omega$ in the original deterministic model.

The expected cost criterion may be difficult to apply since estimating the probabilities $\pi_\omega$ may not be straightforward in practical situations. As discussed above, optimizing the worst-case behavior may therefore be a useful alternative. For our model this so-called ‘robust’ optimization approach comes down to introducing an additional decision variable $Z$, which is to be minimized under the additional constraint

$$
\sum_{i \in I} f^h_i Y^p_i + \sum_{j \in J} f^h_j Y^h_j + \sum_{i \in I} f^r_i Y^r_i + \sum_{k \in K} (c^*_k V^{}_{k\omega} + \sum_{j \in J} c^h_{jk} X^h_{jk\omega})
$$

$$
+ \sum_{i \in I} \sum_{j \in J} c^p_{ij} X^p_{ij\omega} + \sum_{k \in K} (c^c_k X^c_{k\omega}) + \sum_{l \in L} \sum_{i \in I} c^r_{li} X^r_{li\omega} \leq Z \quad \forall \omega \in \Omega.
$$

Analogously one may choose to minimize the maximum regret by combining both of the above approaches. We illustrate the effect of the different cost criteria in Section 4.6.

Finally, it appears useful to take another look at how the scenario is revealed and hence at which information is available for which decision. As explained before, the above formulation implicitly assumes that all location decisions are taken before the actual scenario is known whereas all other decisions are based on its realization. In this sense, the model captures a two-stage decision process. However, as discussed in Section 4.2 the design of a reverse logistics network may involve more stages, in particular if recovery activities are integrated into an existing ‘forward’ distribution network. One way to capture such a sequential decision process is to separate the scenario space into two independent sets $\Omega = \Xi \times \Psi$ concerning demand-related information (captured by parameters $d_{k\xi}$) and return-related information (captured by $u_{k\psi}$ and $\gamma_\psi$), respectively. The degree of information that is available for the different decisions can then be modelled by indexing the decision variables as $Y^p_i, Y^h_j, X^p_{ij\xi}, X^h_{jk\xi}, V^{}_{k\xi}, Y^r_i, X^c_{k\xi\psi}, X^r_{li\xi\psi}, U^{}_{k\xi\psi}$ and modifying (4.1) - (4.11) accordingly.

4.5 Continuous Approximation Models for Reverse Logistics Network Design

4.5.1 Approximating Reverse Logistics Costs and Revenues

MILP-based location models as discussed in the preceding sections provide a powerful tool which can be tailored to a variety of different settings. Yet these approaches have some drawbacks when it comes to establishing general insights into the economics of logistics systems. Capabilities for sensitivity analyses in MILP models are limited and, even more important, the interrelation between various parameters is not made explicit. Therefore, conclusions on the behavior of a given real-life system often rely on extensive numerical experiments rather than on analytic arguments.
In view of this shortcoming Daganzo proposed an alternative approach to investigating logistics costs and optimizing the design of logistics systems, which has become known as the ‘continuous approximation methodology’ (Daganzo, 1999). A key element of this approach is the representation of demand by a continuous density function, as opposed to the discrete demand representations exploited in traditional MILP approaches. Assuming the demand density and other system parameters to be slowly varying across a given service region (which may have spatial and temporal dimensions), logistics costs can be reasonably approximated by appropriate averages, which results in fairly simple expressions in a limited number of parameters. In this way, the cost impact of critical system parameters can be revealed and guidelines for the design of logistics structures can be derived.

In this section we follow the reasoning of Fleischmann (2001b) in applying the ‘continuous approximation’ approach to the analysis of reverse logistics networks. We consider a setting analogous with the one in Section 4.3. However, for the time being we restrict the modeling scope to the ‘reverse’ network part in a strict sense, i.e. the logistics structure conveying used products from collection points via inspection and sorting centers to some given recovery facilities (compare Figure 4.1). An extension of the model to the entire network including the redistribution stage is discussed at the end of this section. Our goal is to approximate the total reverse logistics costs for serving a given area, and eventually to minimize these costs by choosing an appropriate reverse logistics network design. To this end, assume that the return rate of used products per time per unit surface is given by a location–dependent continuous density function, which is slowly varying within the service area. It turns out to be useful to consider the costs per unit returned. The idea of the ‘continuous approximation’ approach then is to express these costs in ‘local’ problem parameters only and to approximate the overall costs by integrating over the service area.

To assess the unit reverse logistics costs it is useful to distinguish two cases, depending on whether the testing and sorting is carried out at the recovery facility or at some distinct location. In what follows we refer to these cases as 'central' and 'local' testing, respectively. For both cases one may split the total reverse logistics costs into a number of cost components, namely inbound transportation costs to the test and sort process, outbound transportation costs after sorting, variable sorting and handling costs, and fixed installation costs for the test facility. In what follows we go through all of these components and discuss which parameters they depend on. In addition to the symbols introduced earlier we use the following notation.

\[
A \quad = \quad \text{overall service area}
\]

\[
\rho(x) \quad = \quad \text{return rate of used products per time per unit surface at location } x \in A
\]

\[
C_R(\ell, \rho) \quad = \quad \text{reverse logistics costs per unit returned for a service area with constant return rate } \rho \text{ at a distance } \ell \text{ from the corresponding recovery facility}
\]

\[
C_{RL}(\ell, \rho) \quad = \quad \text{— in the case of local testing}
\]

\[
C_{RC}(\ell, \rho) \quad = \quad \text{— in the case of central testing}
\]

\[
c_t \quad = \quad \text{low volume vehicle transportation cost per distance}
\]

\[
\tilde{c}_t \quad = \quad \text{high volume vehicle transportation cost per distance}
\]

\[
v \quad = \quad \text{low volume vehicle capacity}
\]

\[
\tilde{v} \quad = \quad \text{high volume vehicle capacity}
\]

\[
A_R \quad = \quad \text{size of a test facility’s service area}
\]

\[
A^*_R \quad = \quad \text{optimal size of a test facility’s service area}
\]

\[
\ell^* \quad = \quad \text{optimal distance for switching from central to local testing}
\]

The inbound transportation costs to the test and sort process concern the collection tours within the corresponding service area. The length of a tour can be approximated based on a probabilistic analysis of the standard vehicle routing problem by the sum of a line-haul distance from and to the test and sort installation and the sum of the expected distances between two


consecutive collection stops (see e.g. Daganzo 1999). Assuming full vehicle loads we get in the case of central testing

\[
\text{unit inbound transportation cost (central)} \approx 2 \frac{c_t}{v} \ell + 0.57 c_t \rho^{-1/2}. \quad (4.16)
\]

In the case of local testing and sorting the line–haul distance depends on the size \(A_R\) of the area covered by the test facility. Assuming this area to have a circle–like shape with the test facility located at its center, the average line–haul distance approximately equals \(2\sqrt{A_R/3\pi}\) and one obtains

\[
\text{unit inbound transportation cost (local)} \approx \frac{4}{3\sqrt{\pi}} \frac{c_t}{v} \sqrt{A_R} + 0.57 c_t \rho^{-1/2}. \quad (4.17)
\]

In the case of central testing the only relevant outbound costs concern disposal costs for rejected products, namely \(c_w(1 - \gamma)\) per unit. For local testing one also needs to consider the flow of accepted products to the recovery facility. Assuming those shipments to be carried out as line-hauls rather than in tours and again assuming full vehicle loads the corresponding costs can be expressed as

\[
\text{unit outbound transportation and disposal cost (local)} \approx 2 \frac{\tilde{c}_t}{v} \ell \gamma + c_w (1 - \gamma). \quad (4.18)
\]

The annualized fixed costs for a local test and sort installation can be approximated on a per product basis by

\[
\text{unit fixed installation cost (local)} \approx \frac{f_r}{\rho A_R}. \quad (4.19)
\]

Finally, any variable handling and processing costs may be aggregated into a term \(c_h\). Summing up, one obtains the following expression for the unit reverse logistics costs in the case of central testing and sorting

\[
C_{RC}(\ell, \rho) = 2 \frac{c_t}{v} \ell + 0.57 c_t \rho^{-1/2} + c_w (1 - \gamma) + c_h. \quad (4.20)
\]

For the local testing case Equations (4.17) and (4.19) characterize the optimal size \(A_R^*\) of the testing service area. First order conditions yield

\[
A_R^* = \left(\frac{3\sqrt{\pi} f_r v}{2 c_t \rho}\right)^{2/3} \approx 1.92 \left(\frac{f_r v}{c_t \rho}\right)^{2/3}. \quad (4.21)
\]

Inserting this expression for \(A_R\) and summing up the different cost components then leads to the following cost function

\[
C_{RL}(\ell, \rho) = 2 \frac{\tilde{c}_t}{v} \ell \gamma + 0.57 c_t \rho^{-1/2} + c_w (1 - \gamma) + c_h + 1.56 \left(\frac{c_t}{v^2 \rho}\right)^{1/3}. \quad (4.22)
\]

Comparing \(C_{RC}(\cdot)\) and \(C_{RL}(\cdot)\) yields an appropriate service area for the central test and sort operation. Specifically, (4.20) and (4.22) define a critical distance \(\ell^*\) from the recovery facility up to which central testing is preferable over local testing. Equating the cost functions yields

\[
\ell^* = 0.78 \left(\frac{f_r v}{c_t \rho}\right)^{1/3} \left(1 - \frac{\tilde{c}_t}{v} \gamma\right)^{-1}. \quad (4.23)
\]

Putting the above results together one finally obtains the overall reverse logistics unit cost function \(C_R(\cdot)\) which can be written as \(C_R(\ell, \rho) = \min\{C_{RC}(\ell, \rho), C_{RL}(\ell, \rho)\}\) and the total reverse logistics costs which, as discussed above, is approximated by integrating over the service area as \(\int_A \rho(x) C_R(\ell(x), \rho(x))dx\).

In Section 4.6 we compare the above cost expressions with the results of the previously discussed discrete models and interpret them in the light of the reverse logistics issues identified in Section 4.2. Before doing so we discuss a number of extensions and refinements to the above approach.
4.5.2 Extensions

It should be clear that the above formulas provide a very basic cost model which can be extended in manifold ways. In particular, we have not included any inventory considerations and we have assumed all vehicles to operate at full capacity. These assumptions can be relaxed by including decisions on lotsizes and dispatching frequencies. Furthermore, the formulas can be extended to a multi-product setting. However, since these refinements do not appear to exhibit any particular reverse logistics elements and since they do not change the core of our argumentation we content ourselves by referring to Daganzo (1999) for a more in-depth discussion of the ‘continuous approximation’ technique.

Analogous with the above analysis one may also derive cost expressions for the ‘forward’ parts of the overall logistics network (see Figure 4.1). Assuming that products are shipped from the factory via distribution centers to the customers one obtains the same formulas as above, where \( \rho \) is replaced by an appropriate demand density \( \delta \) and \( \gamma \) equals one. In fact, in this way one arrives at the original model discussed by Daganzo (1999). Let us denote these ‘forward’ logistics costs by \( C_F(.) \) in what follows.

By putting together \( C_F(.) \) and \( C_R(.) \) one may address the overall network structure. In particular, by considering \( C_R(.) \) as inbound and \( C_F(.) \) as outbound costs and including investments one may assess the size of a factory’s service area. If \( \delta(x) \) and \( \rho(x) \) are roughly proportional one can derive expressions similar to (4.21) with \( \rho \) replaced by \( \delta + \rho \gamma \). However, a critical look seems advisable. On the one hand, the distance approximations may be less accurate since the number of distribution centers and test centers is much smaller than the number of customer locations in the original model. On the other hand, Equation (4.21) assumes the facility to be located close to the center of its service area. While this seems reasonable for a distribution center it may not be evident for the location of a factory, which depends on additional factors such as tax rates and labor costs.

Finally, note that we have assumed return and disposal rates to be given and therefore have not included any revenues in the analysis. However, the above cost expressions can also be used to assess profitability of a recovery operation. In particular, the tradeoff between reverse logistics costs and production cost savings or additional revenues can be made explicit. To this end, denote by \( C_{RN}(.) \) the unit cost for any used product that is not recovered (which may include e.g. lost revenues and/or fees for local recycling). The unit reverse logistics cost function \( C_R(.) \) is then obtained by selecting the cheapest among the three options \( C_{RC}, C_{RL}, \) and \( C_{RN} \) for each value of \( \ell \) and \( \rho \).

4.6 Quantitative Analysis of Reverse Logistics Network Design Issues

Having reviewed alternative modelling approaches for supporting reverse logistics network design decisions, let us now return to the issues highlighted in Section 4.2. In what follows we exploit the above quantitative tools to analyze these issues and highlight the impact of key parameters on the economics of reverse logistics networks.

We illustrate the analysis in a numerical example adapted from Fleischmann et al. (2001). All computational results are based on an installation of the CPLEX 7.0 standard MILP solver on a Pentium 4, 1495 MHz PC. Consider the situation of an electronic equipment manufacturer operating in the European market (recall the case of IBM from Section 4.1; see also the copier business in Chapter 11 and in Thierry et al., 1995). Assume that used equipment, stemming e.g. from expiring lease–contracts is collected from the customers, remanufactured, and resold. To allow for remanufacturing, used equipment must meet certain quality standards. To this end, all collected equipment is inspected and tested. Rejected equipment is disposed of locally, while
Table 4.2: Parameter Settings of Network Design Example

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Model Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>discrete</td>
</tr>
<tr>
<td>Fixed cost per factory</td>
<td>5,000,000</td>
<td>$f^p$</td>
</tr>
<tr>
<td>Fixed cost per warehouse</td>
<td>1,500,000</td>
<td>$f^h$</td>
</tr>
<tr>
<td>Fixed cost per test center</td>
<td>500,000</td>
<td>$f^r$</td>
</tr>
<tr>
<td>Fixed cost per test center</td>
<td>500,000</td>
<td>$f^r$</td>
</tr>
<tr>
<td>Transportation costs per km per product</td>
<td></td>
<td></td>
</tr>
<tr>
<td>factory—warehouse</td>
<td>0.0045</td>
<td>$c^p$</td>
</tr>
<tr>
<td>warehouse—customer</td>
<td>0.0100</td>
<td>$c^h$</td>
</tr>
<tr>
<td>customer—test center</td>
<td>0.0050</td>
<td>$c^c$</td>
</tr>
<tr>
<td>test center—plant</td>
<td>0.0030</td>
<td>$c^c$</td>
</tr>
<tr>
<td>Penalty cost unsatisfied demand</td>
<td>1,000</td>
<td>$c^b$</td>
</tr>
<tr>
<td>Penalty cost uncollected returns</td>
<td>1,000</td>
<td>$c^o$</td>
</tr>
<tr>
<td>Capacity factory</td>
<td>500,000</td>
<td>$\bar{p}$</td>
</tr>
<tr>
<td>Capacity warehouse</td>
<td>150,000</td>
<td>$\bar{h}$</td>
</tr>
<tr>
<td>Capacity test center</td>
<td>150,000</td>
<td>$\bar{r}$</td>
</tr>
<tr>
<td>Low volume vehicle capacity</td>
<td>20</td>
<td>$\bar{v}$</td>
</tr>
<tr>
<td>Demand per 1,000 inhabitants</td>
<td>10</td>
<td>$d_k/#inh.$</td>
</tr>
<tr>
<td>Return ratio</td>
<td>[0;0.9]</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Recovery yield</td>
<td>0.5</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Distance from factory</td>
<td>1,000</td>
<td>$\ell$</td>
</tr>
</tbody>
</table>

the remainder is shipped to the remanufacturing operation, which is co-located with an original manufacturing site.

To implement this example as a MILP model we assume that customers are located in 50 major European cities (capitals plus cities larger than 500,000 inhabitants) and that demand is proportional to the population size. Moreover, we restrict the potential (re-)manufacturing locations to 20 main metropolitan areas, whereas distribution warehouses and test operations may be located in any of the 50 cities. Table 4.2 summarizes the parameter settings for this example.

We assume that all equipment that passes the test operation has a sufficient contribution margin to be remanufactured rather than disposed. However, to avoid the cost figures to be distorted by large blocks of sunk costs we do not include variable (re-)manufacturing, handling, and disposal costs. To assess the overall profitability of the remanufacturing operation these costs as well as sales revenues should be added to the results below.

As a starting point we compute the optimal ‘forward’ distribution network for the above example, ignoring any reverse logistics activities. To this end, we solve the conventional two-level facility location model obtained by setting $u_k = 0$ for all $k$ in the MILP model in Section 4.3. The solid lines in Figure 4.3(a) illustrate the resulting network structure, which includes one central manufacturing site in Frankfurt and seven regional warehouses. For the sake of clarity flows from warehouses to customers are omitted. The corresponding annual costs equal M€ 44.8.

### 4.6.1 Impact of Supply Uncertainty

As discussed in Section 4.2 reverse logistics network design typically faces significant uncertainty concerning the supply of recoverable resources. In the above MILP model the supply side is characterized by the parameters $u_k$ and $\gamma$. In what follows we analyze their impact on the optimal solution.
Table 4.3: Results of Network Design Example

<table>
<thead>
<tr>
<th>Scenario</th>
<th>λ</th>
<th>Test Centers</th>
<th>Min. Cost</th>
<th>Regret in Case of Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>ε · 10^3</td>
<td>Scen. 9</td>
</tr>
<tr>
<td>0</td>
<td>0.0</td>
<td>D</td>
<td>0</td>
<td>4,000</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>GB,D,E</td>
<td>2,600</td>
<td>2,580</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>GB,D</td>
<td>4,700</td>
<td>1,660</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>GB,D,E,I,HU</td>
<td>6,610</td>
<td>933</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>GB,D,E,I,HU</td>
<td>8,140</td>
<td>592</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>GB,D,E,I,HU</td>
<td>9,550</td>
<td>365</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>GB,D,E,I,HU</td>
<td>11,000</td>
<td>139</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>GB,D,E,I,HU</td>
<td>12,200</td>
<td>54</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
<td>GB,D,E,I,HU</td>
<td>13,500</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0.9</td>
<td>GB,D,E,I,HU</td>
<td>14,600</td>
<td>0</td>
</tr>
<tr>
<td>stochastic</td>
<td></td>
<td>GB,D,E,I,HU</td>
<td>ø 8,850</td>
<td>≤ 2,000</td>
</tr>
</tbody>
</table>

In the model formulation (4.1)-(4.11) the volume parameters \( u_k \) occur only in the righthand side. Therefore, standard MILP theory implies that the cost function depends piecewise linearly on them (see e.g. Jenkins, 1982). Moreover, for fixed binary variables, i.e. fixed facility locations, the cost function is convex in \( u_k \) for each \( k \). A parametric analysis can be carried out by means of Jenkins’ heuristic (Jenkins, 1982).

Let us now assume that \( u_k = \lambda d_k \) for all \( k \), i.e. there is a uniform return ratio \( \lambda \) across all locations. Table 4.3 summarizes the results for different values of \( \lambda \). More specifically, we vary the return ratio in steps of 0.1 in the interval \([0;0.9]\) and compute for each scenario the optimal reverse logistics network while keeping the forward network fixed to the above layout. The solution time for each scenario is in the order of a few seconds. The dashed lines in Figure 4.3(a) illustrate the solution for \( \lambda = 0 \), which encompasses five regional test centers.

Not surprisingly, the optimal number of test centers and the relevant reverse logistics costs increase with the return volume. However, as discussed before the actual return volume is not known, in general, when the location decision is to be taken. In Section 4.4 we have discussed modelling approaches that explicitly take this uncertainty into account. The last but one row of Table 4.3 characterizes the network design which minimizes the expected costs for the case of a uniform probability distribution across the above scenarios. The solution turns out to be identical with the optimal design for \( \lambda \in [0.4; 0.6] \). As an alternative to this stochastic approach we also compute an optimal ‘robust’ solution, which minimizes the maximum cost deviation from the scenario-optimal solution across all scenarios (see Section 4.4). Note that this solution is not optimal for any single scenario.

For conventional facility location models it is well known that the cost function is fairly ‘flat’ around its minimum, in the sense that a deviation from the optimal network design entails a rather small cost penalty (see, e.g. Daganzo, 1999). An analysis of the continuous cost model developed in Section 4.5 supports a similar conclusion in a reverse logistics context.

To this end, Figure 4.4 illustrates the relation between the discrete and the continuous model for the above example by depicting the corresponding unit reverse logistics costs per product as a function of the return rate. For the discrete model this cost curve is obtained by dividing the results in Table 4.3 by the return volume. For the continuous model the curves display the functions \( C_{RC} \) and \( C_{RL} \) defined in (4.20) and (4.22). The parameter settings are listed in Table 4.2 above. Note that to make both modelling approaches compatible one needs to adjust \( c_t \) and \( \tilde{c}_t \) to account for vehicle capacities and line-haul return trips. The values of \( \ell \) and \( \delta \) approximate the overall averages. Note that the eventual unit reverse logistics cost function \( C_R \) in the continuous model is a mixture of \( C_{RC} \) and \( C_{RL} \). Since the discrete cost function also lies
Figure 4.3: (a): Optimal Forward and Reverse Network; (b): Optimal Integral Network

in this interval, Figure 4.4 suggests the results of the discrete model and the continuous model to be compatible.

To quantify the impact of supply uncertainty on the network design let $C_{RL}(A_R)$ denote the unit reverse logistics costs as a function of the test service area $A_R$ in the case of local testing. From Equations (4.19) and (4.17) one gets that $C_{RL}(A_R)$ can be written as $a + b\sqrt{A_R} + c/A_R$, with some positive constants $a$, $b$, and $c$. Similar to the well–known case of the EOQ–formula this function is very flat around its minimum. Specifically, for $\varepsilon > 0$ one gets

$$\frac{[C_{RL}((1 + \varepsilon)A_R^*) - C_{RL}(A_R^*)]}{C_{RL}(A_R^*)} \leq \frac{\varepsilon^2}{3(1 + \varepsilon)}.$$ \hspace{3cm} (4.24)

Furthermore, Equation (4.21) implies that a relative error of $\varepsilon$ in $\rho$ causes a relative error of at most $0.67 \varepsilon$ in $A_R^*$ and therefore by (4.24) a relative cost penalty of at most $0.22\varepsilon^2/(1.5 + \varepsilon)$. This implies, e.g. that a forecasting error of 50% in the return rate in the network design decision results in an eventual cost penalty of less than 3%.

For the design parameter $\ell^*$, which characterizes the domain of central testing, one observes a similar, robust behavior. Equation (4.23) shows that the impact on $\ell^*$ of an error in $\rho$ is limited. Moreover, moving to a critical distance $\ell'$ different from $\ell^*$ only affects the costs for customers located at a distance between $\ell'$ and $\ell^*$, which again has a dampening effect on the overall cost deviation.

So far, we have restricted our attention to variations in the return volume. To round off our analysis let us take a brief look at the impact of the return quality, characterized by the yield parameter $\gamma$. An exact sensitivity analysis in the MILP model is more cumbersome in this case, since $\gamma$ occurs in the coefficient matrix and the relation with the optimal cost value may therefore be non–linear. However, the continuous model suggests the network structure to be fairly robust again: the local test area $A_R^*$ turns out even to be independent of $\gamma$, whereas for the cost impact through a deviation from $\ell^*$ the same argument holds as for $\rho$ above.

The above robustness property is good news from a practical perspective in that it documents that supply uncertainty, which is characteristic of many reverse logistics environments, does not really hamper logistics network design. At the same time, one should note that variations in
supply volume and quality do affect total and unit reverse logistics costs, as illustrated in Table 4.3, Figure 4.4, and in Expressions (4.20) and (4.22). Therefore, supply uncertainty is certainly a relevant factor when it comes to estimating the profitability of a reverse logistics operation and taking an overall go/no-go decision. However, its impact is largely independent of the specific network design.

This observation also relates to the proficiency of the different modelling approaches. Specifically, it explains why in this context performance differences tend to be small between the solution of a scenario analysis and those of theoretically more powerful yet computationally more demanding methods, such as stochastic or robust models (see also Table 4.3). It is worth underlining the impact of the scenario selection, though. A scenario analysis may require a much finer gradation of scenarios than a stochastic or a robust model. For an illustration consider the last three columns of Table 4.3. For a given network structure the ‘maximum regret’ will, in general, be assumed for one of the extreme scenarios $\lambda = 0$ or $\lambda = 0.9$. For the robust model a scenario space consisting of these two cases is therefore sufficient. For a conventional scenario analysis this is not true, though. Choosing only between the optimal design for $\lambda = 0$ and $\lambda = 0.9$ respectively, yields a 100% increase in the maximum regret. However, there does exist an intermediate scenario ($\lambda = 0.33$), whose corresponding optimal solution performs equally well across all scenarios as the optimal robust solution.

### 4.6.2 Compliance with Forward Networks

The robustness property analyzed in the previous subsection also plays an important role when it comes to the compliance of reverse logistics networks with ‘forward’ logistics infrastructure already in place. As discussed in Section 4.2 this is an important issue since companies, in many cases, do not set up reverse logistics networks from scratch but on top of an existing ‘forward’ network.

In this vein, the forward and reverse network parts have been optimized sequentially in the above examples. To assess the consequences of such a two–stage approach let us compare its outcome with an integral design, which optimizes both network parts simultaneously. Figure 4.3(b) illustrates the optimal solution in this case for $\lambda = 0.4$. All parameters are kept equal to
the values in Table 4.2. Comparing Figures 4.3(a) and (b) one observes that an integral design approach indeed leads to a different network structure. However, the costs of both solutions are almost identical, namely €52.9 in the sequential approach versus €52.7 in the integral approach. This result generalizes to other values of $\lambda$. Specifically, the cost penalty for adding the reverse logistics network on top of a previously designed forward network rather than optimizing both parts together increases from 0% for $\lambda = 0$ to not more than 1.6% for $\lambda = 0.9$. We have observed similar results for many other parameter settings, including the case that demand and return volumes are not proportional (see Fleischmann et al. (2001)).

One can explain this observation as follows. First, forward flows outweigh reverse goods flows, in general, in terms of volumes and costs. Therefore, the overall optimal solution can be expected to be ‘close’ to the optimal forward network. A deviation from this structure must allow for substantial savings per unit in the reverse channel in order to set off against the resulting increase in distribution costs. Second, the flat cost structure highlighted in the previous subsection results in a very limited cost penalty for deviating from the optimal reverse network structure due to constraints imposed by existing infrastructure. This is the more true if demand and returns have a similar geographical distribution.

This observation is again good news from a business perspective since it suggests that setting up an efficient reverse logistics network in many cases does not require a fundamental redesign of a company’s existing logistics networks. In addition to limiting investment costs this conclusion simplifies the organizational implementation of reverse logistics initiatives. From a modelling perspective this observation results in a significant reduction of complexity by optimizing forward and reverse network structures separately.

Fleischmann et al. (2002) indicate the limits of the above observation by means of an example of a paper recycling network. In that case the recycling operation does have a fundamental impact on the entire logistics network by reducing the impact of virgin raw material sources. While the structure of the original forward network is strongly dominated by pulp wood production close to the Scandinavian forests, recycling ‘pulls’ the business activities closer to the main markets in Western Europe. We illustrate the underlying economics in the context of our
4.7 Conclusions and Outlook

In this chapter we have considered the setting up of appropriate infrastructure by companies engaged in reverse logistics programs. In Sections 4.1 and 4.2 we have motivated that logistics network design is a key determinant of the overall profitability of closed-loop supply chains. By comparing this setting with more traditional production–distribution networks we have distilled three important issues that appear to be characteristic of reverse logistics networks. First, there is significant uncertainty on the supply side. Second, the need for testing and sorting used products before assigning them to an appropriate recovery option entails a particular
centralization–decentralization tradeoff. Third, coordination and integration of different inbound and outbound flows is key to reverse logistics.

The core part of this chapter, encompassing Sections 4.3 through 4.5 reviews quantitative models that have been proposed for supporting reverse logistics network design. Analogous with traditional network design problems many of the approaches rely on MILP formulations. We pointed out that these models in general very much resemble conventional multi–level facility location models. Specific features that can be attributed to reverse logistics are few. They include a set of balance equations that link exogenous conditions on the supply and the demand side and an additional degree of freedom in optimizing the logistics network structure and the recovery policy simultaneously. A few models explicitly incorporate the aspect of uncertainty by taking a stochastic programming approach. The richer solution space of these models comes at a cost in terms of significantly larger problem sizes. We have pointed out that a network design that maximizes average or worst–case performance may not be optimal for any of the underlying scenarios. However, the eventual cost benefit compared to a scenario–based approach seems small in many cases. Finally, we have discussed how approximate continuous cost models may be applied in a reverse logistics context. This approach turns out to be helpful in making explicit the impact of various context parameters. From a mathematical perspective the resulting reverse logistics model again turned out to be very similar with its corresponding ‘forward’ counterpart.

In Section 4.6 we have compared the different modelling approaches on the basis of an extended numerical example. The key observation of this analysis concerns the robustness of reverse logistics costs with respect to moderate changes in the network structure. This result, which concurs with what is known about conventional production–distribution networks has important practical implications. On the one hand, it limits the impact of the aforementioned supply uncertainty when it comes to choosing an appropriate network structure. On the other hand, it indicates that in many cases there is enough flexibility for reverse logistics networks to comply with existing network structures.

Given the short history of reverse logistics’ research, it goes without saying that many issues are yet to be explored. We conclude this chapter by listing some issues that may stimulate further research on reverse logistics network design.

From a methodological perspective, focus to date has been on model formulations and on output analysis. In contrast, little attention has been paid to algorithmic aspects yet. As the field is maturing it seems worthwhile to investigate whether solution methods from traditional location theory are still adequate in a reverse logistics context, and which features may require modified approaches. Although the results in this chapter hint at a close similarity between reverse logistics network design and traditional location models a thorough understanding of algorithmic implications would be helpful.

As to the modelling, an important aspect that appears not to have been addressed in much depth yet concerns the role of inventories. Inventory is well known to be an important parameter in the design of distribution networks. Its role in the tradeoff between centralization and decentralization has been investigated in much detail. A corresponding analysis concerning the effects of inventory considerations on reverse logistics networks has nor yet been pursued. Note, for example, that none of the models reviewed in this chapter explicitly addresses inventory aspects of location decisions. Thus, there is a clear need for additional research here.

Another aspect that certainly deserves more detailed analysis concerns the multi-agent character of reverse logistics network design. All of the network design models presented in this chapter follow the perspective of one central decision-maker. Addressing the network design issue from the perspective of different players with different goals and different market power, as emphasized in today’s supply chain management philosophy, may be worthwhile (see also Chapter 12). To this end, a thorough characterization of the incentives and the power of the different players, such as collectors, logistics service providers, processors, and legislative bod-
4.7. Conclusions and Outlook

ies, seems a first important step. Potential issues to further investigate include their effect on e.g. the reverse logistics network design and on the propagation of the aforementioned supply uncertainty through the network.

Finally, we mention globalization as another issue in reverse logistics that has not yet received the attention it seems to deserve. Global sourcing has become a strong factor in many supply chains. Intuitively, one may doubt whether a comparable globalization is beneficial for reverse chains. Some of the immediate obstacles such as tax issues and cross-border waste transportation have been briefly touched in the preceding discussion. However, there may be even more fundamental arguments concerning the role of the individual players within the chain. A thorough analysis of these issues, which seems instrumental for a good understanding of the differences between forward and reverse channels again calls for a broadening of the modelling approaches available to date.
Bibliography


