Financial crisis resolution

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Abstract

Bank deposit contracts, between a bank and its creditor, typically prescribe minimum bank equity requirements, as in Holmstrom and Tirole (1997). During a banking crisis, when bank equity is low, it is shown that these deposit contracts turn out to be inefficient. A regulator will find it optimal to write a new deposit contract with banks, on behalf of all creditors, that rations bank credit supply in the long run. By promising banks future rents the regulator can relax minimum equity requirements during a banking crisis. The optimal regulation takes a simple form, an upper bound on bank lending, since banks hedge financing risk sufficiently when minimum equity requirements are met.

1 Introduction

When the value of intermediary assets falls suddenly, then investment of borrowers that depend on external financing tends to fall as well.¹ The recent financial crisis, 2008-2009, is a reminder that regulation cannot always prevent a large crisis, nor do we know whether prevention can be achieved at acceptable cost in terms of market distortions. Much recent research has been focussing on the causes of financial crises - less attention has been paid to optimal crisis resolution.

In this paper, I analyze optimal regulation during financial crisis. To this end, I develop a model of an infinite horizon production economy where financial intermediaries (banks) have the special

¹Campello, Graham, and Harvey (2010) analyze survey evidence on how firms’ access to bank financing has been affecting their investment decisions during the recent financial crisis, 2008-2009. They find that firms tend to report a tightening of access to bank financing (compared to pre-crisis times), and that firms that report being financially constrained as a result often report to be forced to forego profitable investment opportunities due to insufficient bank financing. Ivashina and Scharfstein (2010) document how, during the recent financial crisis, banks that experienced difficulties in access to finance themselves tended to reduce lending to firms.
ability to mitigate an agency problem between final borrowers (firms) and final lenders (consumers). I assume that banks cannot commit to use this special ability, which gives rise to endogenous bank capital requirements, very similar to the scenario described in Holmstrom and Tirole (1997). A financial crisis in my model is caused by an exogenous, unexpected decrease in bank capital which leads to a drop in credit supply and hence lower aggregate investment. The main point of this paper is to show that the optimal deposit contract specifying the terms of bank borrowing (i.e. bank capital requirements), ceases to be optimal once the economy experiences an unexpected financial crisis. Intuitively, when banks cannot insure against the event in which the crisis occurs then a new contract will be Pareto-improving. At the time of the banking crisis, however, this better contract will not be agreed upon when banks and its creditors interact competitively in a decentralized fashion. I show that, rather, a bank regulator should write this better contract with banks on behalf of all creditors. This Second-Best contract will distort the equilibrium allocation in a way that alleviates the credit crunch caused by the banking crisis.

In the model, banks and consumers can trade a complete set of contingent claims. It is assumed that when a bank extorts payments (e.g. via a buyout) from its creditors, by threatening to withhold its special intermediation ability, it can be excluded from intermediation in the future. Thus, banks’ minimum capital requirements are implicitly given by a sequence of participation constraints, as described in Kehoe and Levine (1993). These participation constraints limit the bank’s dividend policy and credit supply to ensure bank solvency in all states of the world. Equivalently, one can say banks face endogenous debt constraints, as described in Alvarez and Jermann (2000). These debt constraints limit bank short-selling of available assets to ensure bank solvency in all states of the world.

Since banks’ endogenous debt constraints determine firms’ investment, the mechanics of my model look similar to an economy where firms are debt-constrained, as described in Albuquerque and Hopenhayn (2004). What distinguishes my model from existing models of firm dynamics is the fact that individual allocations depend on aggregate allocations via a perfectly competitive market for bank loans to firms. During a banking crisis bank capital, and thus bank credit to firms, is scarce. Banks can raise funds to lend to firms by selling claims to future bank profits, subject to endogenous short-sale constraints (capital requirements). However, perfect competition on the market for loans to firms implies that future bank profits will be low, and eventually zero. In that sense, bank lending in steady state of the laissez-faire competitive equilibrium is excessive. It acts as a negative pecuniary externality leading to an inefficiently low amount of bank lending during
the banking crisis, when bank loans are more scarce compared to the steady state.

Suppose we introduce a (bank) regulator into the economy that has the ability to regulate deposit contracts. Specifically, define a constrained-efficient allocation as the consumer-welfare maximizing allocation that a regulator can achieve by writing a deposit contract with all banks on behalf of consumers. I show that the constrained efficient deposit contract differs from the competitive-equilibrium deposit contract in a simple way: it prescribes an upper bound on bank loans to firms. While this upper bound may not be binding during a severe credit crunch (when capital requirements constrain bank leverage) it will bind eventually when banks are better capitalized in the long run. On the one hand, distorting bank lending in the long run leads to an obvious efficiency loss. On the other hand, increasing the long run profitability of banks leads to higher bank shareholder value and thus increases banks’ debt capacity during the credit crunch. During a credit crunch such an improvement in banks’ access to external financing directly translates into an increase in bank lending to firms. A regulator thus trades off distorting economic activity in steady state versus alleviating the current credit crunch. The optimal upper bound on bank lending can be implemented by taxing bank lending to firms, in addition to banks and depositors contracting in a decentralized fashion. The proposed tax on bank lending is constant (a feature of the specific economic environment considered) and tax revenues are rebated to banks lump sum.

The paper is organized as follows: this section continues with a literature review, section 2 contains the model, section 3 describes the competitive equilibrium, section 4 derives the constrained-efficient allocation and discusses implementation, and section 5 discusses the need for "macro-prudential" regulation. All proofs and also a discussion of the related empirical literature are deferred to the appendix.

1.1 Related literature

Bernanke (1983) argues that financial crises (bank failures, reductions in bank equity) likely deepened the Great Depression through an increase in the cost of credit. Gorton and Winton (2003) summarize further evidence for the effect of bank balance sheets and banks’ ability to obtain financing on aggregate economic activity. Marcus (1984) presents a model where banks with low shareholder value (charter value) choose more risky lending decisions. Keeley (1990) provides empirical support, linking increased bank competition (lower charter value) after 1980 to increased incidence of bank failures. Beck, Demirguc-Kunt, and Levine (2006) provide recent evidence sup-
porting the robustness this finding. This suggests that different combinations of bank capital and charter value lead to similar resilience of banks. Then a sudden change in one of these measures of bank safety may lead to changes in bank lending. Peek and Rosengren (1995) document how sudden increases in exogenous regulatory capital requirements can reduce bank lending to firms that depend on bank loans.

This paper assumes that the size and aggregate riskiness of banks’ asset portfolio is observable by bank creditors. Flannery (1998) argues that bank creditors generally assess bank risk rather well even though regulators may be able to obtain some additional information, see also Berger, Davies, and Flannery (2000). He proposes further deregulation of banks, in particular a reduction in the degree of deposit insurance: however, as governments probably will not be willing or able to abandon the too-big-to-fail doctrine introduced in the 1980s, deregulation cannot proceed too far. In this paper, it is assumed that the time-inconsistency problem lies with bank creditors (rather than regulators): they cannot commit not to renegotiate during bank insolvency.

Being a small open economy, New Zealand as of 1996 abandoned deposit insurance and reduced government bank supervision to largely prescribing comprehensive disclosure requirements for banks, see Grimes (1996). Barth, Caprio, and Levine (2004) present cross-country evidence suggesting that the banking sector seems to functions better under private governance and market monitoring. This supports the request for an increased role for market monitoring formulated in the third pillar of the Basel II Capital Accord.

In this paper bank moral hazard is modeled in an ad-hoc fashion (“banks can steal”) and will not occur in equilibrium as bank creditors will demand appropriate minimum bank equity to keep the bank honest for given bank shareholder value. Swire (1992) provides the following assessment regarding the incidence of bank moral hazard: "Banking involves transactions that are (1) numerous, (2) in highly liquid form, (3) easily forgeable, and (4) involve large amounts of money which (5) often cross jurisdictional boundaries. Each of these factors make it easier for insiders to steal." Kashyap and Stein (2000) provide evidence for significant differences in liquidity of assets across banks. However, in this paper, it is assumed banks are identical.

This paper assumes that incentives of managers and shareholders of banks are perfectly aligned

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2See Besanko and Kanatas (1996) for an environment where an increase in bank capital may fail to reduce bank riskiness. Bhattacharya, Boot, and Thakor (1998) provide further discussion of bank charter values, in particular in the presence of deposit insurance.

3Diamond and Rajan (2000) show that introducing demandable deposits can eliminate the time-inconsistency problem: depositors can commit not to renegotiate bank debt due to a collective action problem that leads to a ‘run’ once a bank is insolvent.
requiring the availability of appropriate compensation schemes that do not constrain a bank’s asset holdings (in particular, if managers steal from creditors it is to benefit shareholders). When this assumption does not hold additional regulation may become necessary to limit inefficient actions by bank managers. Hellwig (2005) discusses the failure of "market discipline" in the presence of poor corporate governance, and Mehran, Acharya, and Thakor (2010) develop a model where leverage imposes market discipline on managers but may induce owners to engage in asset substitution.

There is a large literature studying the role of "macro-prudential" regulation under symmetric information when financial friction give rise to pecuniary externality in economies with productive assets. The following two papers assume markets are exogenously incomplete. Jeanne and Korinek (2010) study an economy where the aggregate capital stock is fixed and can be used as collateral. A low output realization creates pressure to sell capital and increase borrowing in order to smooth consumption. The fall in the price of capital makes borrowing constraints tighter and thus in turn increases the need to sell capital to smooth consumption. Jeanne and Korinek (2010) show that Pigou taxes that promote higher precautionary savings in times of high output can be used to implement the constrained efficient outcome. Brunnermeier and Sannikov (2010) study a production economy where productive capital can only be adjusted at a cost. They show that the degree of precaution exercised by holders of capital may be excessive. Lorenzoni (2008) and Gersbach and Rochet (2011) develop finite-horizon models with complete contracting in which agents’ investment decisions are inefficient in that they affect asset prices too much in a way that exacerbates financial frictions ("systemic risk"). In this paper, the asset price that is affected by a pecuniary externality is the shareholder value of the bank itself. The need for bank capital requirements that aim at reducing systemic risk (in the sense of Lorenzoni (2008)) does not arise. In particular, it is shown that during a credit crunch credit supply increases gradually in the model. However, credit supply can be very unstable over time in a finite horizon version of the model. Intuitively, with an infinite horizon and complete markets, banks’ shareholder value only depends on the expected number of periods necessary to reach their steady state value. This limits the temptation to react excessively to short term profitable opportunities.

Kehoe and Perri (2004) and Abraham and Carceles-Poveda (2006) build models of infinite-horizon production economies where agents can write complete contracts under symmetric information, subject to respective notions of limited commitment. In both papers, capital accumulation in competitive equilibrium without corrective taxation is too high and contributes to making the value of default inefficiently high. The model in this paper is close to theirs with the main difference that
the pecuniary externality of interest affects the value of staying in the contract, rather than the value of default.

2 Model

Consider an infinite-horizon production economy in discrete time with a single non-storable consumption good, productive capital, and labor.

Uncertainty

At time $t$ the aggregate state $s_t$ can take one of two values with equal probability, $s_t \in \{s_L, s_H\}$. Then $S^t = \prod_{j=0}^{t} S$ is the finite set of date-$t$ events $s^t = (s_0, s_1, s_2, \ldots, s_t)$. The initial state $s_0$ is given. Event $s^\tau$, with $\tau \geq t$, is said to follow event $s^t$ (denoted $s^\tau \succeq s^t$) if $s^\tau = (s^t, s_{t+1}, \ldots, s_\tau)$. At date 0, nature draws a sequence $(s_1, s_2, \ldots)$ and at time $t$ the event $s^t$ is revealed. The date-zero probability that $s^t$ is observed is denoted by $\pi(s^t) = 2^{-t}$.

Agents, endowments, and production

There is a unit measure of identical consumers. Each consumer is endowed with one unit of labor that she supplies inelastically in each event. Consumer preferences over consumption plans $C = \{c_t(s^t)\}_{s^t \in S^t, t=0,1,2,\ldots}$ are represented by

$$U(C) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} u(c_t(s^t)) \pi(s^t),$$

where $\beta \in (0, 1)$ is the consumer’s subjective discount factor, and $u(\cdot)$ is strictly increasing, and concave. There is also a unit measure of identical banks. Each consumer is endowed with one ‘share’ in each bank (each bank is jointly owned by all consumers at date zero).

At the end of each date $t$ a unit measure of identical firms is born. Such a firm can invest $k_{f,t+1}(s^t)$ units of productive capital in $t$ and employ $l_{t+1}(s^{t+1})$ units of labor in event $s^{t+1}$ to produce

$$F(k_{f,t+1}(s^t), l_{t+1}(s^{t+1}), s_{t+1}) = z(s_{t+1})k_{f,t+1}(s^t)^a l_{t+1}(s^{t+1})^{1-a} + (1 - \delta)k_{f,t+1}(s^t)$$

units of the consumption good in event $s^{t+1}$, where $a, \delta \in (0, 1)$ are capital share and capital depreciation rate respectively, and with aggregate total factor productivity $z(s_L) = z_L \in (0, \beta)$, $z(s_H) = z_H = 2 - z_L$. At the end of date $t + 1$ firms die and a unit measure of new firms is born.
Markets

There is a competitive labor market where consumers supply one unit of labor inelastically in exchange for wage $w_t(s^t)$, at each $s^t$ and $t$, and where firms hire labor at this wage rate. There is a competitive market for bank loans to firms with a repayment rate of $R_t(s_t,s^{t-1})$ at date $t$ in event $s^t$ per unit borrowed at date $t-1$ in event $s^{t-1}$. It is assumed that a firm can only borrow productive capital from any one bank and that this bank can monitor the firm at zero cost. When the bank does not monitor the firm it is assumed that the firm can divert fraction $\theta \in (0,\alpha]$ of production excluding undepreciated capital, i.e. $\theta z(s_{t+1})k_{f,t+1}(s^t)s_{t+1}(s^{t+1})^{1-\alpha}$ at date $t+1$ in event $s^{t+1} \succ s^t$. Each bank pays a nonnegative dividend $d_t(s^t)$ per share at date $t$ in event $s^t$, and each consumer can trade her bank shares $\xi_t(s^t)$ (with other consumers) at ex-dividend price $p_t(s^t)$. Consumers and banks have access to a market for contingent claims with given date zero prices $q_0(s^t) = \beta^t \pi(s^t)$ for all $s^t,t$. Conditionally on reaching event $s^t$, conditional future claims have prices $q_t(s^t,s^\tau) = \beta^{\tau-t} \pi(s^\tau|s^t)$ for $\tau > t, s^\tau \succ s^t$, and where $\pi(s^\tau|s^t)$ denotes conditional expectations.

Bank objective and participation

It is assumed that at bank’s preferences over dividend payment plans of the form $D = \{d_t(s^t)\}_{s^t \in S^t,t=0,1,2,...}$ are represented by

$$V(D) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} q_0(s^t) d_t(s^t) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) d_t(s^t).$$  (3)

Call $V(D)$ date zero shareholder value and let shareholder value at date $t$ in event $s^t$ be given by $V_t(s^t) = \sum_{\tau=t}^{\infty} \sum_{s^\tau \in S^\tau} \beta^{\tau-t} \pi(s^\tau|s^t) d_\tau(s^\tau)$. Let $k_{t+1}(s^t)$ be bank lending to firms and let $K_{t+1}(s^t)$ be aggregate investment at date $t$ in event $s^t$. Assumption 1 introduces a notion of outside option for the bank. It implies that the bank can extract an extraordinary dividend payment of $O_{t+1}(s^{t+1},K_{t+1}(s^t),k_{t+1}(s^t))$ to bank shareholders from creditors in the case of bank default at date $t+1$ in event $s^{t+1} \succ s^t$ since bank shareholders are protected by limited commitment.\(^5\)\(^6\)

\(^4\)That is, our economy is a small open economy.

\(^5\)Note that this differs from the delegation cost derived in Diamond (1984) (which would be zero for our fully-diversified bank). Here, the agency friction is due to a possible renegotiation in the event of bank default, rather than the bank misrepresenting the value of its assets. This is similar to the narrative developed in Diamond and Rajan (2000) except that, in my model, bank creditors do not face a collective action problem in the event of default (i.e. no sequential-service induced bank run: depositors in my model are concerned with bank solvency only and not with their place in line).

\(^6\)The payment that bank shareholders can extract from bank creditors will in practice depend on the specifics of bankruptcy regulation. For example, in the US bankruptcy law for banks differs from general corporate bankruptcy law in that shareholder-creditor interaction is limited and rather formalized. See Bliss and Kaufman (2006) for a detailed comparison. James (1991) notes that bank assets generally experience severe reductions in value while in receivership.
**Assumption 1.** (Bank insolvency) At any date $t$, event $s^t$ a bank can default on contingent $s^t$ claims it sold to its creditors. Then its creditors obtain the right to collect on bank loans $k_{t+1}(s^t)$. As only bank can monitor firms, creditors will agree to payment $O_{t+1}(s^{t+1}, K_{t+1}(s^t), k_{t+1}(s^t))$ to bank shareholders in exchange for monitoring service in $s^{t+1}$. It is assumed that this payment captures the entire benefit from monitoring (creditor bargaining power normalized to zero). The bank can be excluded from lending to firms after it defaulted.

Then an (interim) individually rational dividend payment and lending plan $\{d_t(s^t), k_{t+1}(s^t)\}_{s^t \in S^t, t=0,1,2,...}$ for banks must satisfy

\[ V_t(s^t) \geq O_t(s^t, K_t(s^{t-1}), k_t(s^{t-1})), \]  

(4)

for all $t = 1,2,\ldots$, and all $s^t \succ s^{t-1}$, and for given aggregate investment $\{K_t(s^t)\}_{s^t \in S^t, t=0,1,2,...}$. Let $b_{t+1}(s_{t+1}, s^t)$ be the amount the bank at date $t$ in event $s^t$ promises to pay to its creditors at date $t + 1$ in event $s^{t+1} \succ s^t$ and let $a_t(s^t)$ be bank net assets (equity) at date $t$ in event $s^t$. Then the bank budget constraint in $s^t$ is given by

\[ k_{t+1}(s^t) + d_t(s^t) = a_t(s^t) + \sum_{s^{t+1} \succ s^t} \beta \pi(s^{t+1}|s^t) b_{t+1}(s_{t+1}, s^t), \]  

(5)

and bank equity evolves according to

\[ a_{t+1}(s^{t+1}) = R_{t+1}(s^{t+1}) k_{t+1}(s^t) - b_{t+1}(s_{t+1}, s^t), \]  

for all $s^{t+1} \succ s^t$. (6)

$a_0 = a_0(s^0) \geq a_0$ is given initial bank equity. Note that banks do not have any loans outstanding at date zero, i.e. there is no participation constraint of type (4) for $t = 0$.

**First Best**

Finally let us note that the level of aggregate investment that obtains when consumers could directly lend to firms satisfies a simple no-arbitrage condition, see definition 1.

**Definition 1.** (First Best Investment) We say that the capital stock in the economy follows First Best dynamics at $s^t$ if

\[ \sum_{s^{t+1} \succ s^t} \beta \pi(s^{t+1}|s^t) R_{\tau+1}(s^{\tau+1}) = 1, \]  

for all $\tau > t$, $s^\tau \succ s^t$. during bank insolvency. See Swire (1992) for a discussion of how a relatively higher incidence of “insider abuse” influenced bank insolvency law in the US. In practice, regulatory toughness vis-a-vis bank shareholders during bank insolvency may bring forward “insider abuse” to a point in time preceding the establishing of insolvency by the regulator. The purpose of assumption 1 is thus to fix ideas rather than to attribute bank “inside abuse” to a particular point in the life of a bank.
3 Competitive equilibrium

Bank creditors realize that a bank may find it in the best interest of its shareholders to default deliberately in order to extract an extraordinary dividend payment (assumption 1). A bank will thus be required to maintain a certain minimum level of equity that discourages the bank from defaulting. Denote this minimum equity requirement at date \( t \) in event \( s^t \) by \( A_t(s^t) \). For example, a very strict equity requirement that prevents bank default (and in fact any bank borrowing) is given by \( A_t(s^t) = d_t(s^t) + k_{t+1}(s^t) \). While equity requirements could be taken as given by the bank, the example illustrates that banks in general internalize the effect of their choices on equity requirements.

Let \( A = \{A_t(s^t)\}_{s^t \in S^t, t=1,2,...} \) denote a collection of minimum equity requirements.

\textbf{Bank problem}

A bank takes as given initial equity \( a_0 \), returns on bank lending \( \{R_t(s^t)\}_{s^t \in S^t, t=1,2,...} \) and minimum equity requirements \( \{A_t(s^t)\}_{s^t \in S^t, t=1,2,...} \). It then chooses dividend payments, loans to firms, and contingent borrowing \( \{d_t(s^t), k_{t+1}(s^t), b_{t+1}(s_{t+1}, s^t)\}_{s^t \in S^t, t=0,1,2,...} \) to maximize shareholder value (3) subject to dividend nonnegativity, budget balance (5) and (6), and the minimum equity requirement \( a_t(s^t) \geq A_t(s^t) \) for all \( s^t \succ s_0 \). Assumption 2 assures, without loss of generality, that the bank’s shareholder value maximizing policy is unique.

\textbf{Assumption 2.} When indifferent as to the timing of dividend payments, banks distribute dividends sooner rather than later. Note that this is equivalent to an arbitrarily small cost of holding equity.

\textbf{Consumer problem}

A consumer takes initial income \( w_0 \equiv w_0(s_0) \) and initial bank share holding \( \xi_0 = 1 \), and a sequence of dividend payments \( \{d_t(s^t)\}_{s^t \in S^t, t=0,1,2,...} \), wage payments \( \{w_t(s^t)\}_{s^t \in S^t, t=1,2,...} \), and bank share prices \( \{p_t(s^t)\}_{s^t \in S^t, t=0,1,2,...} \) as given. The consumer chooses a sequence of consumption levels \( \{c_t(s^t)\}_{s^t \in S^t, t=0,1,2,...} \) and bank share holdings \( \{\xi_{t+1}(s^t)\}_{s^t \in S^t, t=0,1,2,...} \) to maximize lifetime utility (1) subject to budget balance

\[
\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) \left[ c_t(s^t) + \xi_{t+1}(s^t) p_t(s^t) - \xi_t(s^{t-1})(d_t(s^t) + p_t(s^t)) - w_t(s^t) \right] = 0. \tag{7}
\]

\textbf{Firm problem}

Date \( t \) firm owner realizes that when banks are subject to minimum equity requirements they will always monitor the firm in \( t+1 \). The firm owner borrows from the bank at date \( t \) to finance invest-
ment in physical capital \( k_{f,t+1}(s^t) \) and hires labor \( l_{t+1}(s^{t+1}) \) at date \( t + 1 \) in event \( s^{t+1} \) to maximize expected profit (firm owner consumption)

\[
\sum_{s^{t+1} > s^t} \pi(s^{t+1}|s^t) \left[ z(s_{t+1}) k_{f,t+1}(s^t)^a l_{t+1}(s^{t+1})^{1-a} + (1 - \delta) k_{f,t+1}(s^t) - w_{t+1}(s^{t+1}) l_{t+1}(s^{t+1}) - R_{t+1}(s^{t+1}) k_{f,t+1}(s^t) \right]
\]

subject to

\[
z(s_{t+1}) k_{f,t+1}(s^t)^a l_{t+1}(s^{t+1})^{1-a} + (1 - \delta) k_{f,t+1}(s^t) - w_{t+1}(s^{t+1}) l_{t+1}(s^{t+1}) - R_{t+1}(s^{t+1}) k_{f,t+1}(s^t) \geq 0
\]
in each \( s^{t+1} > s^t \).

**Definition 2.** (Competitive Equilibrium with minimum equity requirements A) An allocation for (i) banks \( \{d_t(s^t), k_{t+1}(s^t), b_{t+1}(s_{t+1}, s^{t+1})\}_{s^t \in S, t = 0, 1, 2, \ldots} \) for (ii) consumers \( \{c_t(s^t), \xi_{t+1}(s^t)\}_{s^t \in S, t = 0, 1, 2, \ldots} \), and (iii) firms \( \{k_{f,t+1}(s^t), l_{t+1}(s^{t+1})\}_{s^t \in S, t = 0, 1, 2, \ldots} \) such that given \( a_0, w_0, A, \) and \( \{w_t(s^t), R_t(s^t)\}_{s^t \in S, t = 1, 2, \ldots} \)

- (i) solves the bank problem

- (ii) solves the consumer problem

- (iii) solves the sequence of firm problems

- markets clear at each date \( t = 0, 1, 2, \ldots \) and in each even \( s^t \geq s_0 \)
  - bank lending: \( k_{f,t+1}(s^t) = k_{t+1}(s^t) \)
  - labor: \( l_{t+1}(s^{t+1}) = 1 \)
  - bank shares: \( \xi_{t+1}(s^t) = 1 \)

**Lemma 1.** In a competitive equilibrium, let \( K_{t+1}(s^t) \) be aggregate investment in the economy at date \( t \) in event \( s^t \). The wage is given by \( w_{t+1}(s^{t+1}) = (1 - \alpha) z(s_{t+1}) K_{t+1}(s^t)^a \), the return on bank lending is given by \( R_{t+1}(s^{t+1}) = \alpha z(s_{t+1}) K_{t+1}(s^t)^{a-1} + 1 - \delta \), and the bank share price is related to bank shareholder value as follows \( p_t(s^t) = V_t(s^t) - d_t(s^t) \). If aggregate investment follows first best dynamics then it is constant at \( K_{FB} = (\alpha / (1/\beta - 1 + \delta))^{1/a} \).

It makes sense to think of equity requirements \( A \) as arising naturally in competitive equilibrium. Banks should be able to borrow as long as the amount of equity they hold discourages them from defaulting. Similar to borrowing limits that are not too tight in Alvarez and Jermann (2000), definition 3 defines minimum equity requirements \( A^* \) that are not too high.
Definition 3. If in a competitive equilibrium with equity requirements $A$ we have that $a_t(s^t) = A_t(s^t)$ implies $V_t(s^t) = O_t(s^t, K_t(s^{t-1}), k_t(s^{t-1}))$ at all dates $t$ and events $s^t$, then $A = A^*$.

Let us from now on assume that equity requirements are not too high in competitive equilibrium. The following lemma 2 further characterizes $A^*$.

Lemma 2. The extraordinary dividend that a bank can extract from its creditors during a default is given by

$$O_t(s^t, K_t(s^{t-1}), k_t(s^{t-1})) = \theta z(s_t) K_t(s^{t-1})^{a-1} k_t(s^{t-1}).$$

Minimum equity requirements that are not too high are given by

$$A^*_t(s^t) = \theta z(s_t) K_t(s^{t-1})^{a-1} k_t(s^{t-1}) - \sum_{\tau=t+1}^\infty \sum_{s^{\tau} > s^t} \beta^{\tau} \pi(s^{\tau}|s^t) \left[ R_{\tau}(s^{\tau}) - \frac{1}{\beta} \right] k_{\tau}(s^{\tau-1}).$$

Note that banks internalize the direct effect of their lending activity \{k_{t+1}(s^t)\}_{s^t \in S, t=0,1,2,...} on these equity requirements.

We see that equity requirements in $s^t$ are increasing in the size of the current bank loan portfolio $k_t(s^{t-1})$. However, $A^*_t(s^t)$ also decreases in aggregate physical capital $K_t(s^{t-1})$. Intuitively, when the aggregate stock is lower then firms are more productive and the bank’s monitoring service becomes more valuable per unit of the loan. Finally, note the decreasing effect of future bank profits on equity requirements. Banks do not internalize the indirect effect of their lending activity on equity requirements via aggregate investment and future profitability.

Definition 4. Banks are profitable at date $t$ in event $s^t$ if $\sum_{s^{t+1} > s^t} \beta \pi(s^{t+1}|s^t) R_{t+1}(s^{t+1}) > 1$.

Definition 5. In competitive equilibrium, the economy experiences a credit crunch at date $t$ in event $s^t$ if $K_{t+1}(s^t) < K_{FB}$.

Lemma 3. Banks are profitable at date $t$ in event $s^t$ if and only if $K_{t+1}(s^t) < K_{FB}$.

Definition 6. In competitive equilibrium, at date $t$ in event $s^t$ the present value of future profits from bank lending activity is

$$\Pi_t(s^t) = \sum_{\tau=t+1}^\infty \sum_{s^{\tau} > s^t} \beta^{\tau-1} \pi(s^{\tau}|s^t) \left[ R_{\tau}(s^{\tau}) - \frac{1}{\beta} \right] k_{\tau}(s^{\tau-1}).$$
3.1 Dynamics of competitive equilibrium

Proposition 1 shows that aggregate investment will be constant at \( K_{FB} \) if initial bank equity is high enough, and characterizes the competitive equilibrium in that case. It also implies that the economy will reach a steady state within one period when initial bank equity is high enough.

**Proposition 1.** Suppose \( a_0 \geq \bar{a}_0 \equiv \frac{\beta}{2-\beta} \theta z_H K_{FB}^a \). Then there exists a unique competitive equilibrium (with equity requirements that are not too high) that takes the following form. At date zero, banks make a dividend payment of \( d_0(s_0) = a_0 - \bar{a}_0 \). At date \( t > 0 \), banks make dividend payments of \( d_t(s^t) = \frac{2(1-\beta)}{2-\beta} \theta z_H K_{FB}^a \) if \( s_t = s_H \) and zero else. At each date \( t = 0, 1, 2, \ldots \) banks lend \( k_t(s^t) = K_{FB} \) to firms. Also, at each date \( t = 0, 1, 2, \ldots \), banks finance lending to firms by selling contingent claims

\[
\begin{align*}
&b_{t+1}(s_L, s^t) = \left[ \frac{\beta}{2(1-\beta)} a (z_H - z_L) + \frac{1}{1-\beta} a z_L - \frac{\beta}{2-\beta} \theta z_H \right] K_{FB}^a - \frac{1}{1-\beta} \delta K_{FB}, \\
&b_{t+1}(s_H, s^t) = \left[ \frac{2 - \beta}{2(1-\beta)} a (z_H - z_L) + \frac{1}{1-\beta} a z_L - \theta z_H \right] K_{FB}^a - \frac{1}{1-\beta} \delta K_{FB}.
\end{align*}
\]

Let \( D_{FB} = \frac{1}{2} \beta (b_{t+1}(s_L, s^t) + b_{t+1}(s_H, s^t)) = K_{FB} - \frac{\beta}{2-\beta} \theta z_H K_{FB}^a \) denote bank debt, then bank equity at dates \( t = 1, 2, \ldots \) is given by

\[
\begin{align*}
a_t(s_L, s^{t-1}) &= K_{FB} - D_{FB}, \\
a_t(s_H, s^{t-1}) &= \theta z_H K_{FB}^a.
\end{align*}
\]

Since banks are not profitable when aggregate investment follows first best dynamics we have \( V_t(s^t) = a_t(s^t) \) at all dates \( t \) and all events \( s^t \).

When initial bank equity is below \( \bar{a}_0 \) then a competitive equilibrium may not exit. To see this note that profits of a hypothetical monopolistic bank would be \( \Pi_M = \frac{\beta}{1-\beta} \left[ a K_{M}^{a-1} + 1 - \delta - \frac{1}{\beta} \right] K_M = \frac{\beta}{1-\beta} a (1-a) K_{M}^{a} \), where \( K_M = \left( \frac{a^2}{\frac{1}{\beta} - 1 + \delta} \right)^{1/(1-\alpha)} \). The monopolist must have initial equity of at least \( a_M = \theta z_H K_{M}^{a} - \Pi_M \), otherwise it is tempted to default (recall that we assumed equity requirements that are not too high). For a competitive bank, future profits are strictly below \( \Pi_M \) such that any \( a_0 \leq -\Pi_M \) implies that banks’ individual rationality constraint cannot be satisfied in competitive equilibrium.\(^7\) However, proposition 2 shows that competitive equilibria in which the economy experiences a credit crunch exist.

\(^7\)Note that initial investment may be well below \( K_M \). The reason is that the present value of bank profits is maximized at initial investment below the monopolistic lending outcome due to future competition.
Proposition 2. For some $a_0$, there exists a unique competitive equilibrium for each $a_0 \in [a_0, \bar{a}_0]$. There also exists a number $T_{a_0}$ such that the economy experiences a credit crunch at date $t$ in event $s^t$ if $\eta(s^t) < T_{a_0}$, with $\eta(s^t) = \sum_{\tau=1}^{t} \mathbb{1}(s_{\tau} = s_H)$. During the credit crunch, for $t > 0$, aggregate investment grows at rate $g = (2 - \beta)/\beta^{1/\beta}$ if $s_t = s_H$ and remains constant else. When $\eta(s^t) = T_{a_0}$ and $s_t = s_H$ then aggregate investment grows at rate $\bar{g} \in (0, g]$. Aggregate investment is then equal to $K_{FB}$ and subsequently stays at that level.

Intuitively, following date zero aggregate investment only increases at times when bank equity requirements bind (i.e. when $z(s^t) = z_H$) we can characterize the competitive equilibrium by focussing on what happens when aggregate productivity is high. Figure 1 illustrates this for a particular numerical example. Note that there are competitive equilibria with negative initial bank equity. Lemma 4 shows that there are no competitive equilibria for zero initial bank equity.

Lemma 4. No competitive equilibrium exists for $a_0 = 0$.

Lemma 5 shows that the competitive equilibrium described in proposition 1 (which reaches a steady state after at most one period) will be reached eventually with probability one for all $a_0 \geq a_0$.

Lemma 5. For any given $a_0 \geq a_0$ and $\varepsilon > 0$ there exists a $t_\varepsilon \geq T_{a_0}$ such that the probability of reaching aggregate investment of $K_{FB}$ within $t_\varepsilon$ periods is greater than $1 - \varepsilon$.

In the presence of financing frictions, a bank - as well as any non-financial firm - must trade off exploiting profitable investment opportunities in the present period against conserving debt capacity for future investments.\(^8\) An interesting question thus arises: to what extent do banks hedge financing risk? Definition 7 specifies what it means to hedge financing risk in the model introduced above. This notion of risk management will be further explored in section 5. Lemma 6 is a corollary to proposition 2 and verifies that banks in the above economy hedge financing risk sufficiently.

Definition 7. A bank hedges financing risk sufficiently if bank lending is non-decreasing over time, $k_{t+1}(s^t) \geq k_{t}(s^{t-1})$ for all $s^t > s^{t-1}$ and all $t \geq 0$. Note that if all banks hedge financing risk sufficiently the return on bank lending will be non-increasing over time, $R^K_{t+1}(s^t) \leq R^K_{t}(s^{t-1})$ for all $s^t > s^{t-1}$ and all $t \geq 0$.


\(^8\)See Froot et al. (1993), Froot and Stein (1998), Lorenzoni (2008), and Rampini and Viswanathan (2010) for a detailed discussion.
3.2 Competitive equilibrium with taxation of bank lending

When bank lending to firms can be taxed at a constant rate \( \tau_0 \), a bank’s net-of-tax return from lending activity is \( (1 - \tau_0)R_t(s^t) \) at date \( t \) in event \( s^t \). Since banks lend to firms only if \( \beta R_t(s^t)(1 - \tau_0) \geq 1 \) we see - using lemma 1 - that aggregate investment cannot exceed

\[
 K_{\tau_0} = \left( \frac{\alpha}{\beta(1 - \tau_0) - 1 + \delta} \right)^{\frac{1}{1 - \delta}}.
\]  

\[\tag{9}\]
It is assumed that any tax revenue is rebated lump sum to banks. When aggregate investment is constant at $K_t$, then a bank’s future profits are given by

$$
\Pi_t = \frac{\beta}{1 - \beta} \tau_0 K_t.
$$

(10)

Proposition 3 verifies that a steady state - at bank lending $K_t$ - is reached within one period when initial bank equity is high enough. This threshold is lower than the one in proposition 1 for two reasons: First, lower bank lending implies a reduced benefit from default, and second, steady state profits increase the cost of default (a loss of future profits). Proposition 4 characterizes competitive equilibria with taxation in which the economy experiences a credit crunch.

**Proposition 3.** Suppose bank lending to firms is taxed at constant rate $\tau_0$, and that $a_0 \geq \bar{a}_0 \equiv \frac{\beta}{2 - \beta} \theta z_H K^a_{t_0} - \Pi_{t_0}$. Then there exists a unique competitive equilibrium (with equity requirements that are not too high) that takes the following form. At date zero, banks make a dividend payment of $d_0(s_0) = a_0 - \bar{a}_0$. At date $t > 0$, banks make dividend payments of $d_t(s^t) = \frac{2(1 - \beta)}{2 - \beta} \theta z_H K^a_{t_0}$ if $s_t = s_H$ and zero else. At each date $t = 0, 1, 2, \ldots$ banks lend $k_t(s^t) = K_{t_0}$ to firms. Also, at each date $t$ and each event $s^t$, $t = 0, 1, 2, \ldots$, banks sell contingent claims

$$
b_{t+1}(s_L, s^t) = \alpha z_L K^a_{t_0} + (1 - \delta) K_{t_0} + \Pi_{t_0} - \frac{\beta}{2 - \beta} \theta z_H K^a_{t_0},
$$

$$
b_{t+1}(s_H, s^t) = \alpha z_L K^a_{t_0} + (1 - \delta) K_{t_0} + \Pi_{t_0} - \theta z_H K^a_{t_0}.
$$

Let $D_{t_0} = \frac{1}{2} \beta (b_{t+1}(s_L, s^t) + b_{t+1}(s_H, s^t)) = K_{t_0} + \Pi_{t_0} - \frac{\beta}{2 - \beta} \theta z_H K^a_{t_0}$ denote bank debt, then bank equity at dates $t = 1, 2, \ldots$ is given by

$$
a_t(s_L, s^{t-1}) = K_{t_0} - D_{t_0},
$$

$$
a_t(s_H, s^{t-1}) = \theta z_H K^a_{t_0} - \Pi_{t_0}.
$$

Since banks earn profits we have $V_t(s^t) = a_t(s^t) + \Pi_t$ at all dates $t$ and in all events $s^t$.

**Proposition 4.** Suppose bank lending is taxed at a constant rate $\tau_0$. Then there exists a value $a_0$ of initial bank equity, $a_0 \in [a_M, \bar{a}_0]$, such that there exists a unique competitive equilibrium for each $a_0 \in [a_M, \bar{a}_0]$. There also exists a number $T_{a_0}$ such that the economy experiences a credit crunch at date $t$ in event $s^t$ if $\eta(s^t) < T_{a_0}$, with $\eta(s^t) = \sum_{T=1}^t 1(s_T = s_H)$. During the credit crunch, for $t > 0$, aggregate investment grows at rate $g = \left((2 - \beta) / \beta\right)^{1/\alpha}$ if $s_t = s_H$ and remains constant else. When $\eta(s^t) = T_{a_0}$ and $s_t = s_H$ then
aggregate investment grows at rate \( \tilde{g} \in (0, g] \). Aggregate investment is then equal to \( K_{0} \) and subsequently stays at that level. The steady state with aggregate investment \( K_{0} \) is reached with probability one.

4 Constrained efficient allocations

The constrained efficient allocation for this economy will in general differ from the allocation in competitive equilibrium. The reason is that competitive banks do not internalize the effect of their lending policy on equilibrium prices, in particular on the return on bank lending.\(^9\) Proposition 5 shows that the constrained efficient allocation can differ quite significantly from the competitive equilibrium allocation.

Definition 8. A constrained-efficient allocation is a policy for banks \( \{d_t(s^t), k_{t+1}(s^t)\}_{t=0,1,2,\ldots, s^t \succ s_0} \) that achieves the highest level of consumer life-time income

\[
W_0 = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) \left( d_t(s^t) + (1 - \alpha)z(s^t)k_t(s^{t-1})^\alpha \right)
\]

subject to dividend non-negativity, given initial bank equity, bank budget balance

\[
\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) \left( d_t(s^t) + k_{t+1}(s^t) - az(s^t)k_t(s^{t-1})^\alpha - (1 - \delta)k_t(s^{t-1}) \right) \leq a_0,
\]

and bank individual rationality

\[
\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^{T-t} \pi(s^T|s^t)d_T(s^T) \geq \theta z(s^t)k_t(s^{t-1})^\alpha.
\]

Proposition 5. Suppose the constrained-efficient allocation enters a steady state following some event \( s^t \), such that aggregate investment is \( k_T(s^{T-1}) \equiv K_{SB} \) for all \( s^T \succ s^t \). If \( a_0 < \bar{a}_0 \) then \( K_{SB} < K_{FB} \).

\(^9\)One could argue that wages are inefficiently high during banking crises such that inefficiently low labor supply by workers (supposing we add that margin) acts as a second pecuniary externality. Here, I make the simplifying assumption that labor is inelastically supplied and cannot be taxed at all. Note that the general argument in the proof of proposition 5 remains valid when labor supply is endogenous and labor income can be taxed. In practice we sometimes observe labor being subsidized during a banking crisis (e.g. recently in Germany).
Proof. The Euler equation to the problem given in definition 8 is

$$\sum_{s^{t+1} > s^t} \beta \pi(s^{t+1} | s^t) \left( az(s^{t+1}) l_{t+1}^{a-1} + 1 - \delta \right) = 1 + \frac{\lambda_0 - 1}{\lambda_0} \sum_{s^{t+1} > s^t} \beta \pi(s^{t+1} | s^t) q_{t+1}(s^t) k_{t+1}(s^t)^{a-1}$$

$$+ \sum_{s^{t+1} > s^t} \beta \pi(s^{t+1} | s^t) \psi_{t+1}(s^{t+1}) z(s^{t+1}) \lambda_0^{-1} k_{t+1}(s^t)^{a-1},$$

where $\lambda_0$ is the Lagrange multiplier on the bank budget constraint and $\psi_{t+1}(s^{t+1})$ are Lagrange multipliers on bank individual rationality constraints. Note that the latter must be zero in a steady state: the reason is that the timing of dividend payments does not affect consumer life-time income $W_0$ directly, such that dividend payments are optimally deferred until the bank’s individual rationality constraints stop binding. Since productivity shocks are i.i.d. $K_{SB}$ can be written as

$$K_{SB} = \left( \frac{\alpha (1 - \kappa_0)}{\frac{1}{\beta} - 1 + \delta} \right)^{1/\pi},$$

where $\kappa_0 = (1 - \alpha) \left( 1 - \frac{1}{\lambda_0} \right)$. By the envelope theorem we have $\lambda_0 = \frac{dW_0}{da_0}$. When $a_0 < a_0$ the equilibrium with first-best investment described in proposition 1 is not feasible. Then banks are constrained such that an additional unit of bank equity increases consumer life-time income more than one-for-one, $\lambda_0 > 1$, implying that $K_{SB} < K_{FB}$.

To see the intuition behind proposition 5 note that a marginal reduction in steady state investment $K_{FB}$ has no direct effect on consumer life-time income: the loss in wage income will be exactly offset by an increase in dividend payments. However, the increase in steady state dividend payments increases bank shareholder value from date zero onwards. In that sense, the constrained-efficient allocation allows for bank profitability in steady state in order to relax bank individual rationality constraints during a credit crunch. As a result, minimum equity requirements decrease and bank leverage increases at date zero, allowing for increased bank lending to firms thus alleviating the credit crunch. A sufficiently small reduction in steady state aggregate bank lending therefore strictly increases consumer life-time income, implying that $K_{SB} < K_{FB}$ in the steady state of a constrained-efficient allocation.

**Proposition 6.** Suppose $a_0 \geq a_M$, then the constrained-efficient allocation can be implemented as a competi-
tive equilibrium with a positive constant tax \( \tau_{SB} \) on bank lending. This tax is given by

\[
\tau_{SB} = \frac{1 - \beta(1 - \delta)}{\frac{1}{\kappa_0} - \beta(1 - \delta)},
\]

where \( \kappa_0 = (1 - \alpha) \left( 1 - \frac{1}{\lambda_0} \right) \) and \( \lambda_0 = \frac{dW_0}{da_0} \) is the social marginal value of bank equity.

A constrained social planner can achieve a Pareto improvement by distorting equilibrium prices. As in Lorenzoni (2008), the channel of interest works through the investing agent’s balance sheet: investment in laissez-faire competitive equilibrium is too high and reduces income of the borrowing-constrained, high return-on-income, investing agent. While in Lorenzoni’s model current investment should be reduced to alleviate a future scarcity of equity, in my model it is future investment that should be reduced to alleviate a current scarcity of equity. (Note that in Lorenzoni’s model the scarcity of equity is alleviated by increasing equity, and it is alleviated in my model by increasing leverage.) By the same logic, Lorenzoni finds that current (i.e. date zero) equity requirements are too low (see his proposition 4), while in my model current equity requirements are too high. Another difference to his results is that the pecuniary externality in my model does not require limited commitment of consumers (but it may be aggravated by it for certain sets of parameters). Assumption 1 ensures that banks care about future profits when trading off costs and benefits of a default. As proposition 4 shows, any competitive equilibrium with or without taxation of bank lending features an aggregate capital stock that is non-decreasing over time. This implies that the "fire sales" of physical capital featured in Lorenzoni (2008) would not arise in my model.

4.1 Providing banks with fresh equity

Suppose we allow the social planer to transfer resources from bank shareholders to banks directly. To make this interesting, assume that there are two types of consumers - bank shareholders and workers - and that worker labor income cannot be taxed. (Assume equal welfare weights and linear utility function \( u \) so that the welfare criterion is unchanged.) Note that we only need to assume worker labor income cannot be taxed when workers default: since they do not own any bank shares and risk sharing is of no value to them (linear utility) any attempt to tax them would make them default. As the example in figure 1 illustrates, asking bank shareholders to increase bank equity is in general not Pareto improving. The reason is that bank shareholders are entitled to future profits of \( \Pi_1 \). In the example, an increase of initial equity of \( a_0 = 0.145 \) to its steady state value \( a_{FB} = 0.645 \)
Table 1: The table shows aggregate investment in the laissez-faire competitive equilibrium (CE), the constrained Second Best (SB), and the competitive equilibrium with a constant tax on bank lending (CE\(_T\)). The constant tax of \(\tau = 0.07\) has been chosen to equate \(K_2(\%H)\) in SB and CE\(_T\). We see from the table that this tax is sufficient to implement the SB as a competitive equilibrium.

<table>
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<tr>
<th></th>
<th>(K_1)</th>
<th>(K_2(%L))</th>
<th>(K_2(%H))</th>
<th>(W_0)</th>
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<td>1.6320</td>
<td>1.1837</td>
<td>2.4081</td>
<td>1.7317</td>
</tr>
</tbody>
</table>

5 A finite horizon economy

Consider a version of the above economy with only three periods, \(t = 0, 1, 2\). At \(t = 1\) aggregate productivity is either \(z_L \in (0, \beta)\) or \(z_H = 2 - z_L\) with equal probability while in \(t = 2\) aggregate productivity is unity irrespective of history. Note that all equity will be paid to shareholders at \(t = 2\) - everything else is as in section 2. Let \(k_1\) denote bank lending in period \(t = 0\) and let \(k_2(s_j)\) denote bank lending in period \(t = 1\) when productivity was \(z_j\) in \(t = 1, j = L, H\). As before, upper case versions of these variable will denote aggregate quantities.

Table 1 shows aggregate investment over time for a particular numerical example. Parameter values used are \(\alpha = \theta = 0.4, \beta = 0.93, \delta = 0.08, z_L = 0.8,\) and initial bank equity is \(a_0 = 0.05\). Note that the first best capital stock in this economy is \(K_{FB} = 4.8412\) and that a monopolistic bank would choose lending to firms of \(K_M = 1.0512\). The example shows that banks do not hedge financing risk sufficiently, in contrast to the infinite horizon version of the economy (see lemma 6). However, we see that the second best can still be decentralized by a constant tax on bank lending, \(\tau = 0.07\). As in the infinite horizon version, the constrained social planner can increase date zero welfare \(W_0\) by smoothing out the scarcity of bank loans to firms over time.

5.1 Irreversible investment

Suppose firms cannot convert physical capital back into the consumption good in \(t = 1\). Specifically, an old firm that produces in \(t = 1\) needs to sell undepreciated capital to new firms that produce in

---

10Equivalently, one could think about issuing new bank share to be acquired by workers. As this reduces bank profitability and hence the value per share, existing shareholders would need to be compensated, for example with the right to buy new shares at a reduced price. However, when the discount for existing shareholders cannot be financed by taxing workers the same problem arises: the fact that bank shareholders are entitled to future profits \(\Pi_1\) rules out eliminating the credit crunch with fresh bank equity. On the other hand, if workers could be taxed, the social planner could achieve the first best as a Pareto improvement by issuing new shares to workers and paying \(\Pi_1\) to existing shareholders.
\( t = 2 \) before repaying its bank loan. When banks do not hedge financing risk sufficiently they are forced to reduce bank lending to firms in certain states of the world. It may then be the case that new firms do not receive enough bank loans to buy all undepreciated capital from old firms, \((1 - \delta)K_1 - K_2(s_j)\) for some \( j = L, H \). Similar to Lorenzoni (2008), assume there is some alternative sector ("scrap yard") which can absorb any left-over physical capital but which is strictly less productive than the technology \( F \). Without modeling the alternative technology explicitly suppose the price for physical capital in \( t = 1 \) is given by

\[
p_1(s_j) = \left(1 + \max(0, (1 - \delta)K_1 - K_2(s_j))\right)^{-\gamma} \quad \text{for } j = L, H \text{ and } \gamma > 0.
\]

Note that when new firms can obtain sufficient loans from banks to buy all undepreciated capital from old firms the price of physical capital is one. On the other hand, if banks do not hedge financing risk sufficiently this may result in "fire sales" of assets (physical capital) at the firm level. In that latter case, banks will receive a reduced repayment on outstanding loans to old firms and an increased return on loans to new firms (new firms can obtain capital more cheaply, a gain which is fully appropriated by banks). The net effect is a reduction in bank equity in states of the world where bank equity is already low.

Lemma 6 tells us that "fire sales" of physical capital would not occur in competitive equilibrium of the infinite-horizon economy augmented by investment irreversibility, since aggregate investment is always non-decreasing. Similar, from propositions 4 and 6 we can infer that a constant tax on bank lending is still sufficient to implement the constrained efficient allocation in the infinite-horizon economy augmented by investment irreversibility. However, as table 1 shows, "fire sales" may occur in the finite-horizon economy implying that the constrained efficient allocation may no longer be implementable by a constant tax on bank lending.

We extend the numerical example of table 1 to the case of investment irreversibility. Table 2 shows that a tax on bank lending that targets the constrained efficient level of bank lending in \((t = 2, s = s_H)\) cannot implement the constrained efficient allocation fully: banks will overinvest in \( t = 0 \) such that the price of physical capital will be too low in \( t = 2 \) when productivity was low in \( t = 1 \). The last column of table 2 shows the price of physical capital when productivity has been low, \( p_1(s_L) \). When productivity has been high then \( p_1(s_H) = 1 \) in all three economies. As in Lorenzoni (2008), a specific regulatory intervention is needed to target the pecuniary externality via \( p_1(s_j) \). For example, the constrained efficient allocation could be implemented by a tax on bank lending that exceeds \( \tau = 0.0706 \) in \( t = 0 \) and then falls to \( \tau = 0.0706 \) in \( t = 1 \).
<table>
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<th>$K_2(s_H)$</th>
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</table>

Table 2: (Irreversible investment) The table shows aggregate investment in the laissez-faire competitive equilibrium (CE), the constrained Second Best (SB), and the competitive equilibrium with a constant tax on bank lending (CE$_T$). The constant tax of $\tau = 0.0706$ has been chosen to equate $K_2(s_H)$ in SB and CE$_T$. We see from the table that this tax is not sufficient to implement the SB as a competitive equilibrium: initial bank lending in CE$_T$ is excessive.

6 Conclusion

The goal of this paper is to find a straightforward way to embed Holmstrom and Tirole’s concern about the necessity of bank capital for economic activity in a real business cycle model. In doing so, I assume that banks obtain proprietary information about its debtors. In equilibrium, they will need to hold own capital as a way to commit themselves to use this information to protect the interests of its creditors. The economy experiences a financial crisis when insufficient bank capital constrains credit supply. The central normative insight gained from the above analysis is that the social cost of financial crises is too high: intense competition on the market for loans in steady state reduces bank debt capacity during the financial crisis. A constrained regulator will find it optimal to reduce credit supply in a steady state in order to increase the bank’s shareholder value and debt capacity during the credit crunch. Optimal steady state reduction of credit to firms can be decentralized by taxing bank lending at a constant rate and rebating proceeds to banks lump sum.

References


Mehran, H., V. Acharya, and A. Thakor (2010). Caught between scylla and charybdis? regulating bank leverage when there is rent seeking and risk shifting.


7 Appendix

7.1 Discussion of related empirical literature

7.1.1 Entry of multinational banks

Entry of foreign banks can help to sustain the flow of credit to domestic firms even as domestic banks face binding capital requirements. Peek and Rosengren (2000) discuss this for Argentina, Brazil, and Mexico, and argue that foreign banks also bring expertise to the domestic banking system. The problem is that severe banking crises may lead to not just insufficient, but actually negative bank net assets. If the country’s government lets foreign banks enter then competition for loans drives down the value of struggling domestic banks to zero. Governments most likely will not allow all struggling domestic banks to file bankruptcy at the same time. In fact, during the Brazilian banking crisis, Brazil made it a condition for entering multinational banks to absorb struggling domestic banks. But then the entering bank likely requires the government to restrict further entry: there need to be sufficient rents from loans to be earned during the transition to earn back the cost of absorbing negative equity of troubled domestic banks. Further, if domestic bank shareholders have a stronger lobby than domestic workers, then a government is unlikely to allow foreign bank capital as it will dilute domestically held equity.

7.1.2 Recapitalization

In the economy studied in this paper a recapitalization of banks would require coordination of existing shareholders to provide fresh equity. If fresh equity would be offered by outsiders then it will be in the interest of existing shareholders to pay out fresh equity as a dividend and default. In addition, special regulatory circumstances may play a role in making a timely recapitalization difficult. Swire (1992) documents how, for the US, regulatory powers have been expanded significantly in the wake of banking crises. He argues that this might lead to a time inconsistency problem. ‘Superpowers’ granted to the FDIC include determining when a bank is insolvent, and subordinating claims of insiders and outsiders to the deposit insurance fund’s claims. In particular, informal agreements will not be honored by the FDIC, which acts as a receiver. Swire (1992) argues that the specialness of bank, compared to nonfinancial corporate, insolvency law leads to a different kind of bank run. Bank creditors as well as debtors will cease business relations with the bank once it has low equity, as the point of insolvency is unclear due to FDIC discretion in that matter. Hence, FDIC’s ex-post toughness on third parties may lead to excessive bank insolvencies ex ante. In particular, recapitalization of banks may become more difficult: potential investors would prefer to wait until after the bank went through an FDIC-orchestrated insolvency as this can eliminate hidden liabilities.11 Hoshi and Kashyap (2010) describe how uncertainty over regulators’ intentions slowed down recovery from the Japanese banking crisis of the 1990s.

7.1.3 Collusion or concentration

The constrained-efficient banking crisis resolution proposed above involves collusion (implemented by a tax on bank lending) rather than concentration on the market for bank loans. However, both are ways to recapitalize banks. Cetorelli and

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11Coates and Scharfstein (2009) argue that attempts to recapitalize banks should involve forgiving debt partially. The idea is to reduce the amount of new private equity needed to avoid a de facto nationalization of banks, given that current regulation (in the US) prohibits individual non-financial investors to hold large stakes in banks.
Gambera (2001) find that bank concentration is positively correlated with growth in fast growing, underdeveloped sectors, while negatively correlated with growth in general. In particular, it may be beneficial if the banks serving an industry that experiences a scarcity of investment have some market power. However, bank concentration differs from collusion in that it may also affect otherwise perfectly competitive product markets on which borrowers are active. Cartelization of firms as a result of bank concentration around 1900 has been discussed by Simon (1998). For a recent example of how debt dependence may increase margins on the product market see Chevalier (1995). Rajan and Zingales (2003) argue that incumbent banks influence regulators to hinder financial reform, and thus keep bank industry concentrated, unless pressures from trade and capital flow liberalization are strong.\footnote{12}

While bank concentration, as opposed to bank collusion, may be be interpreted as a possible ‘third best’ response to a bank crisis, it cannot be cleanly separated from political economy issues. For example, the 1923 Tokyo earthquake cost 38% of Japanese GDP at that time and arguably also represented a large shock to bank net assets. In fact, the number of banks dropped from 2000 before the disaster to about 65 after, while the fraction of total deposits held by the five largest banks increased from 20.5% to 45.7%. In addition, banks became to head bond committees which may have allowed them to exert power over borrowers that had access to direct finance. However, these measures cannot be interpreted solely in the light of optimal regulation, as the Japanese government at that time was also in need of a strong and willing banking sector to finance two wars (1937 war against China, and the second world war).

7.1.4 Causes of financial crises

With respect to the 2007-2009 financial crisis, Acharya, Cooley, Richardson, and Walter (2010) argue that large complex financial institution (LCFI) deliberately exposed themselves to severe tail risk, thereby using their recently\footnote{13} obtained status as ‘too big to tail’ to realize and maximize the option value on their holdings of securitized assets. Wilmarth (2008) argues that LCFI exposed themselves to tail risk as an unintended by-product of their quest to maximize fee income from various financial services derived from an originate to distribute policy. For example, LCFI ended up warehousing a substantial part of the asset-backed securities (ABS) they created, and also provided guarantees to ease passing on these ABS to other investors. Both papers acknowledge special macroeconomic conditions (especially, high international savings and loose monetary policy) leading to exceptionally high demand for ABS by relatively ingenuous investors, as well as a regulatory failure to prescribe adequate minimum equity requirements as essential amplifiers. However, while Wilmarth (2008) believes that LCFI shared the misperception, for example, about the likelihood of a decline in house prices with investors and regulators, Acharya et al. (2010) believe that LCFI intentionally manufactured tail risk.

In the analysis provided in this paper it is assumed that the regulator thinks a future financial crisis is impossible. Hence, the optimal regulation is derived without worrying about its effect on the likelihood or severity of future crises. I think this is a good assumption since it is generally observed that regulators did not make use of the tools they had to avert a disastrous economic outcome: if most market participants do not see the crisis coming, regulators probably will not either.\footnote{14} Conversely, if LCFI are in fact guilty of deliberately bringing about a financial crisis, then regulators are very...
likely guilty of gross negligence. Hence I do not think that regulators should worry about being too lenient in providing assistance to LCFI during banking crises.

### 7.2 Proofs

**Proof of Lemma 1.** At date \( t+1 \), the firm first order condition for labor input \( l_{t+1}(s^{t+1}) \) is given by 
\[
\omega_{t+1}(s^{t+1}) = (1 - a)z(s^{t+1})k_{f,t+1}(s^{t})a l_{t+1}(s^{t+1})^{-a}.
\]
In equilibrium this becomes 
\[
\omega_{t+1}(s^{t+1}) = (1 - a)z(s^{t+1})K_{t+1}(s^{t})a.
\]
Evaluating the firm profit nonnegativity condition in equilibrium yields 
\[
az(s^{t+1})K_{t+1}(s^{t})a + (1 - \delta)K_{t+1}(s^{t}) - R_{t+1}(s^{t+1})K_{t+1}(s^{t}) \geq 0.
\]
The firm first order condition for investment in physical capital is given by 
\[
\sum_{s^{t+1} > s'} \pi(s^{t+1}|s') [az(s^{t+1})k_{f,t+1}(s^{t})a^{-1}l_{t+1}(s^{t+1})^{-a} + 1 - \delta - R_{t+1}(s^{t+1})] = 0,
\]
which in equilibrium becomes
\[
\sum_{s^{t+1} > s'} \pi(s^{t+1}|s') [az(s^{t+1})K_{t+1}(s^{t})a^{-1} + 1 - \delta - R_{t+1}(s^{t+1})] = 0.
\]
This implies that firm profits are zero in each event and that 
\[
R_{t+1}(s^{t+1}) = az(s^{t+1})K_{t+1}(s^{t})a^{-1} + (1 - \delta).
\]
The consumer first order condition for bank shares at date \( t \) in event \( s' \) is given by 
\[
p_t(s') = \sum_{s^{t+1} > s'} \beta \pi(s^{t+1}|s') [d_{t+1}(s^{t+1}) + p_{t+1}(s^{t+1})].
\]
Iterating yields 
\[
p_t(s') = \sum_{t'=t+1}^{\infty} \sum_{s^{t'} > s'} \beta^{t'-1} \pi(s^{t'}|s') d_{t'}(s^{t'}) = V_t(s') - d_t(s').
\]

**Proof of Lemma 2.** At date \( t \) in event \( s' \) the firm can divert \( \theta z(s')k_{f,t}(s^{t-1})a l_t(s^{t-1}) \) per unit borrowed from the bank. Since banks internalize how much they lend to firms (but not how much firms borrow) we have 
\[
O_t(s', K_t(s^{t-1}), k_t(s^{t-1})) = \theta z(s_t)K_t(s^{t-1})a^{-1}k_t(s^{t-1}).
\]
Suppose \( a_t(s') = A_t^*(s') \), then 
\[
V_t(s') = O_t(s', K_t(s^{t-1}), k_t(s^{t-1}))
\]
and note that shareholder value equals the sum of equity and pure profits,
\[
V_t(s') = a_t(s') + \sum_{t'=t+1}^{\infty} \sum_{s^{t'} > s'} \beta^{t'} \pi(s^{t'}|s') \left[ R_{t'}(s^{t'}) - \frac{1}{\beta} \right] k_t(s^{t'-1}).
\]

**Proof of Lemma 3.** This follows immediately from lemma 1

**Proof of Proposition 1.** The proof of existence is by construction. Since consumers and firms have a passive role in this economy it is sufficient to focus on the bank allocation. For the proposed allocation we need to check whether equity requirements hold. Since equity requirements are not too high we can check bank individual rationality constraints instead which is easier. Given the proposed dividend payment plan,
\[
V_t(s_L, s^{t-1}) = \frac{\beta}{2} \frac{1}{1 - \beta} \frac{2(1 - \beta)}{2 - \beta} \theta z_H K^T_{FB} = \frac{\beta}{2 - \beta} \theta z_H K^T_{FB} > \theta z_L K^T_{FB},
\]
crisis began in regulated entities ... This happened right under our noses.
where the strict inequality follows from $z_L < \beta$, and

$$V_t(s_H, s_i^{t-1}) = \left[ 1 + \frac{\beta}{2 - \beta} \right] \frac{2(1 - \beta)}{2 - \beta} \theta z_H K_{FB}^q = \theta z_H K_{FB}^q.$$ 

Since banks are not profitable, shareholder value equals equity. Then future dividends can only be increased by reducing current dividends (and increasing equity) such that the proposed dividend payment plan satisfies assumption 2. That is, assumption 2 guarantees uniqueness. 

Proof of Proposition 2. The proof is by construction. From proposition 1 we know that for $a_t(s^t) \geq \bar{a}_0$ the economy enters a steady state within one period. Working backwards from this steady state we can construct cutoff values for initial bank conditions for bank optimality for some $K$ shock necessary such that aggregate investment reaches $K_{FB}$. For an arbitrary $T$, consider the following allocation for all $s^t$ such that $\eta(s^t) \leq T - 1$,

$$K_{t+1}(s^t) = K_{\eta(s^t)}, \quad \delta_t(s^t) = \delta_{\eta(s^t)}$$

where

$$\hat{K}_{t+1} = \left( \frac{\beta}{2 - \beta} \right)^{(T-\eta)} \hat{K}_{FB}, \quad \hat{\delta} = \hat{K}_{t+1} - \overline{D}_{t+1},$$

$$\hat{\overline{D}}_t = \left( \frac{\beta}{2 - \beta} \right) [2 \hat{a}_{\eta} \hat{K}_{\eta} - 2 \hat{\delta} \hat{K}_{\eta}] + \hat{\overline{D}}_{t+1} - (\hat{K}_{t+1} - \hat{K}_{\eta}), \quad \hat{\overline{D}}_T = K_{FB} - \left( \frac{\beta}{2 - \beta} \right) \theta z_H K_{FB}^q.$$ 

Pick the $\kappa$ such that $a_0 \in [\bar{a}_k, \underline{a}_k]$ and let $T_{a_0} = T - \kappa$. We need to show that the proposed allocation above satisfies the conditions for bank optimality for some $a_0$ and $T_{a_0}$. Let $t$ count the occurrences of the high productivity shock rather that time periods. For $t = 0, 1, 2, \ldots T_{a_0} - 1$ the bank’s problem can be expressed as (for $t < T_{a_0} \Leftrightarrow a_t < \bar{a}_0$, otherwise we are done by proposition 1)

$$V_t(a_t, A_t) = \max_{d_t(a_t, A_t), \hat{k}_{t+1}(a_t, A_t), \hat{b}_{t+1}(a_t, A_t)} \left\{ d_t + \frac{1}{2} \beta \left( V_{t+1}(a_t, A_t) + V_{t+1}(a_t, A_t) \right) \right\}$$

subject to

$$\mu_t: \quad d_t \geq 0$$

$$\lambda_t: \quad d_t + k_{t+1} = a_t + \frac{1}{2} \beta \left( \hat{b}_{t+1}^L + \hat{b}_{t+1}^H \right)$$

$$\hat{a}_{t+1}^j = (a_{L} k_{t+1} - 1 - \delta) k_{t+1} - b_{t+1}^j, \quad j = L, H$$

$$\psi^L_{t+1}: \quad V_{t+1}(a_t, A_t) \geq \beta z_L K_{t+1}^L k_{t+1}$$

$$\psi^H_{t+1}: \quad V_{t+1}(a_t, A_t) \geq \beta z_H K_{t+1}^H k_{t+1}$$

where $V_t$ maps individual bank equity and aggregate bank equity into individual bank shareholder value. Again, working with individual rationality constraints directly is more straightforward. The bank rationally forecasts aggregate investment and aggregate bank equity according to rule

$$\{ K_{t+1}, (A_{t+1}^L, A_{t+1}^H) \} = \Psi_t(A_t).$$

28
Note that while $V_l(a_t, A_t)$ is linear in $a_t$, $V_l(a_t, A_t)|_{a_t=A_t}$ is strictly concave (for $a_0 < \bar{a}_0$) in $A_t$. Let $V'_l(a_t, A_t)$ denote the derivative with respect to $a_t$ and note that the envelope condition yields $V'_l(a_t, A_t)|_{a_t=A_t} = \lambda_t$ for all $t$. Similar, let $\lambda_{t+1,j} = V'_l(A_{t+1}^j, A_{t+1}^j)|_{a_{t+1}=A_{t+1}^j}$ for $j = L, H$. Note that the bank’s (linear) program is convex such that we only need to find suitable Lagrange multipliers that satisfy the bank’s first order conditions. Suppose the first individual rationality constraint never binds strictly for all $t$, $\psi_{t+1}^1 = 0$, then

$$\mu_t = \lambda_t - 1, \quad \lambda_t = \lambda_{t+1,L}, \quad \psi_{t+1,H} = \frac{\beta (\lambda_t)}{2 (\lambda_{t+1,H} - 1)},$$

Note that the denominator cannot be negative: suppose not then bank would perceive (at given aggregate quantities) that individual rationality constraint is not binding such that investment equals $K_{FB}$, a contradiction. Since banks are profitable at date $t$, the return on equity is strictly positive, $\lambda_t > 0$, implying that no dividends are paid out, $d_t = 0$. Strict concavity of $V_l(a_t, A_t)$ implies that bank equity stays constant in case next period’s productivity is low. Since nothing in the bank’s problem depends on time, it will choose the same level of lending and have unchanged shareholder value as long as productivity realizations are low. It is straightforward to check that $\frac{\lambda_t}{\lambda_{t+1,L}}$ is greater than unity. Note that $\lambda_{T_{\bar{a}_k}} = 1$ such that $\lambda_{T_{\bar{a}_k}} > 1$. Then we know that the individual rationality binds in the case of the high productivity shock, $\psi_{t+1,H} > 0$. From the Bellman equation we see that shareholder value grows at rate $\frac{2-\beta}{p}$ when high productivity is realized in the following period. Then $V_{t+1}(A_{t+1}^L, A_{t+1}^L) = V_l(A_t, A_t) = \frac{\beta}{2p} V_{t+1}(A_{t+1}^H, A_{t+1}^H) = \frac{\beta}{2p} \theta z_{t+1} K_{t+1}^a > \theta z_{t} K_{t+1}^a$, verifying that indeed $\psi_{t+1,L} = 0$. Repeating this argument for $t+1$ it is straightforward to see (from the individual rationality constraints) that aggregate investment grows at rate $\left(\frac{2-\beta}{p}\right)^{\frac{1}{2}}$ whenever productivity is high. Note that if $\lambda_t < 0$ for the proposed solution, then $a_0 < \bar{a}_0$: in fact, we define $\bar{a}_0$ as the lowest $a_0$ for which $\lambda_t > 0$. Second, if $a_0 \in (\bar{a}_k, \bar{a}_{k+1})$ then the proposed allocation above will have to be translated such that aggregate investment grows at rate $g < g$ when reaching $K_{FB}$.

**Proof of Lemma 4.** Recall that $V_l(a_t, A_t)$ is linear in $a_t$ and suppose $k$ is a feasible choice for bank lending. When bank equity is zero and borrowing is proportional to $k$ then the bank individual rationality constraint is homogenous of degree one in $k$ (from the viewpoint of the bank, for a given aggregate level of bank lending). Since $k$ was feasible it follows the bank is in fact not constrained, as any multiple of $k$ is feasible as well, provided that $R K^\beta \geq 1$. In an equilibrium it must be the case that $R K^\beta = 1$, but then shareholder value equals equity. It follows that shareholder value is zero in any future period as well implying that $k = K_{FB} > 0$ does not satisfy the bank’s individual rationality constraint.

**Proof of Lemma 5.** From proposition 2 we know that aggregate investment grows either at rate $g$ or remains constant, with equal probability. Then

$$\text{Prob} \left( \eta(s^I) \geq T_{\bar{a}_0} \right) = 1 - \left( \frac{1}{2} \right) \left( \text{Prob} \left( T_{\bar{a}_0} \right) \sum_{t \in 1}^{T_{\bar{a}_0}} \left( t_s - T_{\bar{a}_0} + 1 \right) < 1 - \epsilon \right)$$

for $t_s$ large enough. Note that reaching aggregate investment of $K_{FB}$ within $t_s$ periods is equivalent to reaching a steady state (with aggregate physical capital stock constant at $K_{FB}$) in $t_s + 1$ periods.

**Proof of Proposition 3.** This follows the same steps as in the proof to proposition 1 with $K_{FB}$ replaced by $K_{\bar{a}}$.
Proof of Proposition 4. This follows the same steps as in the proofs to proposition 2 and lemma 5 with $K_{FB}$ replaced by $K_{SB}$ and with banks borrowing against future profits in steady state, $\bar{D}_T = K_{SB} + \Pi_{SB} - \frac{\delta}{2 - \delta} \theta z_h K_{SB}^{*}$. \hfill \square

Proof of Proposition 6. We need to show that the competitive equilibrium with constant taxation of bank lending at rate $\tau_{SB}$, as characterized by propositions 3 and 4, yields an allocation that solves the problem stated in definition 8. For $a_0 \geq \bar{a}_0$ first-best lending is feasible in competitive equilibrium such that the optimal tax on lending is none. For given $a_0 \in [a_M, \bar{a}_0)$ and $T_{a_0}$ let $t$ denote the occurrences of the high productivity shock thus far. Then for $t = 0, 1, 2, \ldots, T_{a_0} - 1$ we can write the problem in definition 8 recursively.

\[
W_t(A_t, V_t) = \max_{d_t, K_t, V'_t, V''_t, B_t, B_{t+1}} \left\{ d_t + \beta(1 - \alpha)K_t^a + \beta \frac{1}{2} (W_t(A'_t, V'_t) + W_{t+1}(A'_{t+1}, V'_{t+1})) \right\}
\]

subject to

\[
\begin{align*}
\mu_t: & \quad d_t \geq 0 \\
\lambda_t: & \quad d_t + K_t \leq A_t + \beta \frac{1}{2} (B_t + B_{t+1}) \\
\varphi'_t: & \quad V'_t \geq \theta z(s_L)K_t^a \\
\varphi''_{t+1}: & \quad V''_{t+1} \geq \theta z(s_H)K_t^a \\
\eta_t: & \quad d_t + \frac{1}{2} \beta (V'_t + V''_t) \geq V_t \\
A'_t: & \quad z(s_L)\alpha K_t^a + (1 - \delta)K_t - B_t \\
A''_{t+1}: & \quad z(s_H)\alpha K_t^a + (1 - \delta)K_t - B_{t+1} \\
V'_t, V''_{t+1}: & \quad \frac{1}{\beta} \Pi_{M}. 
\end{align*}
\]

Note that the constrained social planning problem is here expressed as a dynamic game as in Abreu, Pearce, and Stacchetti (1990). The last line guarantees that the social planner only promises feasible levels of shareholder value to banks. For $W$ concave in $A$ and convex in $V$, together with assumption 2, the following can be established. (i) The social planner smooths the social return on bank equity such that $\lambda_t = \lambda_{T_{a_0}} \equiv \lambda_0$ for all $t$. From the first-order condition for aggregate bank lending in steady state it follows that $\lambda_0$ is related to $K_{SB}$ as given by proposition 5. (ii) The bank individual rationality constraint binds only in the high state, $\varphi'_t = 0$. (iii) With envelope conditions $\frac{dW_t}{dA_t} = \lambda_0$ and $\frac{dW_t}{dV_t} = -\eta_t$, bank equity and shareholder value remain constant when aggregate productivity is low. Since no dividends are paid out during the transition, shareholder value depends only on $K_{SB}$ (through $a_0$). Then $\frac{dW_t}{dV_t}$ does not depend on how often the high shock occurred such that the constrained efficient allocation features an increasing sequence of Lagrange multipliers $\eta_t$. From the first-order condition for dividends in steady state we see that $\eta_{T_{a_0}} = \lambda_0 - 1$. From the first-order condition for promised shareholder value we see that $\varphi''_{t+1} = \eta_{t+1} - \eta_t$. Using these results in the first-order condition for aggregate bank lending we can verify that $\eta_{t+1} > \eta_t$ whenever $K_t < K_{SB}$. \hfill \square