Negative Net Incomes and the Measurement of Poverty: A Note

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Abstract
This note warns about the careless computation of poverty indexes when the welfare of each household is measured by its net income, since this can be negative. As is illustrated in the case of Mexico, even if only a handful of households report negative incomes, the resulting poverty aggregates, when they go beyond a mere headcount measure, can behave rather badly. The note ends with suggestions on how to deal with the problem.

Resumen
Esta nota advierte sobre el cálculo apresurado de índices de pobreza cuando el bienestar de cada hogar es medido por su ingreso neto, pues éste puede ser negativo. Como se ilustra en el caso de México, aun si sólo un puñado de hogares reportan ingresos negativos, los resultantes agregados de pobreza, cuando van más allá de un mero conteo de cabezas, podrían comportarse de manera muy anómala. La nota termina con sugerencias acerca de cómo tratar el problema.

Clasificación JEL: I32, C81  
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1. Introduction
Poverty measurement is a delicate task. To start with, as first noted by Sen (1976), the researcher has to solve two important problems: How to identify the poor among a given population, and how to aggregate the corresponding data into a reasonable index. For instance, rightly or wrongly, in the case of Mexico the official poverty statistics are based on absolute poverty lines, while the only aggregate poverty measure that is employed is headcount (see Comité Técnico para la Medición de la Pobreza, 2002). Once those two fundamental problems mentioned by Sen are solved, there are still other important issues to be dealt with by the researcher. To give just two examples: Should household welfare be measured using income or consumption data? Also, should children

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and adults needs be treated the same? For instance, in the case of Mexico the official methodology identifies household welfare with net income, while it ignores the second question.

To those theoretical problems described above, one can add some statistical issues. Two questions come immediately to mind in particular: How to estimate in a robust way the standard errors of the resulting poverty measures, and, if welfare is identified with income, how to treat the negative net incomes at the bottom of the distribution. The former question is a bit less ordinary than it would seem at first sight (see, e.g., Urzúa et al., 2008, and references therein), while the latter is certainly easy. But, as this note shows, such an obvious point can be nevertheless quite important for applied work.

Section 2 presents the algebraic consequences of calculating poverty measures when some of the net incomes are negative (we shall always assume in this paper that welfare is identified with income, and not with consumption). After that, section 3 illustrates, using several different Mexican income and expenditure surveys, how badly some poverty indexes may behave if the net incomes of some of the households are actually negative. Finally, the last section comments on possible ways to deal with the problem.

2. Negative Net Incomes and Poverty Measurement

There could be several reasons for a household to have a net income that is negative. Aside from the possibility of misreporting, the two most important factors would be, first, that the business of a self-employed member of the household might have made a loss during the period under consideration, and, second, that the transfers made by the household to other households might have exceeded its total income.

But regardless of the reasons that are behind that fact, the point to be made here is that almost all poverty measures cannot cope with the possibility of some incomes being negative. In the case of the headcount index the problem is obviously innocuous, since a household is counted as poor whenever its net income, be that positive, zero or negative, is below some positive number. Going to the other extreme, there are poverty measures that are, from the start, undefined when some incomes are negative. This is the case, for instance, of the Watts index (Watts, 1968), which requires the computation of the logarithm of each income, so that this number cannot be less or equal than zero.

There are, nevertheless, other indices that could be computed numerically even if some of the incomes are negative. Yet, those poverty measures could behave abnormally in such extreme cases. In order to illustrate the point, we will consider in this paper the popular Foster- Greer-Thorbecke (FGT) class of poverty measures (Foster et al., 1984):

\[
P_\alpha(z) = \frac{1}{n} \sum_{i=1}^{q} \left( \frac{z - y_i}{z} \right)^\alpha, \quad \alpha \geq 0,
\]

where \( q \) is the number of poor people in a population of size \( n \), and where the \( i \)-th member has an income \( y_i \) which is less or equal than the poverty line \( z \). The headcount index mentioned earlier is obtained when \( \alpha = 0 \), while the
Negative Net Incomes

(relative) poverty-gap measure is found when \( \alpha = 1 \):

\[
P_1(z) = \frac{1}{n} \sum_{i=1}^{q} \left( \frac{z - y_i}{z} \right).
\]

Already in this case, if \( y_i < 0 \) for some \( i \), then that particular poverty-gap ratio would be (incorrectly) found to be greater than one. This anomalous behavior is exacerbated as \( \alpha \) is further increased, as, for instance, in the case of the squared poverty-gap index given by:

\[
P_2(z) = \frac{1}{n} \sum_{i=1}^{q} \left( \frac{z - y_i}{z} \right)^2.
\]

But the consequences of having negative incomes become really worrisome when one compares different FGT measures. If \( y_i \geq 0 \) for all \( i \), then

\[
P_0(z) \geq P_1(z) \geq P_2(z) \geq P_3(z) \geq \ldots
\]

(1)

(the equalities will hold if \( y_i = 0 \) for all households). This is so simply because

\[
\left( \frac{z - y_i}{z} \right)^\alpha \geq \left( \frac{z - y_i}{z} \right)^\beta \quad \text{if} \quad 0 \leq \alpha < \beta.
\]

However if \( y_i < 0 \) for at least some \( i \), it could be the case, as an example in the next section will illustrate, that some of the inequalities given in (1) above are reversed.

Furthermore, as it will be also exemplified in the next section, if some of the incomes are negative, it could be even the case that \( P_\alpha(z_1) > P_\alpha(z_2) \) when \( z_1 < z_2 \). That is, even though one would expect as a rule that a poverty index would decrease as the poverty line is lowered, in some rare instances the opposite could happen if there were enough negative net incomes. This is so because if \( y < 0 \), then \( (z_1 - y)/z_1 \) would be greater than \( (z_2 - y)/z_2 \).

3. Examples Using Mexican Income and Expenditure Surveys

Before going through the particular examples, it is worth to describe very briefly the way in which the official Mexican poverty statistics are produced. For that end, the authorities use income and expenditure surveys (known as ENIGHs, by their acronym in Spanish), which are mostly made each even year. Given a particular survey, the monetary and non-monetary incomes are first added-up for each household (the latter income includes self-consumption, inkind payments, imputed rents and gifts). After calculating total current income, net income is obtained by subtracting the transfers made by the household to other households. Next, expansion factors are used to represent the entire set of Mexican households or, which is more common, the entire Mexican population (this is the alternative that we will follow here). Finally, the headcount ratio is used to report poverty incidence according to three different poverty definitions. These are: “food poverty”, when net income is too low to cover basic food
necessities; “capabilities poverty”, when net income is insufficient to buy basic food, education and health necessities; and “assets poverty”, when net income is too low to cover basic food, education, health, dressing, housing and public transportation necessities.

Returning to the purpose of this note, one might now wonder whether or not there are households that indeed report negative net incomes in the ENIGHs. For the authorities this issue seems to be irrelevant since their only interest is on poverty incidence. Thus, one has to go back to each of the surveys to identify those outliers. Using the ENIGHs corresponding to the years 1992, 1994, 1996, 1998, 2000, 2002, 2004 and 2005, Table 1 presents the corresponding results.

<table>
<thead>
<tr>
<th>Year</th>
<th>Observations (Households)</th>
<th>Expanded (Individuals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>2</td>
<td>21,660</td>
</tr>
<tr>
<td>1994</td>
<td>10</td>
<td>13,352</td>
</tr>
<tr>
<td>1996</td>
<td>7</td>
<td>10,983</td>
</tr>
<tr>
<td>1998</td>
<td>20</td>
<td>129,961</td>
</tr>
<tr>
<td>2000</td>
<td>4</td>
<td>24,568</td>
</tr>
<tr>
<td>2002</td>
<td>12</td>
<td>34,477</td>
</tr>
<tr>
<td>2004</td>
<td>38</td>
<td>139,134</td>
</tr>
<tr>
<td>2005</td>
<td>9</td>
<td>19,227</td>
</tr>
</tbody>
</table>

Source: Own calculations based on the ENIGHs.

As can be appreciated from the table, even though there are a relatively small number of households reporting negative net incomes in each ENIGH, once the observations are expanded to cover the entire Mexican population, the numbers become sizable. For instance, in the years 1998 and 2004 more than one percent of the population was supposed to have had negative incomes. Taking those observations at their face value, Figure 1 presents three different measures of food poverty in Mexico from 1992 to 2005: the headcount ratio (which coincides with the official estimates), the poverty gap ratio and the squared poverty gap ratio. As can be seen there, we are now able to exemplify with real data the reversal of some of the inequalities in (1) above.

How can we avoid that anomaly? In the last section of this note we present our preferred method for correcting for those negative incomes, but here we simply solve the problem by replacing each negative income with a zero. By doing so, we continue to obtain in particular the official poverty incidence statistics. The contrasting, well-behaved Figure 2 presents the results thus obtained.

An even more anomalous behavior is illustrated in Figure 3, which shows, in the years 1996, 2004 and 2005, how the squared poverty gaps could become larger as the poverty lines become smaller! Once again, the purported paradox could be solved by replacing the negative net incomes with zero values.
There is still another lesson to be drawn from this exercise. Until recently, most studies that reported poverty measures tended to do so without any reference on the precision of those statistics. Fortunately, that practice has become less frequent nowadays, especially after Kakwani (1993) showed, by means of the time honored delta-method, how to derive the approximate sampling variance for the class of FGT poverty indices. The approximation is:

**Figure 1**

*Unadjusted Food Poverty in Mexico, 1992-2005*

![Graph](image1)

**Figure 2**

*Food Poverty in Mexico, 1992-2005*

![Graph](image2)
\[ \var(P_\alpha) \approx \frac{P_{2\alpha} - P_\alpha^2}{n}. \]

Note, however, that Kakwani’s approximation may not be always suitable, since it implicitly assumes a normal distribution for the poverty measures, which actually range only from 0 to 1. Furthermore, if some of the net incomes are negative, then, as we have illustrated before, the FGT measures, and hence its approximate sampling variances and corresponding confidence intervals, could behave rather badly. Using once again real data, Figure 4 illustrates this point. As can be appreciated in the figure, the normal approximation would lead us to have, in the years 1996 and 2005, negative bounds for the confidence intervals of squared poverty gaps! It is worth to mention in passing that, aside from the possible corrections for negative net incomes, the best way to solve this last conundrum is to use resampling methods instead of normal approximations (see, e.g., Deaton, 1977, and Urzúa et al., 2008).

4. Conclusions
Throughout this paper we have shown the anomalous implications that negative net incomes might have in the measurement of poverty. The way in which we solved those anomalies was to set the negative incomes equal to zero. Although this procedure has the advantage of rendering the same poverty incidence statistics as the official ones, it certainly contaminates the poverty measures that go beyond a mere headcount ratio.

What to do then? Clearly, the authorities will have to change their own methodology to account for those outliers. One possibility to do so, the one that we recommend most, is to identify household welfare not with income, but with consumption. This methodological change is also proposed, for even more important reasons, by Ruiz-Castillo (2005).
Barring such a replacement of income with consumption, we would recommend to explore as an alternative procedure the replacement of net incomes with net expenditures in the case of households that report negative values. We believe that Table 2 provides good reasons in favor of such a procedure. As can be appreciated from that table, most of the households that report negative net incomes tend to have expenditure levels that would correspond to the case of much higher income levels. In all the ENIGHs considered here, the mean of net expenditures of those households is substantially higher than the urban (food) poverty line that is given in the last column. Furthermore, in the years 1996, 2000, 2004 and 2005, all the households had expenditure levels above the poverty line. Hardly an evidence of penury.

**Figure 4**

*Confidence Intervals for Unadjusted Food Poverty Gaps*

**Table 2**

*Net Expenditures of Households with Negative Incomes*

<table>
<thead>
<tr>
<th>Year</th>
<th>Observations</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Poverty Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992*</td>
<td>2</td>
<td>$1,221</td>
<td>$169</td>
<td>$1,250</td>
<td>$168</td>
</tr>
<tr>
<td>1994</td>
<td>10</td>
<td>$6,632</td>
<td>$44</td>
<td>$9,292</td>
<td>$193</td>
</tr>
<tr>
<td>1996</td>
<td>7</td>
<td>$3,175</td>
<td>$1,349</td>
<td>$8,255</td>
<td>$389</td>
</tr>
<tr>
<td>1998</td>
<td>20</td>
<td>$5,163</td>
<td>$102</td>
<td>$13,867</td>
<td>$524</td>
</tr>
<tr>
<td>2000</td>
<td>4</td>
<td>$3,352</td>
<td>$2,070</td>
<td>$4,064</td>
<td>$627</td>
</tr>
<tr>
<td>2002</td>
<td>12</td>
<td>$3,187</td>
<td>$264</td>
<td>$10,147</td>
<td>$672</td>
</tr>
<tr>
<td>2004</td>
<td>38</td>
<td>$6,309</td>
<td>$807</td>
<td>$140,358</td>
<td>$740</td>
</tr>
<tr>
<td>2005</td>
<td>9</td>
<td>$10,584</td>
<td>$4,065</td>
<td>$83,783</td>
<td>$791</td>
</tr>
</tbody>
</table>

Source: Own estimates based on the ENIGHs.

*Old pesos converted to new pesos.*
References


