MICROSTRIP ANTENNA WITH MODIFIED RADIATION PATTERN (SLOPING)

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Abstract
In this work the transmission-line model was used, considering the characteristics of the dielectric and that’s of the patch to determine the self and mutual conductance, as well as the input resistance for matching. The cavity model was used to calculate and plot the radiation pattern for E and H planes, both for the radiating slots and for the non radiating ones, obtaining also the directivity of the antenna. The analysis, design and simulation were realized exciting the antenna with the dominant mode \((\text{TM}_{010})\) for different dimensions and the response for different modes of excitation are investigated. The obtained results allow the antenna to be used as a single antenna or as an element for designing antenna arrays.

Keywords — Microstrip antennas, Radiation Patterns, Substrates, Excitation Modes, Transmission Line Model, Cavity Model, Radiating Slots, Self and Mutual Conductance.

I. INTRODUCTION

Nowadays the wireless systems of telecommunications have presented a great development and diversity of applications, as in wireless telephones, mobile telephony, wireless networks, telemetry and systems of measurement.

The microstrip antennas are now very common for adapting easily to different specifications and requirements, numerous geometries and accessibility. Diverse technologies have been developed to improve the efficiency and use of the microstrip antennas [1-6]. Their research has contributed to the development of other sciences, like: medicine, astronomy, biology, aeronautics, etc.

A microstrip antenna is a metallic strip (patch) placed over a ground plane. The microstrip antenna is designed generally to have a pattern with maximum radiation normal to the patch (broadside radiator). Nevertheless for certain applications like receiving aerials placed in irregular surfaces there is wished that the radiation pattern has certain inclination, this is achieved modifying the antenna dimensions and materials an so modifying the mathematical expressions that represent its properties of radiation [7-16].

For the excitation with the dominant mode \((\text{TM}_{010})\), its radiation will present a maximum perpendicular to the patch. Nevertheless, when it is excited with other modes and by choosing the appropriate dimensions, it is possible to modify the direction of its radiation.
II. METHODOLOGY

In order to analyze the global behavior of the microstrip antenna for different modes of excitation, the Transmission Line Model and the Cavity Model are applied to determine the self and mutual conductance of the antenna, its input impedance, the resonant frequency, the characteristics of different modes of excitation, the radiated fields and the directivity. Some programs in MatLab have to be developed to compute or to simulate some equations, to calculate the radiated fields and directivity and to design microstrip antennas. The results are compared to those at the references.

III. RESULTS

III.1. Excitation with the TM010 Mode

A. Edge Effects.

Because the patch has finite dimensions both in length and in width, the fields on the borders produce some overflow. The spillover effect is based on the size of the patch and the height of the substrate, this overflow mainly affects the resonance frequency of the antenna, so that the line appears to be wider electrically than its physical dimensions therefore it is necessary to introduce the concept of effective dielectric constant, \( \varepsilon_{\text{reff}} \). At low frequencies (static values) the effective dielectric constant is given by \cite{17}

\[
\varepsilon_{\text{reff}} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2} \left[ 1 + 12 \frac{h}{\varepsilon_r} \right]^{-1/2} \tag{1}
\]

As mentioned, due to edge effects, patch microstrip antenna seems greater than its physical dimensions. For the main \( E \)-plane (\( x - y \) plane), this effect is illustrated in Fig. 1.

![Fig 1. Physical and effective lengths of a rectangular microstrip patch.](image)

A very popular approximate relation to calculate the standard extension length \( \Delta L \) is \cite{18}

\[
\frac{\Delta L}{h} = 0.412 \frac{\varepsilon_{\text{reff}} + 0.3}{\varepsilon_{\text{reff}} - 0.258} \left( \frac{h}{\varepsilon_r} + 0.264 \right) \tag{2}
\]

Thus the effective length of the patch is
\[ L_e = L + 2\Delta L \]  \hspace{1cm} (3)

Where for the dominant mode \( TM_{010} \), \( L_e = \lambda/2 \), and the resonant frequency of microstrip antenna is a function of the length given by

\[ (f_r)_{010} = \frac{1}{2L\sqrt{\varepsilon_r\mu_0\varepsilon_0}} = \frac{v_0}{2L\sqrt{\varepsilon_r}} \]  \hspace{1cm} (4)

Where \( v_0 \) is the speed of light in free space. Edge effects are incorporated as follows:

\[ (f_{rc})_{010} = \frac{1}{2L_{eff}\sqrt{\varepsilon_{reff}\mu_0\varepsilon_0}} = \frac{1}{2(L + 2\Delta L)\sqrt{\varepsilon_{reff}\mu_0\varepsilon_0}} \]

\[ (f_{rc})_{010} = q \frac{1}{2L\sqrt{\varepsilon_r\mu_0\varepsilon_0}} = q \frac{v_0}{2L\sqrt{\varepsilon_r}} \]  \hspace{1cm} (5)

where

\[ q = \frac{(f_{rc})_{010}}{(f_r)_{010}} \]  \hspace{1cm} (5a)

And \( q \) is known as the edge factor or length reduction factor.

**B. - Design of Rectangular Microstrip Antennas Procedure.**

To design a rectangular microstrip antenna are specified substrate dielectric constant \( \varepsilon_r \), the resonant frequency \( f_r \) and the substrate height \( h \), to determine the dimensions of the antenna: \( W \) and \( L \). The design procedure is:

1.- Calculate the width of the patch \[1\]

\[ W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}} \sqrt{\frac{2}{\varepsilon_r+1}} = \frac{v_0}{2f_r\sqrt{\varepsilon_r+1}} \]  \hspace{1cm} (6)

2.- Determine the effective dielectric constant of microstrip antenna using (1).

3.- Determine the extension of length \( \Delta L \) using (2).

4.- The actual length of the patch is determined by solving for \( L \) in (5), or,

\[ L = \frac{1}{2f_r\sqrt{\varepsilon_{reff}\mu_0\varepsilon_0}} - 2\Delta L \]  \hspace{1cm} (7)

**Design 1.**

Design a rectangular microstrip antenna using a substrate (RT/duroid 5870) with dielectric constant \( \varepsilon_r = 2.33, h = 0.1575 \text{ cm (0.062 inches)} \) such that resonates at 10 GHz.
Solution:

Using (6) the width $W$ of the patch is

$$W = \frac{30}{2(16)} \sqrt{\frac{2}{2.33+1}} = 1.1624 \text{ cm (0.4576 inches)}$$

The effective dielectric constant of the patch is from (1);

$$\varepsilon_{\text{eff}} = \frac{2.33 + 1}{2} + \frac{2.33 - 1}{2} \left[ 1 + \frac{0.1575}{1.1624} \right]^{-1/2} = 2.075$$

The incremental length $\Delta L$ of the patch is using (2)

$$\Delta L = (0.1575)(0.412) \frac{(2.075+0.3)(1.1624+0.264)}{(2.075-0.258)(1.1624+0.8)} = 0.079 \text{ cm (0.0312 inches)}$$

The actual length $L$ of the patch is found using (3), or

$$L = \frac{\lambda}{2} - 2\Delta L = \frac{30}{2(16)\sqrt{2.075}} - 2(0.079) = 0.8833 \text{ cm (0.3477 inches)}$$

Finally, the effective length is

$$L_e = L + 2\Delta L = \frac{\lambda}{2} = 1.041 \text{ cm (0.4099 inches)}$$

C. - Self Conductance with the Transmission Line Model.

Each radiating slot is represented as an admittance equivalent $Y$ in parallel with conductance $G$ and susceptance $B$, as shown in Fig. 2. The equivalent admittance of the slot number 1, based on an infinitely wide uniform slot is obtained as follows [19]:

$$Y_1 = G_1 + jB_1 \quad (8)$$

**Fig 2. Rectangular microstrip antenna and its equivalent circuit model.**
Where for a slot of finite width, \( W \),

\[
G_1 = \frac{W}{120\lambda_0^2} \left[ 1 - \frac{1}{24} (k_0 h)^2 \right] \quad \frac{h}{\lambda_0} < \frac{1}{10}
\]

\[
B_1 = \frac{W}{120\lambda_0^2} \left[ 1 - 0.636 \ln(k_0 h) \right] \quad \frac{h}{\lambda_0} < \frac{1}{10}
\]  

(8a)

(8b)

With a program developed for MatLab, the conductance from (8a) is plotted in terms of the normalized slot width and is shown in Fig. 3. Since both slots are identical,

\[
Y_2 = Y_1, \quad G_2 = G_1, \quad B_2 = B_1
\]

(9)

The conductance of a single slot can also be obtained using the expressions for the calculated fields through the cavity model as follows. In general de conductance is defined as

\[
G_t = \frac{2P_{\text{rad}}}{|V_0|^2}
\]

(10)

So using the expression for the electric field obtained from the cavity model, the radiated power can be written as:

\[
P_{\text{rad}} = \frac{|V_0|^2}{2\pi\eta_0} \int_0^\pi \left[ \sin \left( \frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta d\theta
\]

(11)
Therefore, the conductance from (10), can be expressed as:

$$G_1 = \frac{I_1}{120\pi^2}$$

where

$$I_1 = \int_0^\pi \left[ \sin\left(\frac{k_0 W \cos \theta}{\cos \theta} \right) \right]^2 \sin^3 \theta d\theta$$

$$I_1 = -2 + \cos(X) + XS_1(X) + \frac{\sin(X)}{X}$$

$$X = k_0 W$$

Through a program developed in MatLab, we obtained a graph of $G_1$ as a function of $W/\lambda_0$ using (12) and is shown in Fig. 4.
D.- Input Resistance and Coupling with the Transmission Line Model

The total admittance in slot 1 (input admittance) is obtained by transferring the admittance of the second slot from the output terminals to the input terminals using the theory of transmission lines [17], [20], [21]. Ideally the two slots are separated by \( \lambda/2 \) where \( \lambda \) is the wavelength in the dielectric (substrate). However, due to edge effects, the length of the patch is electrically larger than its actual length. Therefore the separation between slots should be slightly less than \( \lambda/2 \), typically \( 0.48\lambda < L < 0.49\lambda \). The transformed admittance of the second slot is converted to

\[
\tilde{Y}_2 = \tilde{G}_2 + j\tilde{B}_2 = G_1 - jB_1
\]

or

\[
\tilde{G}_2 = G_1 \quad \text{(13a)}
\]
\[
\tilde{B}_2 = -B_1 \quad \text{(13b)}
\]
Therefore the total input admittance at resonance is real and is given by

\[ Y_{in} = Y_1 + \bar{Y}_2 = 2G_1 \quad (14) \]

As the total input impedance is real, the resonant input impedance is also real

\[ Z_{in} = \frac{1}{Y_{in}} = R_{in} = \frac{1}{2G_1} \quad (15) \]

The resonant input resistance, given by (15), does not take into account mutual effects between the slots. This can be achieved by modifying (15) to [11],

\[ R_{in} = \frac{1}{2(G_1G_{12})} \quad (16) \]

Where the sign + is used for modes with an odd resonant voltage distribution (antisymmetric) under the patch and into the slots while the sign - is used for modes with an even resonant voltage distribution (symmetric). The mutual conductance is defined in terms of the fields in far zone as

\[ G_{12} = \frac{1}{|V_0|} \text{Re} \int_S \vec{E}_1 \times \vec{H}_2^* \cdot d\vec{S} \quad (17) \]

Where \( \vec{E}_1 \) is the electric field radiated by the first slot, \( \vec{H}_2 \) is the magnetic field radiated by the second slot, \( V_0 \) is the voltage across the slot, and the integration is performed over a sphere of large radius. It can be shown that \( G_{12} \) can be calculated using [11],

\[ G_{12} = \frac{1}{120\pi^2} \int_0^\pi \left[ \sin\left(\frac{k_0 W}{2} \cos \theta\right) \right]^2 J_0(k_0 L \sin \theta) \sin^3 \theta d\theta \quad (18) \]

Here \( J_0 \) is the Bessel function of first kind and order zero. For typical microstrip antennas, the mutual conductance \( G_{12} \) is small compared with the self conductance \( G_1 \). In Fig. 4 the mutual conductance was plotted to be compared with the self conductance as a function of the width slot. Fig. 5 shows the results obtained with a program developed in MatLab to calculate the variation of \( G_{12} \) (Eq. 18) as a function of the separation \( L_o \) between slots. The curves correspond to the principal planes, for different sizes of the slot.
Fig 5. Mutual conductance as a function of the slots separation for different slot sizes.

As seen from (8a) and (15) and tested using analysis of modal expansion, the input resistance does not depend strongly on the substrate height $h$. In fact, one can see that the input resistance at resonance can be decreased by increasing the width $W$ of the patch as long as the ratio $W/L \leq 2$ to prevent loss of efficiency.

Inset Feed. - The input resistance at resonance can be changed using the inset feed into the patch through a distance $y_0$ from the first slot, as shown in Fig. 6a).

The described technique can be used effectively to match the antenna using a microstrip line as a feeder whose characteristic impedance is given by [17]

$$Z_c = \begin{cases} \frac{60}{\sqrt{\mu_0} \epsilon_0} \ln \left( \frac{8h + W_0}{4h} \right), & \frac{W_0}{h} \leq 1 \\ \frac{W_0}{h}, & \frac{W_0}{h} > 1 \end{cases}$$

Where $W_0$ is the width of the microstrip line. Using modal expansion analysis, the input resistance for the inset feed is approximately given by [11], [20]

$$R_{in} (y = y_0) = \frac{1}{2(G_{11} + G_{12})} \left[ \cos^2 \left( \frac{\pi}{L} y_0 \right) + \frac{G_1^2 + B_1^2}{Y_c^2} \sin^2 \left( \frac{\pi}{L} y_0 \right) - \frac{B_1}{Y_c} \sin \left( \frac{2\pi}{L} y_0 \right) \right]$$

Here, $Y_c = 1/Z_c$. Since for most of the typical microstrip $G_1/Y_c \ll 1$ and $B_1/Y_c \ll 1$, (20) reduces to
Through a program developed in MatLab, in Fig. 6b) is shown a graph of the normalized input resistance of (20a).

The resonant input impedance given by (15) and the one that takes into account mutual effects between the slots given by (16) were calculated as a function of the width of the slot, and the results are plotted in Fig. 7.
Fig 7. Input resistance as a function of the width of the slot without and with mutual effects.

Design 2.

A microstrip antenna with overall dimensions $L = 0.8833$ cm (0.3477 inches) and $W = 1.1624$ cm (0.4576 inches), substrate height $h = 0.1575$ cm (0.062 inches) and dielectric constant of $\varepsilon_r = 2.33$, is operating at 10 GHz, Find:

a) - The input impedance,

b) - The position of the point of the inset feed where the input impedance is 50 Ω.

Solution:

The wavelength in free space is

$$\lambda_0 = \frac{3 \times 10^8}{1 \times 10^10} = 3 \text{ cm}$$

Using (12)

$$G_1 = 0.00157 \text{ siemens}$$
Which coincides well with $G_1 = 0.00328$ calculated with (8a). Using (18) yields

$$G_{12} = 6.1683 \times 10^{-4}$$

Using (16) with the + sign because the field distribution among the radiating slots is odd for the dominant mode $TM_{010}$.

$$R_{in} = 228.3508 \Omega$$

Because the input impedance at the edge of the first radiant slot is 228.3508 $\Omega$ and the desired impedance at the inset point is 50 $\Omega$, the distance $y_0$ at the inset feed point is obtained using (20a)

$$50 = 228.3508 \cos \left( \frac{\pi}{L} y_0 \right)$$

or

$$y_0 = 0.3126 \text{ cm (0.123 inches)}.$$

**E. Radiation Patterns with the Cavity Model**

A microstrip antenna can be modeled as a cavity with upper and lower walls considered as perfect electric conducting walls (the tangential components of electric fields vanish in these walls). As the cavity is filled with a dielectric material, the other four walls (side walls) are considered perfectly conducting magnetic walls (the tangential components of magnetic fields vanish at these walls). On the other hand as the height of the substrate is very small ($h \ll \lambda$) where $\lambda$ is the wavelength within the dielectric, therefore, only field configurations $TM^n$ will be considered within the cavity.

**Field Configurations (modes) $TM^n$.**

The configurations of the fields inside the cavity can be found by considering a rectangular cavity filled with dielectric material with dielectric constant (permittivity) $\varepsilon_r$ and calculating the vector potential $\vec{A}$. The vector potential $A_x$ must satisfy the homogeneous wave equation [1,17,19]

$$\nabla^2 A_x + k^2 A_x = 0 \quad (21)$$

Whose solution using the technique of separation of variables is

$$A_x = [A_1 \cos(k_x x) + B_1 \sin(k_x x)] [A_2 \cos(k_y y) + B_2 \sin(k_y y)] [A_3 \cos(k_z z) + B_3 \sin(k_z z)] \quad (22)$$

Where $k_x$, $k_y$, and $k_z$ are the wave numbers in the directions $x$, $y$, and $z$ respectively, and are determined from boundary conditions. The electric and magnetic fields inside the cavity are related to the vector potential $A_x$ by:
Subject to the boundary conditions:

\[
E_y(x' = 0, 0 \leq y' \leq L, 0 \leq z' \leq W) = E_y(x' = h, 0 \leq y' \leq L, 0 \leq z' \leq W) = 0
\]

\[
H_y(0 \leq x' \leq h, 0 \leq y' \leq L, z' = 0) = H_y(0 \leq x' \leq h, 0 \leq y' \leq L, z' = W) = 0
\]

\[
H_z(0 \leq x' \leq h, y' = 0, 0 \leq z' \leq W) = H_z(0 \leq x' \leq h, y' = L, 0 \leq z' \leq W) = 0
\]

Where primed coordinates \(x', y', z'\) are used to represent the fields inside the cavity. Applying the boundary conditions it is found that:

\[
k_x = \frac{m\pi}{h}, \quad m = 0, 1, 2, \ldots
\]

\[
k_y = \frac{n\pi}{w}, \quad p = 0, 1, 2, \ldots
\]

\[
k_z = \frac{n\pi}{L}, \quad n = 0, 1, 2, \ldots
\]

Where \(m, n, p\) represent respectively the number of half cycles in the field variations in the directions \(x, y, z\). Then the final form for the vector potential \(A_x\) inside the cavity is:

\[
A_x = A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z')
\]

As the wave numbers \(k_x, k_y, \) and \(k_z\) are subject to the constraint equation

\[
k_x^2 + k_y^2 + k_z^2 = \left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{w}\right)^2 + \left(\frac{p\pi}{L}\right)^2 = k_r^2 = \omega_r^2 \mu \varepsilon
\]

The resonance frequencies for the cavity are given by

\[
(f_r)_{mnp} = \frac{1}{2\pi \sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{w}\right)^2 + \left(\frac{p\pi}{L}\right)^2}
\]

Using (26) in (23), the electric and magnetic fields inside the cavity can be written as

\[
E_x = -j \left(\frac{k_z^2 - k_r^2}{\omega \mu \varepsilon}\right) A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z')
\]

\[
E_y = -j \left(\frac{k_x k_y}{\omega \mu \varepsilon}\right) A_{mnp} \sin(k_x x') \sin(k_y y') \cos(k_z z')
\]

\[
E_z = -j \left(\frac{k_x k_z}{\omega \mu \varepsilon}\right) A_{mnp} \sin(k_x x') \cos(k_y y') \sin(k_z z')
\]

\[
H_x = 0
\]

\[
H_y = -k_z A_{mnp} \cos(k_x x') \cos(k_y y') \sin(k_z z')
\]

\[
H_z = k_y A_{mnp} \cos(k_x x') \sin(k_y y') \cos(k_z z')
\]

The mode with the lowest resonant frequency is known as the dominant mode. For all microstrip antennas with \(h \ll L\) and \(h \ll W\) we have:
For $L > W > h$, the dominant mode is the $TM_{010}^X$ with

$$(f_r)_{010} = \frac{1}{2L\sqrt{\mu}} = \frac{\nu_0}{2L\sqrt{\varepsilon}}$$ (30a)

Where $\nu_0$ is the speed of light in free space. For $L > W > L/2 > h$ the second-order mode is the $TM_{001}^X$ with

$$(f_r)_{001} = \frac{1}{2W\sqrt{\mu}} = \frac{\nu_0}{2W\sqrt{\varepsilon}}$$ (30b)

For $L > L/2 > W > h$, the second order mode is the $TM_{020}^X$ with

$$(f_r)_{020} = \frac{1}{L\sqrt{\mu}} = \frac{\nu_0}{L\sqrt{\varepsilon}}$$ (30c)

For $W > L > h$, the dominant mode is the $TM_{001}^X$ with its resonance frequency given by (30a). While for $W > W/2 > L > h$, the second-order mode is the $TM_{002}^X$. Using (29), we calculated the distributions of electric fields tangential to the walls of the cavity for some of the lowest order modes, and the results are presented in Fig. 8.

Figure 8. Modes or configurations of the field for a rectangular microstrip patch.
Equivalent Current Densities.

Using the Huygens principle of equivalence of fields, the microstrip patch is represented by an equivalent current density \( \vec{J}_l \) on the upper surface, there is also an electric current density \( \vec{J}_s \) on the bottom surface of the patch. The four side slots are represented by an electric current density \( \vec{J}_s \) and a magnetic current density \( \vec{M}_s \) as shown in Figure 9a) and are represented by:

\[
\vec{J}_s = \hat{n} \times \vec{H}_a \quad \text{(31a)}
\]

and

\[
\vec{M}_s = -\hat{n} \times \vec{E}_a \quad \text{(31b)}
\]

Where \( \vec{E}_a \) and \( \vec{H}_a \) represents respectively the electric and magnetic fields in the slots. Due to the existence of the ground plane, to the fact that the ratio between height and width of the patch is very small, and because the tangential magnetic fields on the side walls are negligible, all the electrical current density \( \vec{J}_s \) can be considered zero.

So only the magnetic current densities \( \vec{M}_s \) distributed in the side walls radiate in the presence of the ground plane as shown in Figure 9b). Finally, applying image theory by the presence of the ground plane, the equivalent magnetic current density radiating in free space as shown in Figure 9c) is

\[
\vec{M}_s = -2\hat{n} \times \vec{E}_a \quad \text{(31c)}
\]
Of the four slots of the antenna both with width $L$, will not contribute significantly to radiation from the antenna, while the two width $W$ and separated by a distance $L \approx \lambda/2$ are the radiant slots that act as an array of two elements. For the dominant mode it can be seen that the fields in the radiating apertures have opposite polarizations as shown in Figure 8a).

Considering the dominant mode $TM_{010}$ inside the cavity of the components of electric and magnetic fields from (29) reduce to

$$
\begin{align*}
E_x &= E_0 \cos \left( \frac{\pi y'}{L} \right) \\
H_x &= H_0 \sin \left( \frac{\pi y'}{L} \right) \\
E_y &= E_x = H_x = H_y = 0
\end{align*}
$$

(32)

Where $E_0 = -j \omega A_{010}$ and $H_0 = (\pi/\mu L)A_{010}$. The electric field distribution is shown in Figure 8a) which shows that suffers an inversion along its length and is uniform in its width. It is due to the phase inversion that the array radiates in the direction perpendicular to the patch (broadside).

**Radiating Slots.**

For the fundamental mode $TM_{010}$ in a microstrip antenna, the two radiating slots can be analyzed using the geometry shown in Fig. 10.
The field in the apertures is constant and given by:

$$E_a = -\hat{a}_x E_\psi \begin{cases} \frac{h}{2} \leq x' \leq \frac{h}{2} \\ -\frac{W}{2} \leq z' \leq \frac{W}{2} \end{cases}$$  (33)

Where $E_\psi$ is a constant. In the other aperture, the only change is the direction of the field. The radiated fields are calculated through the following equivalent problem.

Choosing an area from $-\infty \leq x, z \leq \infty$, the problem is equivalent to have the following current distributions, radiating into free space:

$$M_s = \begin{cases} -2\hat{n} \times \vec{E}_a = -2\hat{a}_y \times (-\hat{a}_x E_\psi) = -\hat{a}_x 2E_\psi, & \frac{h}{2} \leq x' \leq \frac{h}{2} \\ -\frac{W}{2} \leq z' \leq \frac{W}{2} \end{cases}$$

0, else where

$$\vec{j}_s = 0, \text{ every where}$$  (34b)

Using far-field approximations, the vector potential functions $\vec{A}$ and $\vec{F}$ and the fields $\vec{E}$ and $\vec{H}$ respective, are calculated as follows:

$$\vec{A} \approx \frac{ue^{-jkr}}{4\pi r} \vec{N}$$  (35a)

$$\vec{F} \approx \frac{ee^{-jkr}}{4\pi r} \vec{L}$$  (35b)

Where the normalized functions are:

$$N_\theta = \int_S \left[ J_x \cos \theta \cos \phi + J_y \cos \theta \sin \phi - J_z \sin \theta \right] e^{+jkr'\cos \psi} ds'$$  (36a)

$$N_\Phi = \int_S \left[ -J_x \sin \phi + J_y \cos \phi \right] e^{+jkr'\cos \psi} ds'$$  (36b)

$$L_\theta = \int_S \left[ M_x \cos \theta \cos \phi + M_y \cos \theta \sin \phi - M_z \sin \theta \right] e^{+jkr'\cos \psi} ds'$$  (36c)

$$L_\Phi = \int_S \left[ -M_x \sin \phi + M_y \cos \phi \right] e^{+jkr'\cos \psi} ds'$$  (36d)

Then:

$$E_r \approx 0$$  (37a)

$$E_\theta \approx -\frac{jke^{-jkr}}{4\pi r} \left( L_\Phi + \eta N_\theta \right)$$  (37b)

$$E_\phi \approx +\frac{jke^{-jkr}}{4\pi r} \left( L_\theta - \eta N_\Phi \right)$$  (37c)

$$H_r \approx 0$$  (37d)

$$H_\theta \approx +\frac{jke^{-jkr}}{4\pi r} \left( N_\Phi - \frac{L_\theta}{\eta} \right)$$  (37e)
\[ H_\phi \approx -\frac{jke^{-jkr}}{4\pi r} \left( N_\theta + \frac{L_\phi}{\eta} \right) \] (37f)

For the geometry in Fig. 10:

\begin{align*}
    r' \cos \Psi &= r' \cdot \hat{a}_r = (\hat{a}_x x' + \hat{a}_z z') \cdot \left( \hat{a}_x \sin \theta \cos \phi + \hat{a}_y \sin \phi \sin \theta + \hat{a}_z \cos \theta \right) \\
    r' \cos \Psi &= x' \sin \theta \cos \phi + z' \cos \theta \\
    ds' &= dx'dz'
\end{align*}

(38a)

Using (36a-36d) for an aperture at the center of coordinates,

\begin{align*}
    N_\theta &= 0 \\
    N_\phi &= 0 \\
    L_\theta &= \int_{-h/2}^{h/2} \int_{-W/2}^{W/2} (2E_0) \sin \theta \ e^{jk(x' \sin \theta \cos \phi + z' \cos \theta)} \ dx' \ dz' \\
    L_\phi &= 0
\end{align*}

(39a)

(39b)

(39c)

(39d)

So,

\[ L_\theta = 2E_0 \sin \theta \ \int_{-h/2}^{h/2} \int_{-W/2}^{W/2} e^{jk \sin \theta \cos \phi x'} dx' \int_{-W/2}^{W/2} e^{jk \cos \theta z'} dz' \]

It can be shown that:

\[ \int_{-c/2}^{c/2} e^{j \alpha x} dx = c \left[ \frac{\sin(\alpha c)}{\alpha c} \right] \]

(40)

Thus,

\[ L_\theta = 2hW E_0 \sin \theta \left( \sin \left( \frac{k h}{2} \sin \theta \cos \phi \right) \sin \left( \frac{k W}{2} \cos \theta \right) \right) \]

or

\[ L_\theta = 2hW E_0 \left[ \sin \theta \frac{\sin(X) \sin(Z)}{X} \right] \]

(41a)

\[ X = \frac{k h}{2} \sin \theta \cos \phi \]

(41b)

\[ Z = \frac{k W}{2} \cos \theta \]

(41c)

Using (36a-36d) y (10-10b) in (37a-37c), the electric field components are:

\[ E_r = 0 \]

(42a)
\[ E_\theta = 0 \quad (42b) \]

\[ E_\phi = \frac{j k h W e_\theta e^{-jkr}}{2\pi r} \left\{ \sin \theta \frac{\sin(X) \sin(Z)}{X Z} \right\} \quad (42c) \]

Since there are two radiating slots placed along the \( y \) axis, the array factor with the slots placed symmetrically about the origin of coordinates is

\[ AF = a_1 e^{jk d \cos \gamma} + a_2 e^{-jk d \cos \gamma} \quad (43) \]

For equal amplitude excitation, separation \( d = L_e \) and since, \( \cos \gamma = \sin \theta \sin \phi \), the array factor reduces to

\[ AF = e^{jk L_e \sin \theta \sin \phi} + e^{-jk L_e \sin \theta \sin \phi} \]

\[ AF = 2 \cos \left( k \frac{L_e}{2} \sin \theta \sin \phi \right) \quad (44) \]

Finally, the total electric field is given by the product of (42c) and (44),

\[ E_\phi^t = \frac{j k h W e_\theta e^{-jkr}}{2\pi r} \left\{ \sin \theta \frac{\sin(X) \sin(Z)}{X Z} \right\} \cos \left( k \frac{L_e}{2} \sin \theta \sin \phi \right) \quad (45) \]

Plane-\( \vec{E} \): (\( \theta = 90^\circ \), \( 0^\circ \leq \phi \leq 90^\circ \), \( 270^\circ \leq \phi \leq 360^\circ \)).

For the microstrip antenna, the \( x - y \) plane is the principal \( E \)-plane. For this plane, the expressions for the radiated fields are reduced from (35) to

\[ E_\phi^t = j k a_1 W V e^{-j k_0 \rho} \left\{ \sin \left( \frac{k_0 h \cos \phi}{2} \right) \right\} \cos \left( \frac{k_0 h}{2} \sin \theta \sin \phi \right) \quad (46) \]

Where \( V_0 = h E_0 \) is the voltage across the slot.

Plane-\( \vec{H} \): (\( \phi = 0^\circ \), \( 0^\circ \leq \theta \leq 180^\circ \)).

The principal \( H \)-plane of the microstrip antenna is the \( x - z \) plane, here the radiated fields reduce from (45) to

\[ E_\phi^t \approx j k a_1 W V e^{-j k_0 \rho} \left\{ \sin \theta \frac{\sin \left( \frac{k_0 h \sin \phi}{2} \right)}{\frac{k_0 h}{2} \sin \theta} \sin \left( \frac{k_0 W \cos \phi}{2} \right) \right\} \quad (47) \]

A program was developed to calculate the fields at the principal planes \( E \)- and \( H \)- and are shown in Figs. 11 and 12 respectively.
**Nonradiating Slots.**

The fields radiated by the non-radiating slots with length $L_e$ and height $h$ are obtained as follows. For the fundamental mode $TM_{010}$ in a microstrip antenna, the two non-radiating slots have the geometry shown in Fig. 13.
Fig 13.- Geometry for a non-radiating slot radiating with the fundamental mode $TM_{010}$.

For the fundamental mode, $TM_{010}$, the electric field at the aperture, centered about the coordinates system is

$$
\vec{E}_a = -\hat{a}_x E_0 \cos \left( \frac{\pi}{l_e} \left( y' + \frac{l_e}{2} \right) \right) = \hat{a}_x E_0 \sin \left( \frac{\pi}{l_e} y' \right), \quad \left\{ \begin{array}{c}
\frac{-h}{2} \leq x' \leq \frac{h}{2} \\
\frac{-l_e}{2} \leq y' \leq \frac{l_e}{2}
\end{array} \right. (48)
$$

And the equivalent current distributions, radiating into free space are:

$$
\vec{M}_s = \begin{cases} 
-2\hat{n} \times \vec{E}_a = -2(-\hat{a}_x) \times \left( \hat{a}_x E_0 \sin \left( \frac{\pi}{l_e} y' \right) \right) = \hat{a}_y 2E_0 \sin \left( \frac{\pi}{l_e} y' \right), \\
0, \text{ elsewhere}
\end{cases} \quad (49a)
$$

$$
\vec{J}_s = 0, \text{ everywhere} \quad (49b)
$$

Following a similar procedure as for the radiating slots, the normalized components of the electric field in far zone are given by:

$$
E_\theta = -\frac{k_0 h l_e E_0 e^{-jk_0 r}}{2\pi r} \left( Y \cos \phi \sin \frac{X}{X} \cos \frac{Y}{X} \right) e^{i(X+Y)} \quad (50a)
$$

$$
E_\phi = \frac{k_0 h l_e E_0 e^{-jk_0 r}}{2\pi r} \left( Y \cos \phi \sin \frac{X}{X} \cos \frac{Y}{X} \right) e^{i(X+Y)} \quad (50b)
$$

where $X$ is given by (41b) and

$$
Y = \frac{k_0 l_e}{2} \sin \theta \sin \phi \quad (50c)
$$
Since the two non-radiating slots form an array of two elements of the same magnitude but opposite phase, separated along the $z$ axis by a distance $W$, the array factor is

\[
(AF)_z = 2j \sin \left( \frac{k_0 W \cos \theta}{2} \right) \tag{51}
\]

Therefore the total far zone field is the product of (50a) and (50b) with (51) but it can be seen that for the $H$-plane, (50a) and (50b) are zero because the fields radiated by each quarter cycle of each slot is canceled with the fields radiated by the other quarter. The fields in the $E$-plane also are zero because (51) becomes zero. Outside the principal planes, there is some radiation.

In order to control the radiation pattern of the microstrip antenna, the size dimensions $W$ and $L_\varepsilon$ were changed and the electric fields radiated were calculated. In Figs. 14 and 15 some results are presented for the total radiated field, and for the fields from the radiating and for the nonradiating slots to see the behavior of the antenna.

Fig. 14. Radiation patterns of the microstrip antenna as a function of the width $W$ and the tilted planes. a) magnitude of electric fields at $E$-Plane, for radiating slots (magenta) and for nonradiating slots (blue for theta component and black for phi component), the red pattern is the total field for non radiating slots. b) shows the total radiated field at $E$-Plane tilted $\theta = 20^\circ$ and for $W = \lambda/4$. c) magnitude of electric fields at $H$-Plane, for radiating slots (magenta) and for nonradiating slots (blue for theta component and black for phi component), the red pattern is the total field for non radiating slots. d) shows the total radiated field at $H$-Plane tilted $\phi = 80^\circ$ and for $W = \lambda$. 
Fig. 15. Radiation patterns of the microstrip antenna as a function of the length $L_e$ and the tilted planes. a) magnitude of electric fields at E-Plane, for radiating slots (magenta) and for nonradiating slots (blue for theta component and black for phi component), the red pattern is the total field for non radiating slots. b) shows the total radiated field at E-Plane tilted $\theta = 50^\circ$ and for $L_e = \lambda$. c) magnitude of electric fields at H-Plane, for radiating slots (magenta) and for nonradiating slots (blue for theta component and black for phi component), the red pattern is the total field for non radiating slots. d) shows the total radiated field at H-Plane tilted $\phi = 90^\circ$ and for $L_e = 3\lambda/4$.

**F.- Directivity with the Cavity Model**

The directivity is a very important feature of any antenna and is calculated as

$$D_0 = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}} \quad (52)$$

For a single narrow slot ($k_0h \ll 1$) the maximum radiation and the radiated power can be written, respectively, using (42c) as:
Thus the directive would be

$$D_0 = \left( \frac{2\pi W}{\lambda_1} \right)^2 \frac{1}{I_1}$$

(55)

where

$$I_1 = \int_0^\pi \left[ \sin \left( \frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta d\theta = \left[ -2 + \cos(k_0 W) + k_0 W S_1(k_0 W) + \frac{\sin(k_0 W)}{k_0 W} \right]$$

(56)

and $S_1(z)$ is the sine integral function given by

$$S_1(z) = \int_0^z \frac{\sin t}{t} dt$$

(57)

For the two small slots with $(k_0 h \ll 1)$, using (45), the directivity can be written as:

$$D_2 = \left( \frac{2\pi W}{\lambda_0} \right)^2 \frac{1}{I_2} = \frac{2}{15 G_{rad} \lambda_0^2}$$

(58)

where $G_{rad}$ is the radiating conductance, and

$$I_2 = \int_0^{\pi} \int_0^{\pi} \left[ \sin \left( \frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta \cos^2 \left( \frac{k_0 \phi}{2} \sin \theta \sin \phi \right) d\theta d\phi$$

(59)

The total broadside directivity $D_2$ of the two radiating slots separated by the dominant mode field $TM_{010}^*$ which has an antisymmetric voltage distribution, can be calculated [11], as

$$D_2 = D_0 D_{AF} = D_0 \frac{2}{1 + g_{12}}$$

(60a)

$$D_{AF} = \frac{2}{1 + g_{12}}$$

(60b)

Where:

- $D_0$ - is the directivity of a single slot, given by (55) and (56),
- $D_{AF}$ - is the directivity of the AF, given by (44),
- $g_{12}$ - is the normalized mutual conductance, $g_{12} = G_{12}/G_1$.

In Fig 16, it is shown a graph of the directivity of one and two slots as a function of the width $W$. 

$$U_{max} = \frac{\left| V_0 \right|^2}{2 \eta_0 \pi \left( \frac{\pi W}{\lambda_0} \right)^2}$$

(53)

$$P_{rad} = \frac{\left| V_0 \right|^2}{2 \eta_0 \frac{1}{\pi}} \int_0^{\pi} \left[ \sin \left( \frac{k_0 W \cos \theta}{2} \right) \right]^2 \sin^3 \theta d\theta$$

(54)
IV. CONCLUSIONS

In this paper several results concerning with the behavior of the microstrip antenna are presented. The transmission line model was used to calculate the self and mutual admittances as well as the input impedance to match properly the antenna. The cavity model was used to calculate the modes inside the antenna and by using an equivalent problem, the radiated fields were calculated. Through the variations of the antenna size, material and exciting modes, the radiation can be modified thus, the microstrip antenna can radiate in other directions besides the normal one and in this way, the microstrip antenna can be used efficiently as a radiating element alone or as an element in an array. The effect of the materials and the excitation with higher modes are still under investigation.

Fig. 16. Calculation of directivity for one and two radiating slots in terms of width W.
REFERENCES