RAFAŁ RAKOCZY, STANISŁAW MASIUÇÃO, MARIAN KORDAS

ANALYSIS OF THE MIXING PROCESS OF A GRANULAR MATERIAL WITH THE GRINDING EFFECT BY USING THE PRINCIPLE OF MAXIMUM ENTROPY

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A new approach of the mathematical description of the mixing process with the grinding effect has been presented by means of the MEP approach based on the maximization of informational entropy. The experimental analysis was carried out by using the probability size distribution (PSD) for the analysis of the size distribution of particles at various moments of the mixing process. The results confirmed the adequacy of the theoretical background and supported the possibility of appropriate prediction of PSD resulting from the experimental investigations.

1. INTRODUCTION

Owing to their importance in the field of many industrial applications such as manufacturing of chemicals, pharmaceuticals, drugs and food, mixing operations of granular material have been the subject of many experimental investigations. The relevant reports have been focused on the studies of mixing of granular materials as well on the predictive mathematical modelling of such processes and devices [1–3]. Various units have been used to carry out the processes such as jet contractors [4], horizontal cylindrical shell mixers [5], rotating cylinders [6], ribbon or multi-ribbon mixers [7–10] and vibration mixers [11]. Much of the research on continuous mixing

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process of granular material focused on extending various mathematical descriptions, balances or models such as the mixing intensity criterion [12], Markov chains theory [13], population balances [14], dispersed-flow model [15] and dynamical behaviour of the process [16]. The random nature of mixing process of granular materials may be also examined by using statistical characteristics and methods. The integral characterisation of the state can be done within the entropy approach, in which the informational entropy may be used as a measure of disorder in a statistical ensemble of particles. This approach enable one to connect the statistical analysis with the informational characteristics as an interesting alternative to the description of mixing process of granular materials [9, 10].

The grinding effect is inescapably activated during the mixing process of a granular material. This effect changes the particle size distribution of the mixed batch and generates a new daughter collection of particle sizes. Modelling of this process is very well defined in relevant literature [17, 18].

The main objective of the present study was to develop a convenient and useful mathematical tool for approximation analysis of the grinding process accompanied by the mixing of granular material in a multi-ribbon blender. The assessment of mixing process of a granular material with the grinding effect encompasses two aspects: mathematical formulation of the particle size distribution and the formulation of the breakage function. The temporal description of this process is usually based on the sieve analysis of the collections of particles obtained from representative samples. Then the temporal variation of a mixing process may be estimated by means of the mixing degree employing the standard variation. This approach is usually limited to a pure physical mixing process and the breakage function is not taken into consideration.

The paper reports an analysis of the mixing process with the grinding effect of granular material (activated carbon). The probabilistic distributions of the product size distribution (PSD) are needed in the evaluation of the efficiency of the mixing process. The resultant PSD may be correlated by means of various empirical relationships. Among the empirical correlations, a well-defined analytical distribution has been the most popular one due to its ability to fit most accurately the variety of product of grinding process of granular material at various moments of the process. This experimental study develops some theoretical approach to the analytical determination of PSD for the grinding process of granular material through application of the maximum entropy principle (MEP). The principle of maximum uncertainty is well developed and utilized within the classical information theory. The MEP method is a general technique to assign values to probability distributions based on the partial information about the process. It is formulated, in generic terms, as the following optimization problem: determine the probability distribution that maximizes the Shannon entropy [19] subject to given constraints which usually express partial information about probability distribution, as well as general axioms of the probability theory. The typical constraints employed in practical applications of this MEP are expected values of the
random variables [20]. According to this principle, introduced by Jaynes [21], the maximum entropy distribution is uniquely determined as the one which is maximally non-committal with regard to missing information, and that it agrees with what is known, but expresses maximum uncertainty with respect to all other matters.

This approach is based on the informational entropy notion as a tool to describe various nonequilibrium processes in many engineering applications [22]. It is clear that the MEP is applied to define a very complicated tasks such as the statistical model of turbulence [23], the growth rate of crystal [24] and the description of particle size reduction in grinding processes by using the population balance model [25]. Recently, MEP have been employed in different industrial applications in order to describe, approximate or model complex processes as shown in Table 1.

<table>
<thead>
<tr>
<th>Application studies</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A new approach to estimate mixture models. It is shown that that this estimation generally yields better results than maximum likelihood estimation.</td>
<td>[26]</td>
</tr>
<tr>
<td>A new mathematical tool for enhanced hot spot detection (a relatively small, localized area in which a contaminant concentrations exceeds a certain numerical standard with high probability) based on composite sampling with no or minimal requirement for composite break down.</td>
<td>[27]</td>
</tr>
<tr>
<td>A new approach of reduced dynamics which correctly reproduced the two limiting cases: fully deterministic and fully stochastic. The Brownian particle in an external field can provided a testing ground for this approach.</td>
<td>[28]</td>
</tr>
<tr>
<td>The study of the MEP on the thermodynamic behaviour of gases.</td>
<td>[29]</td>
</tr>
<tr>
<td>A method for modelling a comminution process. A random function describing particle size distribution is used and calculated by the maximizing entropy.</td>
<td>[30]</td>
</tr>
<tr>
<td>The MEP approach applied to study a general class of deterministic fractal sets.</td>
<td>[31]</td>
</tr>
<tr>
<td>A theoretical approach to the analytical determination of wind speed data measured at various geographical locations.</td>
<td>[32]</td>
</tr>
<tr>
<td>An information-theoretical method to quantify the effect of uncertainty at smaller scales on macroproperties. Uncertainties are manifested in the form of a distribution of microstructural samples generated using the maximum entropy scheme.</td>
<td>[33]</td>
</tr>
<tr>
<td>The method of the description of the aggregation kinetics based on the maximum entropy of aggregation probability distribution makes it possible to determine the self-preserving aggregate size distribution of an irreversible aggregation mechanism.</td>
<td>[34]</td>
</tr>
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</table>

2. EXPERIMENTAL

Experimental investigations were carried out using a set-up with a multi-ribbon mixer shown schematically in Fig.1a. The experimental equipment consisted of an electric motor with a variable speed drive and a safety coupling, a stainless steel tank in the form of partly penetrating horizontal cylinders. Mixing was performed by
means of two co-operating helical multi-ribbon agitators [35]. More information about this mixer is given in elsewhere [9, 10].

The examined system used in this work consisted of activated carbon particles of various longitudinal dimensions. In Figure 2, the probabilities of size distributions of granular particles at the initial state of the mixing process compared with the grinding effect predicted and measured. In the figure, particle size dimensions $x$ are related to the maximum particle size $x_{\text{max}}$. The particle size distribution of the feed fraction is presented in the coordinate system relative particle size $x^* = x/x_{\text{max}}$, and the particle size distribution $f(x^*)$.

In the present report, the scatter of experimental points at the initial state of mixing process of a granular material with a grinding effect is described by means of the following two parameter beta-distribution function:

$$
\left( f(x^*) \right)_{\text{r}=0} = A_0 \frac{1}{\beta(p_0,q_0)} \left( x^* \right)^{p_0-1} \left( 1-x^* \right)^{q_0-1} = 0.05 \frac{1}{2 \cdot 10^{-5}} \left( x^* \right)^7 \left( 1-x^* \right)^7
$$

(1)

The mixer was loaded with 3.6 kg of a granular material. The agitators were set in motion with the rotational speeds of 2 rpm, 50 rpm and 100 rpm. During the sampling,
the agitators were stopped and the whole mixer bulk was analysed by using the sieve analysis. The PSD functions for the investigated processes at various sampling moments are shown in Fig. 3. To keep the picture readable, the graph was divided for three sub-plots: for the speed rotation of multi-ribbon agitator equal to $N = 2$ rpm (Fig. 3a), $N = 50$ rpm (Fig. 3b) and $N = 100$ rpm (Fig. 5c), respectively. The mixing process of granular material is realized at the time period $t_0 < t_i < t_k$, where $t_k$ is the final moment of the sampling.

![Fig. 3. PSD for the various values of rotational speed of multi-ribbon agitators](image)

The number of smaller particles continuously increases during the mixing process. At the same time (Figs. 3b, c), the effect of mixing intensity on PSD of granular particles is clearly visible. Note that for the analysed results very small particles of the material are formed. The number of these elements would increase with the mixing intensity. The continuous description of the grinding kinetic may be obtained when the values of the probability density calculated from the measurements are approximated by the continuous probability density function. It is obviously noticed that the evolution of the PSD for the experimental results at various times can be described using the proposed beta-type distribution function by the following relationship in a general form:
The PSD of mixed granular material obtained by the grinding effect can be usually well fitted by means of the following equation:

\[ f(t;x^*,N) = A(t;N) \frac{1}{\beta[p(t;N),q(t;N)]} (x^*)^{p(t;N)-1} (1-x^*)^{q(t;N)-1} \]  

(2)

3. RESULTS AND DISCUSSION

In this section, the application of the concept of maximum entropy principle (MEP) to analyze the obtained results for the mixing process of granular material with the grinding effect is presented. This approach used Shannon’s concept of entropy to assign the probability density with the minimum incorporation of prior knowledge by maximizing informational entropy, subject to any specific prior knowledge. The method presented in this paper based on the maximum entropy makes it possible to determine the mixing influence on the structure of the granular material, characterized by the PSD variation. Practically, the dynamical behaviour of granular material may be successfully approximated by a traditional description based on the Markov chain theory or special random transient operators [9]. The proposed mathematical description provided useful insight into mixing process of a granular material with the grinding effect and proposed a new approach to the description of dynamical behaviour of this process instead of the classical description based on well-known theories.

3.1. THEORETICAL BACKGROUND

The maximum entropy principle (MEP) has been successfully applied to many engineering problems arising in a wide variety of fields. According to the MEP, introduced by Jaynes [21] and based on the concept of informational entropy, which may be treated as a quantitative measure of uncertainty associated with the probability distribution, this method allows one to determine the least biased probability distribution function when the information available is limited by some constraints. It is recognized that many physical systems may be described by averages that may be known for particular systems. The maximum entropy method is a useful tool for creating the function, which approximates the unknown probability distribution [36].

In general, let \( E \) be \((a, b), -\infty < a < B < \infty, \) or \((0, \infty)\) or \((-\infty, \infty)\) and the probability density function over \( E \) may be written as \( f(x) \). That is, \( f(x) \geq 0 \) for \( x \in E \) and \( f(x) = 0 \) if \( x \not\in E \). The probability density function (PDF) is not known but some prior information is given in the form of mean and variance of this distribution. The decision prob-
lem is to find the best PDF $f(x)$ subject to the constraints given in the information about this distribution. The informational entropy for this problem may be defined in the following form:

$$H(f(x)) \overset{\text{def}}{=} -\int_E f(x) \ln(f(x)) \, dx \quad (4)$$

Let $F$ denote the set of feasible PSD which contain all functions $f(x)$ satisfying the constraints connected with the prior information about the distribution. Accordingly with the MEP approach, it is possible to find the PSD which has the maximum informational entropy. The found PSD is subject to $f(x) \in F$ and defines:

$$\max_E \left( -\int_E f(x) \ln(f(x)) \, dx \right) \quad (5)$$

It should be noticed that the maximum of informational entropy over all probability densities $f(x)$ satisfying the following conditions:

$$f(x) \geq 0 \quad (6a)$$
$$\int_E f(x) \, dx = 1 \quad (6b)$$
$$\int_E r_i(x) f(x) \, dx = \alpha_i \quad \text{for} \quad 1 \leq i \leq m \quad (6c)$$

Thus $f(x)$ is the PDF on support $E$ meeting certain moment constraints $\alpha_1, \alpha_2, \ldots, \alpha_m$. The differential entropy $H(f(x))$ is a concave function over a convex set which form the following functional:

$$L(f(x)) = -\int_E f(x) \ln(f(x)) \, dx - \lambda_0 \int_E f(x) \, dx - \sum_{i=1}^m \lambda_i \int_E r_i(x) f(x) \, dx \quad (7)$$

The differentiation with respect to $f(x)$ yields:

$$\frac{\partial L(f(x))}{\partial f(x)} = -\ln(f(x)) - 1 - \lambda_0 - \sum_{i=1}^m \lambda_i r_i(x) \quad (8)$$

Setting this equal to zero, the following form of the maximizing density may be obtained:

$$f(x) = \exp\left( -\lambda_0' - \sum_{i=1}^m \lambda_i r_i(x) \right) \quad (9)$$
where \( \lambda_0, \lambda_1, ..., \lambda_m \) may be treated as the Lagrange multipliers and these are chosen so that \( f(x) \) satisfies the constraints. It should be noticed that \(-1\) is included in \( \lambda'_0 \).

The MEP approach may be used to approximate the most likely probability distribution or PDF for practical problems of mixing of a granular material. The main objective of this paper is to estimate the distribution based on summary statistics. From the analytical point of view, the well-known distributions may be described as maximum entropy densities subject to moment constraints (characterizing moments). The resultant PDF may be successfully summarized by these moments.

In the case of this experimental work, the size distribution of particles of a granular material (see Fig. 3) obtained by the mixing process with grinding effect can be usually well fitted by means of the two-parameter beta distribution in the form proposed earlier (Eq. (3)). This distribution may be rewritten in the following equivalent form:

\[
f(x^*) = A \frac{1}{\beta(p,q)} (x^*)^{p-1} (1-(x^*))^{q-1} 
\]

\[
\Leftrightarrow \exp \left( \ln(A) - \ln(\beta(p,q)) - (1-p)\ln(x^*) - (1-q)\ln(1-(x^*)) \right) \tag{10}
\]

for \( 0 \leq x^* \leq 1 \) and \( p, q > 0 \).

It should be noticed that this distribution (Eq. (10)) is the maximum entropy density with the following characterizing moments: \( E(\ln x) \) and \( E(\ln(1-x)) \). Moreover, Eq. (10) may be treated as beta PSD, maximizing the informational entropy. It is clear that the practically usable description of the grinding process may be based on this type of PDF.

### 3.2. MATHEMATICAL DESCRIPTION OF EXPERIMENTAL RESULTS

The parametrical identification of the present MEP type function for the obtained PDFs of granular material sizes was carried out based on the experimental measurements (Fig. 3). It can be shown that a general form of maximizing density given by Eq. (5), and the parameters on which the distribution functions depends \((\lambda'_0, \lambda_1, \lambda_2)\) are related to the characterizing moments of the beta distribution:

\[
f(x^*) = \exp(-\lambda'_0 - \lambda_1 r_1(x^*) - \lambda_2 r_2(x^*)) \tag{11}
\]

where characterizing moments and their functional forms, \( r_i(x^*) \) for the proposed beta distribution, are expressed as follows:

\[
\int_{\tilde{E}} r_i(x^*) f(x^*) \, dx = E(\ln(x^*)) \tag{12a}
\]
\[ \int_E r^2(x^*) f(x^*) \, dx = E \left( \ln(1 - x^*) \right) \]  

(12b)

Taking into account Eqs. (10) and (11), the following relationships describing the operational behaviour of the beta distribution parameters \((A, p, q)\) may be obtained in a simple form:

\[
\lambda'_0 = \ln(\beta(p, q)) - \ln(A) \\
\lambda_1 = 1 - p \\
\lambda_2 = 1 - q
\]

(13a)-(13c)

The proposed non-linear Eq. (10) has no analytical solutions, and thus, it has to be resolved numerically. The numerical optimisation was carried out to achieve the best accordance between the calculation function – the PSD of granular material and the experimental dependence. In this paper, the algorithm with the minimum of the sum of the squares of the relative errors was exploited. A solution is satisfied with \(\varepsilon\) being the accepted error, which is taken \(10^{-5}\) in this paper. The numerical procedure was solved by using the Matlab software. The correlation coefficient was used to obtain the performance of the MEP approach. The definition of this coefficient has the following form:

\[
R^2 = 1 - \frac{\sigma_y^2}{\sigma'_y^2}
\]

(14)

where the standard deviation \(\sigma_y^2\) and \(\sigma'_y^2\) of the measured data may be calculated from the equations:

\[
\sigma_y^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1} 
\]

(15a)

\[
\sigma'_y^2 = \frac{\sum_{i=1}^{n} (y_i - y_{ei})^2}{n-2} 
\]

(15b)

It should be noticed that a larger value of the coefficient (Eq. (14)) indicates a better fitting between the proposed MEP approach and the obtained experimental data. The computed values of parameters \((\lambda'_0, \lambda_1, \lambda_2)\) of Eq.(11) and the values of the correlation coefficient are collected in Table 2.
Table 2. The computed parameters of Eq. (7) and the values of the correlation coefficient

<table>
<thead>
<tr>
<th>$N$ [rpm]</th>
<th>$t_i$ [min]</th>
<th>Computed parameters</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\lambda_0$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>13</td>
<td>0.7</td>
</tr>
<tr>
<td>120</td>
<td>12</td>
<td>12</td>
<td>0.8</td>
</tr>
<tr>
<td>240</td>
<td>10</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>9</td>
<td>0.45</td>
</tr>
<tr>
<td>75</td>
<td>7.2</td>
<td>7.2</td>
<td>0.5</td>
</tr>
<tr>
<td>135</td>
<td>7</td>
<td>7</td>
<td>0.55</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
<td>8</td>
<td>0.6</td>
</tr>
<tr>
<td>75</td>
<td>7.5</td>
<td>7.5</td>
<td>0.65</td>
</tr>
<tr>
<td>135</td>
<td>7</td>
<td>7</td>
<td>0.7</td>
</tr>
</tbody>
</table>

To demonstrate the suitability of the analysed MEP modelling, a comparison was made between the obtained predictions and the results based on the experimental investigations. The results of the MEP analysis for the mixing process of granular material with the grinding effect and the experimental measurements are presented in Fig. 4.
It can be seen that the scatter of the experimental values is closely connected with the solid line represented by the MEP approach. The parameters $(\lambda_0', \lambda_1, \lambda_2)$ may be treated as the variables describing the operational conditions for the mixing process of granular material with the grinding effect. Based on the above considerations, the functional form of kinetic relationship of the grinding process in the mixing of granular material is formulated as follows:

$$\lambda_0'(t; N) \lor \lambda_1(t; N) \lor \lambda_2(t; N) = p_t(N) \exp\left(-p_2(N)t\right)$$  \hspace{1cm} (16)$$

The set of values obtained from the analysis of experimental measurements (see Table 1) can be strictly approximated by means of the following relationships:

$$\lambda_0'(t; N) = (-0.062N + 13.59)\exp\left(-(-4 \cdot 10^{-6}N + 0.0018)t\right)$$ \hspace{1cm} (17a)$$

$$\lambda_1(t; N) = (-0.0007N + 0.61)\exp\left(-(-6 \cdot 10^{-8}N + 0.0014)t\right)$$ \hspace{1cm} (17b)$$

$$\lambda_2(t; N) = (-0.18N + 31)\exp\left(-(-1 \cdot 10^{-5}N + 0.002)t\right)$$ \hspace{1cm} (17c)$$

The plots of parameters $\lambda_i$ in function of rotational speed of multi-ribbon agitators and time duration of mixing process are shown in Fig. 5.

Figure 6 gives an overview result, in the form of the PSD of granular material by using the MEP approach for the experimental investigations. The first conclusion drawn from the inspection of this graph is that the proposed MEP approach fits the analysed experimental data very well. Therefore, this result suggests that the PSD for the mixing process with the grinding effect may be successfully approximated by means of the MEP approach.

Based on Eqs. (2) and (11), the shape parameters $(p, q)$ and the ratio $(A(t, N)/\beta p(t, N), q(t, N))$ may be determined. Simple transformations give the following equations describing the operational changes of the parameters of beta type distri-
bution. Notice that $A$, $p$ and $q$ depend on the speed rotation $N$ and the time duration of the mixing process with the grinding effect $t$:

\[
\ln\left( A(t; N) \right) - \ln\left( \beta\left( p(t; N), q(t; N) \right) \right) = - \lambda'_0(t; N) \quad (18a)
\]

\[
p(t; N) = 1 - \lambda_1(t; N) \quad (18b)
\]

\[
q(t; N) = 1 - \lambda_2(t; N) \quad (18c)
\]

Finally, Eq. (18a) can be rewritten in the following form:

\[
\frac{A(t; N)}{\beta\left( p(t; N), q(t; N) \right)} = \exp\left(-\lambda'_0(t; N)\right) \quad (19)
\]

It can be observed that the kinetic equation used to describe the mixing process of the grinding effect may be accurately defined by the following general relationship:

\[
f\left(t; x^*, N\right) = \exp\left(-\lambda'_0(t; N)\right) \left(x^*\right)^{-\lambda_1(t; N)} \left(1 - x^*\right)^{-\lambda_2(t; N)} \quad (20)
\]
A typical plot of Eq. (20) is given in Fig. 7. It can be seen that the scatter of the experimental results (marked points) corresponds to the proposed analytical approximation (marked solid line). The correlation is valid for the whole range of the granular particle sizes for the chosen rotational speed of multi-ribbon agitator (\(N = 2\) rpm), within the root mean square error equal to 0.0612. The proposed analytical description enables estimation of the condition of a granular mixture at the various sampling moments.

4. CONCLUSIONS

This experimental study presents the application of MEP approach for the description of PSD of mixing process of granular material with the grinding effect. This MEP type distribution is developed by modifying the beta type distribution that is derived from maximization of Shannon’s entropy based on the maximum entropy distribution. It is shown that the description of the mixing process with grinding effect may be realized by means of the proposed analysis. The proposed MEP correlation allows describing temporal changes of PSD. We have demonstrated that this approach can be successfully applied to describe the real functioning blender. This conclusion is supported by the following results:

- The MEP approach takes into account the operational variation of the mixing process of granular material with the grinding effect.
- The obtained PSD from the MEP approach adapts to variety of shapes and scales of the obtained histograms.
- The proposed MEP method describes with an acceptable degree of accuracy experimental results, constituting a useful tool for evaluating temporal changes of PSD of granular materials.
- An equation is proposed (Eq. (20)) to describe the grinding process of a mixed granular material based on the MEP method and its applicability has been for an experimental multi-ribbon mixer.

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SYMBOLS

\[ A \] – parameter of \( \beta \) distribution (Eq. (1))

\[ f(x) \] – particle size distribution

\[ H(x) \] – informational entropy, nit

\[ N \] – speed rotation of multi-ribbon agitator, rpm

\[ p \] – parameter of \( \beta \) distribution (Eq. (1))
$r(x)$ — functional forms of characterizing moments

$R$ — correlation coefficient

$q$ — parameter of $\beta$ distribution (Eq. (1))

$t$ — time, min

$x$ — particle size, mm

$x^*$ — relative particle size

$x_{\text{max}}$ — maximum particle size, mm

$y_{ci}$ — $i$th value of PSD computed form the correlation equation

$y_i$ — $i$th measured data of PSD during the experiment procedure

$\overline{y}$ — mean value of measured data

GREEK LETTERS

$\beta(p,q)$ — beta function

$\lambda, \lambda_m$ — Lagrange multipliers

$\sigma^2$ — standard deviation of the measured data

$\sigma^*_{yi}$ — standard deviation of the measured and computed data

ABBREVIATIONS

PDF — probability density function

PSD — probability size distribution

MEP — maximum entropy principle

REFERENCES

Mixing process of a granular material with the grinding effect


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ANALIZA PROCESU MIESZANIA MATERIAŁU ZIARNISTEGO Z EFEKTEM MIELENIA Z WYKORZYSTANIEM ZASADY MAKSYMALNEJ ENTROPII

Proces mieszania lub mielenia materiału ziarnistego pozostaje niezmiennie ważnym problemem badawczym z powodu występowania w wielu gałęziach przemysłu. Ważność tego procesu podkreśla dość znaczną liczbę publikacji naukowych, które poruszają przede wszystkim problematykę matematycznego modelowania procesu rozdrabniania substancji stałych. Modelowanie jednokrotnego rozdrabniania jest zazwyczaj oparte na podejściu energetycznym, które umożliwia określenie związku pomiędzy energią rozdrabniania a stopniem rozdrobnienia. Można również spotkać doniesienia dotyczące zastosowania termodynamicznej hipotezy rozdrabniania, polegającej na prognozowaniu składu granulometrycznego na podstawie rozkładów empirycznych. Popularne jest również podejście stochastyczne, oparte na bilansie populacji lub procesach Markowa. Bardziej szczegółowy opis matematyczny wyników doświadczalnych uzyskuje się na podstawie charakterystyk stosowanych powszechnie w teorii informacji.


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