Chapter 2

ECONOMIC OPTIMIZATION

QUESTIONS & ANSWERS

Q2.1  What is the difference between global and partial optimization?

Q2.1  ANSWER

The complexity of a completely integrated decision analysis approach—or global optimization—sometimes confines its use to major planning decisions. For many day-to-day operating decisions, managers often employ much less complicated partial optimization techniques. Partial optimization abstracts from the complexity of a completely integrated decision process by concentrating on more limited objectives within the firm’s various operating departments. For example, the marketing department is usually required to determine the price and advertising policy that will achieve some sales goal given the firm’s current product line and marketing budget. Alternatively, a production department might be expected to minimize the cost of a specified quantity of output at a stated quality level. In both instances, the fundamentals of economic analysis provide the basis for optimal managerial decisions.

Q2.2  Why are computer spreadsheets a popular means for expressing economic relations?

Q2.2  ANSWER

When tables of economic data are displayed electronically in the format of an accounting income statement or balance sheet, such tables are often referred to as spreadsheets. Microsoft Excel and other spreadsheet software programs are popular means for expressing economic relations because they incorporate methods for manipulating and analyzing economic data. When the underlying relation between economic data is very simple, tables and spreadsheets by themselves may be sufficient for analytical purposes. In other instances, a simple graph or visual representation of the data can provide valuable insight. With spreadsheet software, creating graphs is quick and easy. When the complex nature of economic relations requires that more sophisticated methods of expression be employed, spreadsheet formulas can be used to generate equations, or analytical expressions of functional relationships, that offer a very useful means for characterizing the connection among economic variables. Equations are frequently used to express both simple and complex economic relations. When the underlying relation among economic variables is uncomplicated, equations offer a useful compact means for data description. When underlying relations are complex, equations are helpful because they permit the powerful tools of mathematical and statistical analysis to be employed.
Q2.3  Describe the relation between totals and marginals, and explain why the total is maximized when the marginal is set equal to zero.

Q2.3  ANSWER

A total reflects the sum or whole of an important economic variable. The marginal is the change in the total for a one-unit expansion in the activity level. Just as there is this simple arithmetic relation between totals and marginals, so too there is a corresponding geometric relation. For example, the rise (or fall) in the total profit associated with a one-unit increase in output is marginal profit and the slope of the total profit curve at that point. The marginal concept is critical in managerial economics because the optimization process requires an analysis of change. Typically, fluctuations in independent variables are studied to learn what effect these changes have on the dependent variable. The purpose of this analysis is to locate the set of values for the independent, or decision, variables that will optimize a given objective function. The marginal value of a function will always equal zero at local extremes, maxima and minima, of a total value function. This fact enables one to analyze marginal relations in order to solve optimization problems. A profit function, for example, will be maximized when marginal profits equal zero, $M\pi = 0$, so long as total profit is falling as output expands beyond that point.

Q2.4  Why must a marginal curve always intersect the related average curve at either a maximum or a minimum point?

Q2.4  ANSWER

So long as the marginal value of a function is greater than the average value, the average must be rising. Similarly, if the marginal is less than the average, the average will be declining. At points where the average reaches an extreme (maxima or minima) and is neither rising nor falling, the marginal will equal the average.

Q2.5  Would you expect total revenue to be maximized at an output level that is typically greater or less than the profit-maximizing output level? Why?

Q2.5  ANSWER

Marginal revenue equals zero at the output level where total revenue is maximized. On the other hand, marginal revenue equals marginal cost at the output where profit is maximized. Given a typically downward sloping demand curve and positive marginal costs, it is reasonable to expect that the revenue maximizing output level (where $MR = 0$) will be greater than the profit maximizing output level (where $MR = MC > 0$).
Q2.6  Does the point of minimum long-run average costs always represent the optimal activity level?

Q2.6  ANSWER

No, the point of minimum long-run average costs, where MC = AC, simply indicates the point of lowest average production and/or distribution costs. Determination of the optimal activity level requires that both revenue (demand) and cost (supply) conditions be considered. For example, it would be inefficient to produce an average-cost minimizing level of output if such output could only be sold at such low prices that MR < MC. Similarly, production beyond the average-cost minimizing level of output can be justified so long as MR > MC.

Q2.7  Distinguish the incremental concept from the marginal concept.

Q2.7  ANSWER

Marginal cost refers to the increase in total costs following a single-unit increase in output. On the other hand, incremental cost refers to the increase in total costs due to a relevant managerial decision that may involve a multiple-unit expansion in output. Either cost concept can be relevant for pricing purposes, depending on output production and demand relations.

Q2.8  Economists have long argued that if you want to tax away excess profits without affecting allocative efficiency, you should use a lump-sum tax instead of an excise or sales tax. Use the concepts developed in the chapter to support this position.

Q2.8  ANSWER

Lump-sum taxes only affect total fixed costs. They are invariant with respect to the activity level of the firm. Thus, lump-sum taxes will not appear in the marginal revenue, marginal cost, or marginal profit functions of the firm. They cannot affect the determination of the optimal output level in the short run, as would a sales tax, an excise tax, or any such tax tied to the level of production. Of course, to the extent that lump-sum taxes reduce profits below a risk-adjusted normal rate of return, they too have the potential to affect the level of output. In the extreme, if a lump-sum tax reduced profits below the minimum required level, the firm’s very existence could be imperiled in the long run.

Q2.9  “It is often impossible to obtain precise information about the pattern of future revenues, costs, and interest rates. Therefore, the process of economic optimization is futile.” Discuss this statement.
Q2.9  ANSWER

A view of the process of economic optimization as futile, given the obvious uncertainty regarding the future pattern of economic activity, is plainly incorrect. Economic decisions concerning investment projects, for example, are made on the basis of expected rather than actual values. Indeed, decision making based upon expectations is necessary because it is impossible to learn future values before the fact. Importantly, the costs of information gathering, a key element of the process of forming accurate expectations, is explicitly incorporated into the optimization process through the impact of search on marginal costs. Far from futile, the process of economic optimization provides a practical guide for understanding the basis for managerial decision making.

Q2.10  ANSWER

In estimating regulatory benefits, the Environmental Protection Agency (EPA) assigns a value of $4.8 million to each life saved. What factors might the EPA consider in arriving at such a valuation? How would you respond to criticism directed at the EPA that life is precious and cannot be valued in dollar terms?

Q2.10  ANSWER

From an economic standpoint, the effectiveness of regulatory policy can and should be measured in terms of resulting costs and benefits. When clean-air standards result in a reduction of smog and other pollutants, important benefits are experienced in terms of improved public health and safety. When sickness is avoided, social benefits are measured in terms of reduced health care expenses, cutbacks in the number of sick days for affected workers, and so on. Placing appropriate values on the social benefits enjoyed when the general public simply feels better is much harder to accomplish, of course.

When deaths rates fall following an improvement in clean-air standards, for example, important economic and personal benefits are realized. In light of current interest rates and employment opportunities, an EPA estimate of $4.8 million per each life saved represents the agency’s present-value estimate of the dollar value derived from a typical person’s gainful economic activity. In other words, a life saved is “worth” $4.8 million in terms of preserved economic activity.

To be sure, adoption of such an approach is not to deny the sanctity of human life. It is merely a practical means to ensure that government, business, and the public consider both large and small social benefits when judging the cost effectiveness of health and safety regulation.
SELF-TEST PROBLEMS AND SOLUTIONS

ST2.1 Profit versus Revenue Maximization. Presto Products, Inc., manufactures small electrical appliances and has recently introduced an innovative new dessert maker for frozen yogurt and tofu that has the clear potential to offset the weak pricing and sluggish volume growth experienced during recent periods.

Monthly demand and cost relations for Presto’s frozen dessert maker are as follows:

\[ P = 60 - 0.005Q \]
\[ TC = 100,000 + 5Q + 0.0005Q^2 \]

\[ MR = \frac{\partial TR}{\partial Q} = 60 - 0.01Q \]
\[ MC = \frac{\partial TC}{\partial Q} = 5 + 0.001Q \]

A. Set up a table or spreadsheet for Presto output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), total profit (π), and marginal profit (Mπ). Establish a range for Q from 0 to 10,000 in increments of 1,000 (i.e., 0, 1,000, 2,000, ..., 10,000).

B. Using the Presto table or spreadsheet, create a graph with TR, TC, and π as dependent variables, and units of output (Q) as the independent variable. At what price/output combination is total profit maximized? Why? At what price/output combination is total revenue maximized? Why?

C. Determine these profit-maximizing and revenue-maximizing price/output combinations analytically. In other words, use Presto’s profit and revenue equations to confirm your answers to part B.

D. Compare the profit-maximizing and revenue-maximizing price/output combinations, and discuss any differences. When will short-run revenue maximization lead to long-run profit maximization?

ST2.1 SOLUTION

A. A table or spreadsheet for Presto output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), total profit (π), and marginal profit (Mπ) appears as follows:
B. Using the Presto table or spreadsheet, a graph with TR, TC, and $\pi$ as dependent variables, and units of output (Q) as the independent variable appears as follows:

The price/output combination at which total profit is maximized is $P = $35 and $Q = 5,000$ units. At that point, $MR = MC$ and total profit is maximized at $\$37,500$. 

---

<table>
<thead>
<tr>
<th>Units</th>
<th>Price</th>
<th>Total Revenue</th>
<th>Marginal Revenue</th>
<th>Total Cost</th>
<th>Marginal Cost</th>
<th>Total Profit</th>
<th>Marginal Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$60</td>
<td>$0</td>
<td>$60</td>
<td>$100,000</td>
<td>$5</td>
<td>($100,000)</td>
<td>$55</td>
</tr>
<tr>
<td>1,000</td>
<td>55</td>
<td>55,000</td>
<td>50</td>
<td>105,500</td>
<td>6</td>
<td>(50,500)</td>
<td>44</td>
</tr>
<tr>
<td>2,000</td>
<td>50</td>
<td>100,000</td>
<td>40</td>
<td>112,000</td>
<td>7</td>
<td>(12,000)</td>
<td>33</td>
</tr>
<tr>
<td>3,000</td>
<td>45</td>
<td>135,000</td>
<td>30</td>
<td>119,500</td>
<td>8</td>
<td>15,500</td>
<td>22</td>
</tr>
<tr>
<td>4,000</td>
<td>40</td>
<td>160,000</td>
<td>20</td>
<td>128,000</td>
<td>9</td>
<td>32,000</td>
<td>11</td>
</tr>
<tr>
<td>5,000</td>
<td>35</td>
<td>175,000</td>
<td>10</td>
<td>137,500</td>
<td>10</td>
<td>37,500</td>
<td>0</td>
</tr>
<tr>
<td>6,000</td>
<td>30</td>
<td>180,000</td>
<td>0</td>
<td>148,000</td>
<td>11</td>
<td>32,000</td>
<td>(11)</td>
</tr>
<tr>
<td>7,000</td>
<td>25</td>
<td>175,000</td>
<td>(10)</td>
<td>159,500</td>
<td>12</td>
<td>15,500</td>
<td>(22)</td>
</tr>
<tr>
<td>8,000</td>
<td>20</td>
<td>160,000</td>
<td>(20)</td>
<td>172,000</td>
<td>13</td>
<td>(12,000)</td>
<td>(33)</td>
</tr>
<tr>
<td>9,000</td>
<td>15</td>
<td>135,000</td>
<td>(30)</td>
<td>185,500</td>
<td>14</td>
<td>(50,500)</td>
<td>(44)</td>
</tr>
<tr>
<td>10,000</td>
<td>10</td>
<td>100,000</td>
<td>(40)</td>
<td>200,000</td>
<td>15</td>
<td>(100,000)</td>
<td>(55)</td>
</tr>
</tbody>
</table>
The price/output combination at which total revenue is maximized is $P = $30 and $Q = 6,000$ units. At that point, $MR = 0$ and total revenue is maximized at $180,000$.

C. To find the profit-maximizing output level analytically, set $MR = MC$, or set $M\pi = 0$, and solve for $Q$. Because

$$MR = MC$$

$$60 - 0.01Q = 5 + 0.001Q$$

$$0.011Q = 55$$

$$Q = 5,000$$

At $Q = 5,000$,

$$P = 60 - 0.005(5,000)$$

$$= 35$$

$$\pi = -100,000 + 55(5,000) - 0.0055(5,000^2)$$

$$= 37,500$$

(Note: $\frac{\partial^2 \pi}{\partial Q^2} < 0$. This is a profit maximum because total profit is falling for $Q > 5,000$.)

To find the revenue-maximizing output level, set $MR = 0$, and solve for $Q$. Thus,

$$MR = 60 - 0.01Q = 0$$

$$0.01Q = 60$$

$$Q = 6,000$$

At $Q = 6,000$,

$$P = 60 - 0.005(6,000)$$

$$= 30$$
\[ \pi = TR - TC \]
\[ = (\$60 - 0.005Q)Q - \$100,000 - 5Q - 0.0005Q^2 \]
\[ = -\$100,000 + 55Q - 0.0055Q^2 \]
\[ = -\$100,000 + 55(6,000) - 0.0055(6,000^2) \]
\[ = \$32,000 \]

(Note: \( \partial^2 TR/\partial Q^2 < 0 \), and this is a revenue maximum because total revenue is decreasing for output beyond \( Q > 6,000 \).)

D. Given downward sloping demand and marginal revenue curves, and positive marginal costs, the profit-maximizing price/output combination is always at a higher price and lower production level than the revenue-maximizing price/output combination. This stems from the fact that profit is maximized when \( MR = MC \), whereas revenue is maximized when \( MR = 0 \). It follows that profits and revenue are only maximized at the same price/output combination in the unlikely event that \( MC = 0 \).

In pursuing a short-run revenue rather than profit-maximizing strategy, Presto can expect to gain a number of important advantages, including enhanced product awareness among consumers, increased customer loyalty, potential economies of scale in marketing and promotion, and possible limitations in competitor entry and growth. To be consistent with long-run profit maximization, these advantages of short-run revenue maximization must be at least worth Presto’s short-run sacrifice of \$5,500 (= \$37,500 - \$32,000) in monthly profits.

**ST2.2 Average Cost-Minimization.** Pharmed Caplets, Inc., is an international manufacturer of bulk antibiotics for the animal feed market. Dr. Indiana Jones, head of marketing and research, seeks your advice on an appropriate pricing strategy for Pharmed Caplets, an antibiotic for sale to the veterinarian and feedlot-operator market. This product has been successfully launched during the past few months in a number of test markets, and reliable data are now available for the first time.

The marketing and accounting departments have provided you with the following monthly total revenue and total cost information:

\[ TR = 900Q - 0.1Q^2 \]
\[ TC = 36,000 + 200Q + 0.4Q^2 \]

\[ MR = \partial TR/\partial Q = 900 - 0.2Q \]
\[ MC = \partial TC/\partial Q = 200 + 0.8Q \]

A. Set up a table or spreadsheet for Pharmed Caplets output (\( Q \)), price (\( P \)), total revenue (\( TR \)), marginal revenue (\( MR \)), total cost (\( TC \)), marginal cost (\( MC \)),
average cost (AC), total profit ($\pi$), and marginal profit ($M\pi$). Establish a range for $Q$ from 0 to 1,000 in increments of 100 (i.e., 0, 100, 200, ..., 1,000).

**B.** Using the Pharmed Caplets table or spreadsheet, create a graph with AC and MC as dependent variables and units of output ($Q$) as the independent variable. At what price/output combination is total profit maximized? Why? At what price/output combination is average cost minimized? Why?

**C.** Determine these profit-maximizing and average-cost minimizing price/output combinations analytically. In other words, use Pharmed Caplets’ revenue and cost equations to confirm your answers to part B.

**D.** Compare the profit-maximizing and average-cost minimizing price/output combinations, and discuss any differences. When will average-cost minimization lead to long-run profit maximization?

**ST2.2 SOLUTION**

**A.** A table or spreadsheet for Pharmed Caplets output ($Q$), price ($P$), total revenue ($TR$), marginal revenue ($MR$), total cost ($TC$), marginal cost ($MC$), average cost ($AC$), total profit ($\pi$), and marginal profit ($M\pi$) appears as follows:

<table>
<thead>
<tr>
<th>Units</th>
<th>Price</th>
<th>Total Revenue</th>
<th>Marginal Revenue</th>
<th>Total Cost</th>
<th>Marginal Cost</th>
<th>Average Cost</th>
<th>Total Profit</th>
<th>Marginal Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$900</td>
<td>$0</td>
<td>$900</td>
<td>$36,000</td>
<td>$200</td>
<td>---</td>
<td>($36,000)</td>
<td>$700</td>
</tr>
<tr>
<td>100</td>
<td>$890</td>
<td>89,000</td>
<td>$880</td>
<td>$60,000</td>
<td>$280</td>
<td>600.00</td>
<td>29,000</td>
<td>600</td>
</tr>
<tr>
<td>200</td>
<td>$880</td>
<td>176,000</td>
<td>$860</td>
<td>$92,000</td>
<td>$360</td>
<td>460.00</td>
<td>84,000</td>
<td>500</td>
</tr>
<tr>
<td>300</td>
<td>$870</td>
<td>261,000</td>
<td>$840</td>
<td>$132,000</td>
<td>$440</td>
<td>440.00</td>
<td>129,000</td>
<td>400</td>
</tr>
<tr>
<td>400</td>
<td>$860</td>
<td>344,000</td>
<td>$820</td>
<td>$180,000</td>
<td>$520</td>
<td>450.00</td>
<td>164,000</td>
<td>300</td>
</tr>
<tr>
<td>500</td>
<td>$850</td>
<td>425,000</td>
<td>$800</td>
<td>$236,000</td>
<td>$600</td>
<td>472.00</td>
<td>189,000</td>
<td>200</td>
</tr>
<tr>
<td>600</td>
<td>$840</td>
<td>504,000</td>
<td>$780</td>
<td>$300,000</td>
<td>$680</td>
<td>500.00</td>
<td>204,000</td>
<td>100</td>
</tr>
<tr>
<td>700</td>
<td>$830</td>
<td>581,000</td>
<td>$760</td>
<td>$372,000</td>
<td>$760</td>
<td>531.43</td>
<td>209,000</td>
<td>0</td>
</tr>
<tr>
<td>800</td>
<td>$820</td>
<td>656,000</td>
<td>$740</td>
<td>$452,000</td>
<td>$840</td>
<td>565.00</td>
<td>204,000</td>
<td>(100)</td>
</tr>
<tr>
<td>900</td>
<td>$810</td>
<td>729,000</td>
<td>$720</td>
<td>$540,000</td>
<td>$920</td>
<td>600.00</td>
<td>189,000</td>
<td>(200)</td>
</tr>
<tr>
<td>1,000</td>
<td>$800</td>
<td>800,000</td>
<td>$700</td>
<td>$636,000</td>
<td>$1,000</td>
<td>636.00</td>
<td>164,000</td>
<td>(300)</td>
</tr>
</tbody>
</table>

**B.** Using the Pharmed Caplets table or spreadsheet, a graph with AC, and MC as dependent variables and units of output ($Q$) as the independent variable appears as follows:
The price/output combination at which total profit is maximized is \( P = 830 \) and \( Q = 700 \) units. At that point, \( MR = MC \) and total profit is maximized at \$209,000.

The price/output combination at which average cost is minimized is \( P = 870 \) and \( Q = 300 \) units. At that point, \( MC = AC = 440 \).

C. To find the profit-maximizing output level analytically, set \( MR = MC \), or set \( M\pi = 0 \), and solve for \( Q \). Because

\[
MR = MC
\]

\[
900 - 0.2Q = 200 + 0.8Q
\]

\[
Q = 700
\]

At \( Q = 700 \),

\[
P = TR/Q
\]

\[
= (900Q - 0.1Q^2)/Q
\]

\[
= 900 - 0.1(700)
\]

\[
= 830
\]
\[ \pi = TR - TC \]
\[ = $900Q - 0.1Q^2 - $36,000 - $200Q - 0.4Q^2 \]
\[ = -$36,000 + $700(700) - 0.5(700^2) \]
\[ = $209,000 \]

(Note: \[ \frac{\partial^2 \pi}{\partial Q^2} < 0 \], and this is a profit maximum because profits are falling for \( Q > 700 \).)

To find the average-cost minimizing output level, set \( MC = AC \), and solve for \( Q \). Because

\[ AC = TC/Q \]
\[ = ($36,000 + $200Q + 0.4Q^2)/Q \]
\[ = $36,000Q^{-1} + $200 + 0.4Q, \]

it follows that:

\[ MC = AC \]
\[ $200 + 0.8Q = $36,000Q^{-1} + $200 + 0.4Q \]
\[ 0.4Q = 36,000Q^{-1} \]
\[ 0.4Q^2 = 36,000 \]
\[ Q^2 = 36,000/0.4 \]
\[ Q^2 = 90,000 \]
\[ Q = 300 \]

At \( Q = 300 \),

\[ P = $900 - 0.1(300) \]
\[ = $870 \]
\[ \pi = -36,000 + 700(300) - 0.5(300^2) \]
\[ = 129,000 \]

(Note: \( \frac{\partial^2 AC}{\partial Q^2} > 0 \), and this is an average-cost minimum because average cost is rising for \( Q > 300 \).)

D. Given downward sloping demand and marginal revenue curves and a U-shaped, or quadratic, AC function, the profit-maximizing price/output combination will often be at a different price and production level than the average-cost minimizing price-output combination. This stems from the fact that profit is maximized when MR = MC, whereas average cost is minimized when MC = AC. Profits are maximized at the same price/output combination as where average costs are minimized in the unlikely event that MR = MC and MC = AC and, therefore, MR = MC = AC.

It is often true that the profit-maximizing output level differs from the average cost-minimizing activity level. In this instance, expansion beyond \( Q = 300 \), the average cost-minimizing activity level, can be justified because the added gain in revenue more than compensates for the added costs. Note that total costs rise by \( $240,000 \), from \( $132,000 \) to \( $372,000 \) as output expands from \( Q = 300 \) to \( Q = 700 \), as average cost rises from \( $440 \) to \( $531.43 \). Nevertheless, profits rise by \( $80,000 \), from \( $129,000 \) to \( $209,000 \), because total revenue rises by \( $320,000 \), from \( $261,000 \) to \( $581,000 \). The profit-maximizing activity level can be less than, greater than, or equal to the average-cost minimizing activity level depending on the shape of relevant demand and cost relations.

**PROBLEMS & SOLUTIONS**

**P2.1 Graph Analysis**

A. Given the output (\( Q \)) and price (\( P \)) data in the following table, calculate the related total revenue (TR), marginal revenue (MR), and average revenue (AR) figures:
<table>
<thead>
<tr>
<th>Q</th>
<th>P</th>
<th>TR</th>
<th>MR</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$10</td>
<td>$0</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>9</td>
<td>$9</td>
<td>$9</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>16</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>21</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>24</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>24</td>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>21</td>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>16</td>
<td>-5</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>9</td>
<td>-7</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>-9</td>
<td>0</td>
</tr>
</tbody>
</table>

B. Graph these data using “dollars” on the vertical axis and “quantity” on the horizontal axis. At what output level is revenue maximized?

C. Why is marginal revenue less than average revenue at each price level?

**P2.1 SOLUTION**

A. 

<table>
<thead>
<tr>
<th>Q</th>
<th>P</th>
<th>TR=P×Q</th>
<th>MR=δTR/δQ</th>
<th>AR = TR/Q = P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$10</td>
<td>$0</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>9</td>
<td>$9</td>
<td>$9</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>16</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>21</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>24</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>25</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>24</td>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>21</td>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>16</td>
<td>-5</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>9</td>
<td>-7</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>-9</td>
<td>0</td>
</tr>
</tbody>
</table>

B. Revenue is maximized at an output level of Q = 5, where MR = 0.
C. At every price level, price must be cut by $1 in order to increase sales by an additional unit. This means that the “benefit” of added sales from new customers is only gained at the “cost” of some loss in revenue from current customers. Thus, the net increase in revenue from added sales is always less than the change in gross revenue. Therefore, marginal revenue is always less than average revenue (or price).

P2.2 A. Fill in the missing data for price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), profit (π), and marginal profit (Mπ) in the following table:
Chapter 2 Economic Optimization

<table>
<thead>
<tr>
<th>Q</th>
<th>P</th>
<th>TR</th>
<th>MR</th>
<th>TC</th>
<th>MC</th>
<th>π</th>
<th>Mπ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$160</td>
<td>$0</td>
<td>$---</td>
<td>$0</td>
<td>$---</td>
<td>$0</td>
<td>$---</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>25</td>
<td>25</td>
<td>125</td>
<td>125</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
<td>55</td>
<td>30</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>390</td>
<td>35</td>
<td>300</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>130</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>550</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>175</td>
</tr>
<tr>
<td>6</td>
<td>600</td>
<td>50</td>
<td>55</td>
<td>370</td>
<td>370</td>
<td>370</td>
<td>370</td>
</tr>
<tr>
<td>7</td>
<td>630</td>
<td>290</td>
<td>60</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>640</td>
<td>355</td>
<td>285</td>
<td>285</td>
<td>285</td>
<td>285</td>
</tr>
<tr>
<td>9</td>
<td>75</td>
<td>-85</td>
<td>75</td>
<td>-85</td>
<td>-85</td>
<td>-85</td>
<td>-85</td>
</tr>
<tr>
<td>10</td>
<td>600</td>
<td>525</td>
<td>525</td>
<td>525</td>
<td>525</td>
<td>525</td>
<td>525</td>
</tr>
</tbody>
</table>

**B.** At what output level is profit maximized?

**C.** At what output level is revenue maximized?

**D.** Discuss any differences in your answers to parts B and C.

**P2.2 SOLUTION**

**A.**

<table>
<thead>
<tr>
<th>Q</th>
<th>P</th>
<th>TR=P×Q</th>
<th>MR=∂TR/∂Q</th>
<th>TC</th>
<th>MC</th>
<th>π=TR-TC</th>
<th>Mπ=∂π/∂Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$160</td>
<td>$0</td>
<td>--</td>
<td>$0</td>
<td>$0</td>
<td>$0</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>150</td>
<td>$150</td>
<td>26</td>
<td>$25</td>
<td>125</td>
<td>$125</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
<td>280</td>
<td>130</td>
<td>55</td>
<td>30</td>
<td>225</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>130</td>
<td>390</td>
<td>110</td>
<td>90</td>
<td>35</td>
<td>300</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>480</td>
<td>90</td>
<td>130</td>
<td>40</td>
<td>350</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>550</td>
<td>70</td>
<td>175</td>
<td>45</td>
<td>375</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>600</td>
<td>50</td>
<td>230</td>
<td>55</td>
<td>370</td>
<td>-5</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>630</td>
<td>30</td>
<td>290</td>
<td>60</td>
<td>340</td>
<td>-30</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>640</td>
<td>10</td>
<td>355</td>
<td>65</td>
<td>285</td>
<td>-55</td>
</tr>
<tr>
<td>9</td>
<td>70</td>
<td>630</td>
<td>-10</td>
<td>430</td>
<td>75</td>
<td>200</td>
<td>-85</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>600</td>
<td>-30</td>
<td>525</td>
<td>95</td>
<td>75</td>
<td>-125</td>
</tr>
</tbody>
</table>

**B.** Profit increases so long as MR > MC and Mπ > 0. In this problem, profit is maximized at Q = 5 where π = $375 (and TR = $550).
C. Total Revenue increases so long as MR > 0. In this problem, revenue is maximized at Q = 8 where TR = $640 (and π = $285).

D. Given a downward sloping demand curve and MC > 0, as is typically the case, profits will be maximized at an output level that is less than the revenue-maximizing level. Revenue maximization requires lower prices and greater output than would be true with profit maximization.

The potential long-run advantage of a revenue-maximizing strategy is that it might generate rapid market expansion and long-run benefits in terms of customer loyalty and future unit-cost reductions. The cost is, of course, measured in terms of lost profits in the short-run (here the loss is $90 in profits).

**P2.3 Marginal Analysis.** Characterize each of the following statements as true or false, and explain your answer.

A. If marginal revenue is less than average revenue, the demand curve will be downward sloping.

B. Profits will be maximized when total revenue equals total cost.

C. Given a downward-sloping demand curve and positive marginal costs, profit-maximizing firms will always sell less output at higher prices than will revenue-maximizing firms.

D. Marginal cost must be falling for average cost to decline as output expands.

E. Marginal profit is the difference between marginal revenue and marginal cost and will always equal zero at the profit-maximizing activity level.

**P2.3 SOLUTION**

A. True. The demand curve is the average-revenue curve. Since average revenue is falling along a downward sloping demand curve, marginal revenue is less than average revenue.

B. False. Profits are maximized when marginal revenue equals marginal cost. Profits equal zero at the breakeven point where total revenue equals total cost.

C. True. Profit maximization involves setting marginal revenue equal to marginal cost. Revenue maximization involves setting marginal revenue equal to zero. Given a downward-sloping demand curve and positive marginal costs, revenue-maximizing firms will charge lower prices and offer greater quantities of output than will firms that seek to maximize profits.
D. False. Average cost will fall as output expands so long as marginal cost is simply less than average cost. If this condition is met, average cost will decline whether marginal costs are falling, rising, or constant.

E. True. Marginal profit equals marginal revenue minus marginal cost and will equal zero at the profit-maximizing activity level.

P2.4 Marginal Analysis: Tables. Sarah Berra is a regional sales representative for Dental Laboratories, Inc. Berra sells alloy products created from gold, silver, platinum, and other precious metals to several dental laboratories in Maine, New Hampshire, and Vermont. Berra’s goal is to maximize her total monthly commission income, which is figured at 10% of gross sales. In reviewing her monthly experience over the past year, Berra found the following relations between days spent in each state and monthly sales generated:

<table>
<thead>
<tr>
<th>Days</th>
<th>Maine Gross Sales</th>
<th>Days</th>
<th>New Hampshire Gross Sales</th>
<th>Days</th>
<th>Vermont Gross Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4,000</td>
<td>0</td>
<td>$0</td>
<td>0</td>
<td>$2,500</td>
</tr>
<tr>
<td>1</td>
<td>10,000</td>
<td>1</td>
<td>3,500</td>
<td>1</td>
<td>5,000</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>2</td>
<td>6,500</td>
<td>2</td>
<td>7,000</td>
</tr>
<tr>
<td>3</td>
<td>19,000</td>
<td>3</td>
<td>9,000</td>
<td>3</td>
<td>8,500</td>
</tr>
<tr>
<td>4</td>
<td>22,000</td>
<td>4</td>
<td>10,500</td>
<td>4</td>
<td>9,500</td>
</tr>
<tr>
<td>5</td>
<td>24,000</td>
<td>5</td>
<td>11,500</td>
<td>5</td>
<td>10,000</td>
</tr>
<tr>
<td>6</td>
<td>25,000</td>
<td>6</td>
<td>12,000</td>
<td>6</td>
<td>10,000</td>
</tr>
<tr>
<td>7</td>
<td>25,000</td>
<td>7</td>
<td>12,500</td>
<td>7</td>
<td>10,000</td>
</tr>
</tbody>
</table>

A. Construct a table showing Berra’s marginal sales per day in each state.

B. If administrative duties limit Berra to only ten selling days per month, how should she spend them?

C. Calculate Berra’s maximum monthly commission income.
P2.4 SOLUTION

A.

<table>
<thead>
<tr>
<th>Days</th>
<th>Maine Marginal Sales</th>
<th>New Hampshire Marginal Sales</th>
<th>Vermont Marginal Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>$6,000</td>
<td>$3,500</td>
<td>$2,500</td>
</tr>
<tr>
<td>2</td>
<td>5,000</td>
<td>3,000</td>
<td>2,000</td>
</tr>
<tr>
<td>3</td>
<td>4,000</td>
<td>2,500</td>
<td>1,500</td>
</tr>
<tr>
<td>4</td>
<td>3,000</td>
<td>1,500</td>
<td>1,000</td>
</tr>
<tr>
<td>5</td>
<td>2,000</td>
<td>1,000</td>
<td>500</td>
</tr>
<tr>
<td>6</td>
<td>1,000</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>750</td>
<td>0</td>
</tr>
</tbody>
</table>

B. The maximum commission income is earned by allocating selling days on the basis of obtaining the largest marginal sales for each additional day of selling activity. Using the data in part A, we see that five days should be spent in Maine, three days in New Hampshire, and two days should be spent in Vermont.

C. Given this time allocation, Berra’s maximum commission income is

<table>
<thead>
<tr>
<th>State</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maine (5 days)</td>
<td>$24,000</td>
</tr>
<tr>
<td>New Hampshire (3 days)</td>
<td>9,000</td>
</tr>
<tr>
<td>Vermont (2 days)</td>
<td>7,000</td>
</tr>
<tr>
<td>Total</td>
<td>$40,000</td>
</tr>
</tbody>
</table>

\[ \times \text{Commission rate} \times 0.1 \]

\[ \text{$4,000 per month} \]

P2.5 Marginal Analysis: Tables. Climate Control Devices, Inc., estimates that sales of defective thermostats cost the firm an average of $25 each for replacement or repair. An independent engineering consultant has recommended hiring quality control inspectors so that defective thermostats can be identified and corrected before shipping. The following schedule shows the expected relation between the number of quality control inspectors and the thermostat failure rate, defined in terms of the percentage of total shipments that prove to be defective.
The firm expects to ship 250,000 thermostats during the coming year, and quality control inspectors each command a salary of $30,000 per year.

A. Construct a table showing the marginal failure reduction (in units) and the dollar value of these reductions for each inspector hired.

<table>
<thead>
<tr>
<th>Number of Quality Control Inspectors</th>
<th>Thermostat Failure Rate (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.0</td>
</tr>
<tr>
<td>1</td>
<td>4.0</td>
</tr>
<tr>
<td>2</td>
<td>3.2</td>
</tr>
<tr>
<td>3</td>
<td>2.6</td>
</tr>
<tr>
<td>4</td>
<td>2.2</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

B. How many inspectors should the firm hire?

C. How many inspectors would be hired if additional indirect costs (lost customer goodwill and so on) were to average 30% of direct replacement or repair costs?

P2.5 SOLUTION

A.

<table>
<thead>
<tr>
<th>Inspectors (1)</th>
<th>Failure Rate (2)</th>
<th>Failures (=250,000×(2))</th>
<th>Marginal Failure Reduction (4)</th>
<th>Marginal Value (5) = $25×(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.050</td>
<td>12,500</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>0.040</td>
<td>10,000</td>
<td>2,500</td>
<td>$62,500</td>
</tr>
<tr>
<td>2</td>
<td>0.032</td>
<td>8,000</td>
<td>2,000</td>
<td>50,000</td>
</tr>
<tr>
<td>3</td>
<td>0.026</td>
<td>6,500</td>
<td>1,500</td>
<td>37,500</td>
</tr>
<tr>
<td>4</td>
<td>0.022</td>
<td>5,500</td>
<td>1,000</td>
<td>25,000</td>
</tr>
<tr>
<td>5</td>
<td>0.020</td>
<td>5,000</td>
<td>500</td>
<td>12,500</td>
</tr>
</tbody>
</table>

B. I = 3. With a $30,000 inspector salary, the firm will enjoy a net marginal return of $7,500 (= $37,500 - $30,000) from hiring a third inspector. Hiring a fourth inspector would result in a marginal loss of $5,000 (= $25,000 - $30,000).

C. I = 4. If additional indirect costs total 30% of direct replacement costs, the marginal value of inspectors (column 5) would rise by 30%. Under these circumstances, the marginal value of a fourth inspector would rise from $25,000.
to $32,500 (= 1.3 \times 25,000), and hiring four inspectors could be justified since doing so would increase profits by $2,500 (= $32,500 - $30,000).

**P2.6 Profit Maximization: Equations.** Rochester Instruments, Inc., operates in the highly competitive electronics industry. Prices for its RII-X control switches are stable at $50 each. This means that \( P = MR = $50 \) in this market. Engineering estimates indicate that relevant total and marginal cost relations for the RII-X model are:

\[
TC = $78,000 + 18Q + 0.002Q^2 \\
MC = \frac{\partial TC}{\partial Q} = 18 + 0.004Q
\]

**A.** Calculate the output level that will maximize RII-X profit.

**B.** Calculate this maximum profit.

**P2.6 SOLUTION**

**A.** To find the profit-maximizing level of output, set \( MR = MC \) and solve for \( Q \):

\[
MR = MC \\
$50 = 18 + 0.004Q \\
0.004Q = 32 \\
Q = 8,000
\]

*(Note: \( \frac{\partial^2 \pi}{\partial Q^2} < 0 \), and this is a profit maximum because profits are decreasing for \( Q > 8,000 \).)*

**B.** The total revenue function for Rochester is:

\[
TR = P \times Q = $50Q
\]

Then, total profit is:

\[
\pi = TR - TC \\
= $50Q - 78,000 - 18Q - 0.002Q^2 \\
= -$0.002Q^2 + 32Q - 78,000
\]
Chapter 2  Economic Optimization  

P2.7  Profit Maximization: Equations. 21st Century Insurance offers mail-order automobile insurance to preferred-risk drivers in the Los Angeles area. The company is the low-cost provider of insurance in this market but doesn’t believe its $750 annual premium can be raised for competitive reasons. Its rates are expected to remain stable during coming periods; hence, \( P = MR = $750 \). Total and marginal cost relations for the company are as follows:

\[
TC = $2,500,000 + $500Q + $0.005Q^2 \\
MC = \frac{\partial TC}{\partial Q} = $500 + $0.01Q
\]

A. Calculate the profit-maximizing activity level.

B. Calculate the company’s optimal profit and return-on-sales levels.

P2.7  SOLUTION

A. Set \( MR = MC \) and solve for \( Q \) to find the profit-maximizing activity level:

\[
MR = MC \\
\$750 = $500 + $0.01Q \\
0.01Q = $250 \\
Q = 25,000
\]

(Note: \( \frac{\partial^2 \pi}{\partial Q^2} < 0 \), and this is a profit maximum because profits are decreasing for \( Q > 25,000 \).)

B. The total revenue function for 21st Century Insurance is:

\[
TR = P \times Q = $750Q
\]
Then, total profit is:

\[ \pi = TR - TC \]

\[ = 750Q - 2,500,000 - 500Q - 0.005Q^2 \]

\[ = 750(25,000) - 2,500,000 - 500(25,000) - 0.005(25,000)^2 \]

\[ = 625,000 \]

**Return on Sales** = \[\frac{\pi}{TR} \]

\[ = \frac{625,000}{18,750,000} \]

\[ = 0.033 \text{ or } 3.3\% \]

**P2.8 Not-for-Profit Analysis.** The Denver Athlete’s Club (DAC) is a private, not-for-profit athletic club located in Denver, Colorado. DAC currently has 3,500 members but is planning on a membership drive to increase this number significantly. An important issue facing Jessica Nicholson, DAC’s administrative director, is the determination of an appropriate membership level. In order to efficiently employ scarce DAC resources, the board of directors has instructed Nicholson to maximize DAC’s operating surplus, defined as revenues minus operating costs. They have also asked Nicholson to determine the effects of a proposed agreement between DAC and a neighboring club with outdoor recreation and swimming pool facilities. Plan A involves paying the neighboring club $100 per DAC member. Plan B involves payment of a fixed fee of $400,000 per year. Finally, the board has determined that the membership fee for the coming year will remain constant at $2,500 per member irrespective of the number of new members added and whether plan A or plan B is adopted.

In the calculations for determining an optimal membership level, Nicholson regards price as fixed; therefore, \( P = MR = 2,500 \). Before considering the effects of any agreement with the neighboring club, Nicholson projects total and marginal cost relations during the coming year to be as follows:

\[ TC = 3,500,000 + 500Q + 0.25Q^2 \]

\[ MC = \frac{\partial TC}{\partial Q} = 500 + 0.5Q \]

where \( Q \) is the number of DAC members.
A. Before considering the effects of the proposed agreement with the neighboring club, calculate DAC’s optimal membership and operating surplus levels.

B. Calculate these levels under plan A.

C. Calculate these levels under plan B.

P2.8 SOLUTION

A. Set MR = MC and solve for Q to find the operating surplus (profit)-maximizing activity level:

MR = MC

$2,500 = $500 + $0.5Q

0.5Q = 2,000

Q = 4,000

Surplus = P × Q - TC

= $2,500(4,000) - $3,500,000 - $500(4,000)
- $0.25(4,000^2)

= $500,000

(Note: $\frac{\partial^2 \pi}{\partial Q^2} < 0$, and this is a profit maximum because surplus is decreasing for Q > 4,000.)

B. When operating costs increase by $100 per member, the marginal cost function and optimal activity level are both affected. Under plan A we set MR = MC + $100, and solve for Q to find the new operating surplus (profit)-maximizing activity level.

MR = MC + $100

$2,500 = $500 + $0.5Q + $100

0.5Q = 1,900

Q = 3,800
Surplus = P × Q - TC - Plan A cost

= $2,500(3,800) - $3,500,000 - $500(3,800)
- $0.25(3,800^2) - $100(3,800)

= $110,000

C. When operating costs increase by a flat $400,000 per year, the marginal cost function and operating surplus (profit)-maximizing activity level are unaffected. As in part A, Q = 4,000.

The new operating surplus (profit) level is:

Surplus = PQ - TC - Plan B cost

= $500,000 - $400,000

= $100,000

Here, the DAC would be slightly better off under plan A. In general, a fixed-sum increase in costs will decrease the operating surplus (profit) by a like amount, but have no influence on price nor activity levels in the short-run. In the long run, however, both price and activity levels will be affected if cost increases depress the operating surplus (profit) below a normal (or required) rate of return.

P2.9 Revenue Maximization. Desktop Publishing Software, Inc., develops and markets software packages for business computers. Although sales have grown rapidly during recent years, the company’s management fears that a recent onslaught of new competitors may severely retard future growth opportunities. Therefore, it believes that the time has come to “get big or get out.”

The marketing and accounting departments have provided management with the following monthly demand and cost information:

\[ P = 1000 - 1Q \quad TC = 50000 + 100Q \]

\[ MR = \frac{\partial TR}{\partial Q} = 1000 - 2Q \quad MC = \frac{\partial TC}{\partial Q} = 100 \]

A. Calculate monthly quantity, price, and profit at the short-run revenue-maximizing output level.

B. Calculate these same values for the short-run profit-maximizing level of output.
C. When would short-run revenue maximization lead to long-run profit maximization?

P2.9 SOLUTION

A. To find the revenue-maximizing output level, set MR = 0 and solve for Q. Thus,

\[
MR = 1,000 - 2Q = 0
\]

\[
2Q = 1,000
\]

\[
Q = 500
\]

At Q = 500,

\[
P = \$1,000 - \$1(500)
\]

\[
= \$500
\]

\[
\pi = TR - TC
\]

\[
= (\$1,000 - \$1Q)Q - \$50,000 - \$100Q
\]

\[
= -\$50,000 + \$900Q - \$1Q^2
\]

\[
= -\$50,000 + \$900(500) - \$1(500^2)
\]

\[
= \$150,000
\]

(Note: \(\frac{\partial^2 TR}{\partial Q^2} < 0\), and this is a revenue maximum since revenue is decreasing for Q > 500.)

B. To find the profit-maximizing output level set MR = MC, or M\(\pi\) = 0, and solve for Q. Since,

\[
MR = MC
\]

\[
1,000 - 2Q = 100
\]

\[
2Q = 900
\]

\[
Q = 450
\]
At $Q = 450$,

$$P = $1,000 - $1(450)$$

$$= $550$$

$$\pi = -$50,000 + $900(450) - $1(450^2)$$

$$= $152,500$$

(Note: $\frac{\partial^2 \pi}{\partial Q^2} < 0$, and this is a profit maximum since profit is decreasing for $Q > 450$.)

C. In pursuing a short-run revenue rather than a profit-maximizing strategy, Desktop Publishing can expect to gain a number of important advantages, including: enhanced product awareness among consumers, increased customer loyalty, potential economies of scale in marketing and promotion, and possible limitations in competitor entry and growth. To be consistent with long-run profit maximization, these advantages of short-run revenue maximization must be at least worth the sacrifice of $2,500 per outlet in monthly profits.

P2.10 **Average Cost Minimization.** Giant Screen TV, Inc., is a San Diego-based importer and distributor of 60-inch screen, high-resolution televisions for individual and commercial customers. Revenue and cost relations are as follows:

$$TR = $1,800Q - $0.006Q^2$$

$$MR = \frac{\partial TR}{\partial Q} = $1,800 - $0.012Q$$

$$TC = $12,100,000 + $800Q + $0.004Q^2$$

$$MC = \frac{\partial TC}{\partial Q} = $800 + $0.008Q$$

A. Calculate output, marginal cost, average cost, price, and profit at the average cost-minimizing activity level.

B. Calculate these values at the profit-maximizing activity level.

C. Compare and discuss your answers to parts A and B.
P2.10 SOLUTION

A. To find the average cost-minimizing level of output, set MC = AC and solve for Q. Because,

\[ AC = \frac{TC}{Q} \]
\[ = \frac{($12,100,000 + $800Q + $0.004Q^2)}{Q} \]
\[ = $12,100,000Q^{-1} + $800 + $0.004Q \]

Therefore,

\[ MC = AC \]
\[ $800 + $0.008Q = $12,100,000Q^{-1} + $800 + $0.004Q \]
\[ 0.004Q = \frac{12,100,000}{Q} \]
\[ Q^2 = \frac{12,100,000}{0.004} \]
\[ Q = \sqrt{\frac{12,100,000}{0.004}} \]
\[ = 55,000 \]

And,

\[ MC = $800 + $0.008(55,000) \]
\[ = $1,240 \]

\[ AC = \frac{$12,100,000}{55,000} + $800 + $0.004(55,000) \]
\[ = $1,240 \]

\[ P = \frac{TR}{Q} \]
\[ = \frac{($1,800Q - $0.006Q^2)}{Q} \]
\[ = \$1,800 - \$0.006Q \]
\[ = \$1,800 - \$0.006(55,000) \]
\[ = \$1,470 \]
\[
\pi = P \times Q - TC
\]
\[ = \$1,470(55,000) - \$12,100,000 - \$800(55,000) - \$0.004(55,000^2) \]
\[ = \$12,650,000 \]

(Note: \(\frac{\partial^2 AC}{\partial Q^2} > 0\), and average cost is rising for \(Q > 55,000\).)

B. To find the profit-maximizing level of output, set \(MR = MC\) and solve for \(Q\) (this is also where \(M\pi = 0\)):

\[ MR = MC \]
\[ \$1,800 - \$0.012Q = \$800 + \$0.008Q \]
\[ 0.02Q = 1,000 \]
\[ Q = 50,000 \]

And

\[ MC = \$800 + \$0.008(50,000) \]
\[ = \$1,200 \]
\[ AC = \frac{\$12,100,000}{50,000} + \$800 + \$0.004(50,000) \]
\[ = \$1,242 \]
\[ P = \$1,800 - \$0.006(50,000) \]
\[ = \$1,500 \]
\[
\pi = TR - TC
\]
\[ = \$1,800Q - \$0.006Q^2 - \$12,100,000 - \$800Q - \$0.004Q^2 \]
\[ \pi = -0.01Q^2 + 1,000Q - 12,100,000 \]

\[ = -0.01(50,000^2) + 1,000(50,000) - 12,100,000 \]

\[ = 12,900,000 \]

(Note: \( \frac{\partial^2 \pi}{\partial Q^2} < 0 \), and profit is falling for \( Q > 50,000 \).)

C. Average cost is minimized when \( MC = AC = $1,240 \). Given \( P = $1,470 \), a $230 profit per unit of output is earned when \( Q = 55,000 \). Total profit \( \pi = $12.65 \) million.

Profit is maximized when \( Q = 50,000 \) since \( MR = MC = $1,200 \) at that activity level. Since \( MC = $1,200 < AC = $1,242 \), average cost is falling. Given \( P = $1,500 \) and \( AC = $1,242 \), a $258 profit per unit of output is earned when \( Q = 50,000 \). Total profit \( \pi = $12.9 \) million.

Total profit is higher at the \( Q = 50,000 \) activity level because the modest $2 (= $1,242 - $1,240) decline in average cost is more than offset by the $30 (= $1,500 - $1,470) price cut necessary to expand sales from \( Q = 50,000 \) to \( Q = 55,000 \) units.

**CASE STUDY FOR CHAPTER 2**

*A Spreadsheet Approach to Finding the Economic Order Quantity*

A spreadsheet is a table of data that is organized in a logical framework similar to an accounting income statement or balance sheet. At first, this marriage of computers and accounting information might seem like a minor innovation. However, it is not. For example, with computerized spreadsheets it becomes possible to easily reflect the effects on revenue, cost, and profit of a slight change in demand conditions. Similarly, the effects on the profit-maximizing or breakeven activity levels can be easily determined. Various “what if?” scenarios can also be tested to determine the optimal or profit-maximizing activity level under a wide variety of operating conditions. Thus, it becomes easy to quantify in dollar terms the pluses and minuses (revenues and costs) of alternate decisions. Each operating and planning decision can be easily evaluated in light of available alternatives. Through the use of spreadsheet formulas and so-called “macros,” managers are able to locate maximum or minimum values for any objective function based on the relevant marginal relations. Therefore, spreadsheets are a very useful tool that can be employed to analyze a variety of typical optimization problems.

To illustrate the use of spreadsheets in economic analysis, consider the case of The Neighborhood Pharmacy, Inc. (NPI), a small but rapidly growing operator of a number of large-scale discount pharmacies in the greater Boston, Massachusetts, metropolitan area. A key contributor to the overall success of the company is a system of tight controls over inventory acquisition and carrying costs. The company’s total annual costs for acquisition and inventory of
pharmaceutical items are composed of the purchase cost of individual products supplied by wholesalers (purchase costs); the clerical, transportation, and other costs associated with placing each individual order (order costs); and the interest, insurance, and other expenses involved with carrying inventory (carrying costs). The company’s total inventory-related costs are given by the expression:

\[ TC = P \times X + \Theta \times X/Q + C \times Q/2 \]

where \( TC \) is inventory-related total costs during the planning period, \( P \) is the purchase price of the inventory item, \( X \) is the total quantity of the inventory item that is to be ordered (used) during the planning period (use requirement), \( \Theta \) is the cost of placing an individual order for the inventory item (order cost), \( C \) is inventory carrying costs expressed on a per unit of inventory basis (carrying cost), and \( Q \) is the quantity of inventory ordered at any one point in time (order quantity). Here \( Q \) is NPI’s decision variable, whereas each other variable contained in the total cost function is beyond control of the firm (exogenous). In analyzing this total cost relation, NPI is concerned with picking the order quantity that will minimize total inventory-related costs. The optimal or total cost minimizing order quantity is typically referred to as the “economic order quantity.”

During the relevant planning period, the per unit purchase cost for an important prescribed (ethical) drug is \( P = $4 \), the total estimated use for the planning period is \( X = 5,000 \), the cost of placing an order is \( \Theta = $50 \); and the per unit carrying cost is \( C = $0.50 \), calculated as the current interest rate of 12.5% multiplied by the per unit purchase cost of the item.

A. Set up a table or spreadsheet for NPI’s order quantity \( Q \), inventory-related total cost \( TC \), purchase price \( P \), use requirement \( X \), order cost \( \Theta \), and carrying cost \( C \). Establish a range for \( Q \) from 0 to 2,000 in increments of 100 (i.e., 0, 100, 200, ..., 2,000).

B. Based on the NPI table or spreadsheet, determine the order quantity that will minimize the company’s inventory-related total costs during the planning period.

C. Placing inventory-related total costs, \( TC \), on the vertical or y-axis and the order quantity, \( Q \), on the horizontal or x-axis, plot the relation between inventory-related total costs and the order quantity.

D. Based on the same data as previously, set up a table or spreadsheet for NPI’s order quantity \( Q \), inventory-related total cost \( TC \) and each component part of total costs, including inventory purchase (acquisition) costs, \( P \times X \); total order costs, \( \Theta \times X/Q \); and total carrying costs, \( C \times Q/2 \). Placing inventory-related total costs, \( TC \), and each component cost category as dependent variables on the vertical or y-axis and the order quantity, \( Q \), as the independent variable on the horizontal or x-axis, plot the relation between inventory-related cost categories and the order quantity.
CASE STUDY SOLUTION

A. The table or spreadsheet for NPI’s order quantity (Q), inventory-related total cost (TC), purchase price (P), use requirement (X), order cost (Θ), and carrying cost (C) appears as follows:

<table>
<thead>
<tr>
<th>Quantity (Q)</th>
<th>Total Cost (TC)</th>
<th>Purchase Price (P)</th>
<th>Use Requirement (X)</th>
<th>Order Cost (Θ)</th>
<th>Carrying Cost (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0</td>
<td>$4</td>
<td>5,000</td>
<td>$50</td>
<td>$0.50</td>
</tr>
<tr>
<td>100</td>
<td>22,525</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>200</td>
<td>21,300</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>300</td>
<td>20,908</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>400</td>
<td>20,725</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>500</td>
<td>20,625</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>600</td>
<td>20,567</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>700</td>
<td>20,532</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>800</td>
<td>20,513</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>900</td>
<td>20,503</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,000</td>
<td>20,500</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,100</td>
<td>20,502</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,200</td>
<td>20,508</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,300</td>
<td>20,517</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,400</td>
<td>20,529</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,500</td>
<td>20,542</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,600</td>
<td>20,556</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,700</td>
<td>20,572</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,800</td>
<td>20,589</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>1,900</td>
<td>20,607</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
<tr>
<td>2,000</td>
<td>20,625</td>
<td>4</td>
<td>5,000</td>
<td>50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

B. Based on the NPI spreadsheet, the order quantity that will minimize the company’s inventory-related total costs during the planning period is Q = 1,000, the total cost-minimizing order level.
C. Using inventory-related total costs, TC, on the vertical $Y$ axis, and the order quantity, $Q$, on the horizontal $X$ axis, a plot of the relation between inventory-related total costs and the order quantity appears as follows:

![Total Cost vs Order Quantity](image)

D. A plot of inventory-related total costs, TC, and each component cost category as dependent variables on the vertical $Y$ axis, and the order quantity, $Q$, as the independent variable on the horizontal $X$ axis appears as follows:
Inventory-Related Cost Categories vs Order Quantity

- Total Cost
- Purchase Cost
- Total Order Costs
- Total Carrying Costs

Order Quantity

$
**PROBLEM AND SOLUTION**

2A.1 Lagrangian Multipliers. Amos Jones and Andrew Brown own and operate Amos & Andy, Inc., a Minneapolis-based installer of conversion packages for vans manufactured by the major auto companies. Amos & Andy has fixed capital and labor expenses of $1.2 million per year, and variable materials expenses average $2,000 per van conversion. Recent operating experience suggests the following annual demand relation for Amos & Andy products:

\[ Q = 1,000 - 0.1P \]

where \( Q \) is the number of van conversions (output) and \( P \) is price.

A. Calculate Amos & Andy’s profit-maximizing output, price, and profit levels.

B. Using the Lagrangian multiplier method, calculate profit-maximizing output, price, and profit levels in light of a parts shortage that limits Amos & Andy’s output to 300 conversions during the coming year.

C. Calculate and interpret \( \lambda \), the Lagrangian multiplier.

D. Calculate the value to Amos & Andy of having the parts shortage eliminated.

**P2A.1 SOLUTION**

A. Since \( Q = 1,000 - 0.1P \),

\[ P = $10,000 - $10Q \]

\[ TR = PQ \]

\[ = $10,000Q - $10Q^2 \]

Furthermore, given fixed expenses of $1,200,000 per year and average variable costs of $2,000 per unit, the relevant total cost function for the coming year is:

\[ TC = $1,200,000 + $2,000Q \]

The Amos & Andy profit function is

\[ \pi = TR - TC \]

\[ = $10,000Q - $10Q^2 - $1,200,000 - $2,000Q \]
\[ \pi = -10Q^2 + 8000Q - 1200000 \]

\[ \frac{d\pi}{dQ} = -20Q + 8000 = 0 \]

\[ 20Q = 8000 \]

\[ Q = 400 \]

\[ P = 10000 - 10Q \]

\[ = 10000 - 10(400) \]

\[ = 6000 \]

\[ \pi = -10(400^2) + 8000(400) - 1200000 \]

\[ = 400000 \]

(Note: \( \frac{d^2\pi}{dQ^2} = -20 < 0 \), and \( Q = 400 \) is a profit maximum.)

B. With Amos & Andy output limited to \( Q = 300 \), the constraint \( 0 = 300 - Q \) becomes active. Amos & Andy's constrained optimization problem can then be written

\[ L_\pi = -10Q^2 + 8000Q - 1200000 + \lambda(300 - Q) \]

where

(1) \( \frac{\partial L_\pi}{\partial Q} = -20Q + 8000 - \lambda = 0 \)

(2) \( \frac{\partial L_\pi}{\partial \lambda} = 300 - Q = 0 \)

Multiplying (2) by 20 and subtracting from (1) provides:

\[ \begin{align*}
(1) & \quad -20Q + 8000 - \lambda = 0 \\
\text{minus 20 x (2)} & \quad -20Q + 6000 = 0 \\
& \quad 2000 - \lambda = 0 \\
& \quad \lambda = 2000
\end{align*} \]

Then substituting \( \lambda = 2000 \) into (1) yields:

\[ -20Q + 8000 - 2000 = 0 \]
\[ 20Q = 6,000 \]
\[ Q = 300 \]
\[ P = 10,000 - 10Q \]
\[ = 10,000 - 10(300) \]
\[ = 7,000 \]
\[ \pi = -10Q^2 + 8,000Q - 1,200,000 \]
\[ = -10(300^2) + 8,000(300) - 1,200,000 \]
\[ = 300,000 \]

C. From part B, note that \( \lambda = \frac{\partial \pi}{\partial Q} = 2,000 \), which simply means that profits would increase by $2,000 if output were to expand by one unit.

D. With no vehicle shortage (part A) Amos & Andy earned $400,000, but only $300,000 with a shortage (part B). Thus, having the vehicle shortage eliminated has a maximum value of $100,000 (= $400,000 - $300,000) to Amos & Andy.