Analytical solutions and typical characteristics of part-load performances of single shaft gas turbine and its cogeneration

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Abstract

Explicit analytical solutions of part-load performances of constant rotating speed single shaft gas turbine and its cogeneration are derived based on analytical typical component performances. Their typical part-load operation regularities are summarized as well. For the gas turbine installation, it is found out that the relative gas turbine efficiency variation can almost be summarized as a single curve that is consistent with the practical data. The part-load performances of gas turbine cogeneration are investigated for the industrial process heat supply with saturated steam. It is found out that the variations of the steam flow rate and work/heat ratio are almost not influenced by the gas turbine design values, and it is worth emphasizing that the approach temperature difference in a heat recovery steam generator is likely to be negative in low load condition with high steam parameter and low ambient temperature. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Analytical solution; Part-load performance; Gas turbine; Cogeneration

1. Introduction

Most power plants often run under off-design situations due to change of the load or ambient conditions or both. Particular attention therefore should be given to part-load study. There have been numerous research contributions in this field [1–4]. However, compared with that of design point performance, the understanding of part-load behavior, especially some quantitative rules, is far from enough because of its complexity.

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Various numerical methods and computer codes have been proposed since the development of modern power systems. They have promoted greatly the performance analysis and the practice of

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<td><strong>c</strong> compressor</td>
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<td><strong>f</strong> fuel</td>
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<td><strong>gt</strong> gas turbine</td>
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<td><strong>s</strong> saturated steam</td>
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<td><strong>t</strong> turbine</td>
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<td><strong>w</strong> water</td>
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<td><strong>o</strong> design value</td>
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<td><strong>l</strong> inlet of compressor, atmospheric</td>
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<td><strong>4</strong> outlet of turbine</td>
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<td><strong>5</strong> outlet of HRSG gas side</td>
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power plants. Their effectiveness and accuracy are verified usually by experimental results. However, the practical experimental data have scarcely ever been published in open literature due to commercial reasons. Besides, the experimental data cannot be exactly accurate owing to experimental errors, and it is difficult to research the computational skills from the generally limited and scattered experiment data.

Analytical solution is an effective analysis means in many disciplines. For example, many analytical solutions of steady incompressible potential flow played a key role in the early development of fluid mechanics [5]. Owing to their mathematical precision, analytical solutions can serve as the standard solutions to check numerical solutions and develop skills of various numerical computational methods [6]. Moreover, they also have important theoretical meanings in power plant performance analysis. If the assumed component analytical performances are close enough to the typical real performances, the analytical solution for the whole plant performances will naturally be able to represent the typical performances of the practical systems. Systematical and thorough analysis can then be made with the analytical solutions from which the typical characteristics of part-load performance can be summarized. To demonstrate the idea of this new analysis method, analysis has been made in this paper for the constant rotating speed single shaft gas turbine and its cogeneration. The authors hope it could put some new insights into the field of part-load research. According to the knowledge of the authors, there is no such analytical solution of part-load performances of power plants, including the gas turbine and its cogeneration, published so far.

2. Analytical performance expressions of components and their simultaneous solving

To describe the part-load performance of turbine and compressor, generally the reduced parameters \( G_c \sqrt{T_1/P_1}, n_c/\sqrt{T_1}, \pi_c, \eta_c \) and \( G_t \sqrt{T_3/P_3}, n_t/\sqrt{T_3}, \pi_t, \eta_t \) are used and often drawn into curves as those shown in Figs. 1 and 2.

Proper analytical performances for components are the prerequisite to the analytical performances of the power plant. Firstly, they must be concise enough, otherwise it may be impossible to derive an explicit analytical solution of part-load performances for the whole gas turbine set. Secondly, to serve as the standard solution, the analytical performances of components should be as close to the typical practical performances as possible. To derive the feasible analytical solution for the whole plant, the key point is how to make a compromise between these two factors. There may be more than one format for the component performance expression. In this paper, the relative expressions of reduced parameters (for example, \( G_c = (G_c \sqrt{T_1/P_1})/(G_c \sqrt{T_1/P_1})_0 \)) are used and the following performance formulas for compressor and turbine are proposed after repeated deliberation.

**Performance formulas for compressor**

\[
\dot{n}_c = c_1(n_c \dot{G}_c^2 + c_2(n_c \dot{G}_c + c_3(n_c),
\]

\[
= [1 - c_4(1 - \dot{n}_c)^2](n_c/\dot{G}_c)(2 - \dot{n}_c/\dot{G}_c),
\]

(1)

(2)
where $c_i(n)$:

$$
c_1 = \dot{n}_c/\left[ p(1 - n/\dot{n}_c) + \dot{n}_c(\dot{n}_c - m)^2 \right],
$$

$$
c_2 = \left( p - 2m\dot{n}_c^2 \right)/\left[ p(1 - n/\dot{n}_c) + \dot{n}_c(\dot{n}_c - m)^2 \right],
$$

$$
c_3 = \left( p\dot{n}_c - m^2\dot{n}_c^3 \right)/\left[ p(1 - n/\dot{n}_c) + \dot{n}_c(\dot{n}_c - m)^2 \right].
$$

The values of $m$ and $p$ should satisfy the following equation to insure that the shape and the position of the curves are appropriate:

$$
\sqrt{p} \geq 2m/3.
$$

Actually Fig. 1 is a compressor performance map with $m = 1.06$, $p = 0.36$, $c_4 = 0.3$.

*Performance formulas for turbine*

An improved Flügel formula is used to approximately describe the mass flow characteristics of the turbine:

$$
G_t/G_{t0} = \alpha \sqrt{T_{t0}/T_3} \sqrt{(\pi_t^2 - 1)/(\pi_{t0}^2 - 1)},
$$

where $\alpha = \sqrt{1.4 - 0.4n_t/n_{t0}}$ is adopted to consider the influence of rotating speed.

The efficiency characteristic is defined as

$$
\dot{\eta}_t = [1 - t(1 - \dot{n}_t)^2] (\dot{n}_t/\dot{G}_t) (2 - \dot{n}_t/\dot{G}_t).
$$

The curves in Fig. 2 are drawn with $t = 0.3$. 

![Fig. 1. Compressor characteristics map.](image-url)
The part-load performance of the combustor is considered simply with constant combustor efficiency and pressure loss coefficient.

Gas turbine steady running should maintain the following equilibrium conditions: \( n_t = n_c \), \( G_t/G_{t0} = \mu(G_c/G_{c0}) \), \( \tilde{n}_t = \tilde{\phi}\tilde{n}_c \). The pressure recovery coefficient \( \phi \) and the mass flow ratio \( \mu \) are usually chosen to be constants, i.e. \( \tilde{\phi} = \tilde{\mu} = 1.0 \), in preliminary studies.

The part-load performance calculation can be carried out after two independent variables are given. For example, when relative reduced rotating speed and mass flow rate are the independent variables, the following analytical expression of relative reduced temperature ratio can be obtained by substituting Eqs. (1) and (5) into the equilibrium conditions:

\[
\tau = \frac{x^2[\alpha^2\pi_{c0}^2(c_1\dot{G}_c^2 + c_2\dot{G}_c + c_3)^2 - 1]}{\beta^2\mu^2\dot{G}_c^2(\pi_{c0}^2 - 1)} \frac{T_{t0}}{T_t},
\]

where \( \beta = (P_t/P_{t0}) \sqrt{T_{t0}/T_t} \).

The values of \( \pi_c \) and \( \pi_t \) can then be solved through Eq. (1) or (5). The explicit expressions of other parameters including gas turbine efficiency and power output can be found with Eqs. (2) and (6) and conventional thermal cycle analysis routine.

One calculation example of the analytical solution is shown in Fig. 3. It represents the typical part-load performances of a constant speed single shaft gas turbine. The input data for the calculation are as follows: \( \pi_{c0} = 12 \), \( \tau_0 = 5.0 \), \( \eta_{c0} = \eta_{t0} = 0.88 \), \( \eta_b = 0.99 \), pressure losses are...
represented by the pressure recovery coefficient $\varphi = 0.97$, the processes of compression, heating and expansion are calculated by means of average specific heat.

3. Typical part-load performances of constant speed single shaft gas turbine

Owing to the simplicity of analytical solutions and their possibility to bear many strict mathematical operations, systematic study can be made and typical characteristics of part-load operation can be summarized through a great amount of calculations under various cases. It should be pointed out that the regularities summarized this way may not suit a specific case exactly, but they do represent the general and typical performances of a kind of gas turbine set.

In the following text, most parameters are represented by the ratios to their design values (for example, $\bar{G}_c = G_c/G_{c0}$). Then, within the range of design temperature ratio and design pressure ratio in the state of the art, the relative variation of gas turbine efficiency and fuel consumption versus power output can almost be summarized into single curves respectively as shown in Figs. 4 and 5; the fitting formulas are:

$$\bar{\eta}_{gt} = 3.18N - 4.69N^2 + 3.69N^3 - 1.18N^4,$$

$$\bar{G}_f = 0.288 + 0.624N + 0.088N^2.$$ 

Comparing with practical data also shown in Fig. 4, the analytical solution is proven to be typical enough to represent the performances of present gas turbine sets.

Generally speaking, higher design temperature ratio results in higher part-load efficiency and lower fuel consumption. The influence of design pressure ratio can almost be neglected. For the
constant speed single shaft gas turbine set designed in recent years, the relative no-load fuel consumption is about 0.28–0.3. The value will be higher for older plants with lower design temperature ratio.

The variation of relative pressure ratio $\pi = \pi_0$ under off-design operation with different $\tau_0$ and $\pi_0$ is shown in Fig. 6. $\pi$–$N$ lines are approximately straight lines for every pair of given $\tau_0$ and $\pi_0$. The value of no-load $\pi$ is about 0.74–0.79; it will be lower for higher $\tau_0$ and lower $\pi_0$.

Fig. 4. Relative gas turbine efficiency variation and its comparison with practical data.

Fig. 5. Relative fuel consumption variation.
The variation of relative temperature ratio $\tau = \tau_0/\tau$ under off-design operation is shown in Fig. 7. For given $\tau_0$ and $\pi_0$, $\tau = \bar{N}$ lines are approximately straight lines too but slightly bent upward. The value of no-load $\tau$ is about 0.53–0.6.

The variation of relative exhaust gas temperature $\bar{T}_4 = T_4/T_{40}$ is shown in Fig. 8. For constant speed single shaft gas turbines, $\bar{T}_4$ drops fast in part load and it reaches 0.61–0.67 under no load, which means that the temperature $T_4$ would be lower than 200 °C. It is not favorable to heat
recovery. This is the shortcoming of the constant speed shingle shaft gas turbine with respect to heat recovery.

The influence of ambient temperature on gas turbine efficiency and power output with constant $T_3$ and $n$ is shown in Fig. 9 with no consideration for the limit of machine strength and the output of the electrical generator. $\bar{N}-T_1$ and $\bar{\eta}-T_1$ are approximately linear functions and the design parameters of the gas turbine have only a small influence. The negative slopes are higher with lower $\tau_0$. The average values are: $d \bar{\eta}/dT_1 \approx -0.0023$ $K^{-1}$, $d\bar{N}/dT_1 \approx -0.0097$ $K^{-1}$.

Fig. 8. Relative exhaust gas temperature variation.

Fig. 9. The influence of ambient temperature on gas turbine performance.
4. Typical part-load performances of constant speed single shaft gas turbine cogeneration set

A single pressure level heat recovery steam generator (HRSG) is considered in this preliminary analysis. The analytical solution of part-load performances of single pressure HRSG given by Cai and Hu [7] is applied. Its basic simplified assumption is that the heat transfer coefficients in superheater, evaporator and economizer do not vary with load. It has been proven with good enough accuracy for preliminary schematic calculation for near-constant gas flow rate. When saturated steam is generated, the independent variables are the temperature and mass flow rate of the inlet flue gas. Therefore, the explicit analytical solution for the gas turbine cogeneration set can be obtained simply by combining the analytical solutions of HRSG performance with that of gas turbine derived in the preceding text.

Saturated steam supply for industrial process heating is considered in this paper. The parameters of steam side chosen to evaluate are as follows: exhaust gas temperature $T_s$, approach temperature difference $\Delta T_a$, power/heat ratio $r$ and relative steam mass flow rate $G_s = G_s/G_{s0}$. Their typical variations are all described in Fig. 10. Some input data for this calculation are: $\Delta T_{p0} = 20$ K, $P_{s0} = 4$ bar, $T_{w0} = 378$ K, $\Delta T_{a0} = 20$ K. The exhaust gas temperature varies only a little bit.

To describe the performance of the gas turbine cogeneration system, several evaluation criteria are used in this paper. The common definition can be given as

$$ \eta = (N + BQ)/G_fH. \quad (10) $$

When $B$ is the ratio of heat exergy to heat energy, $\eta$ is the equivalent exergy efficiency $\eta_e$ proposed by El-Masri [8]; when $B$ is the price ratio between heat and power, it is the economic exergy coefficient $\theta$ [9]; $\eta$ is energy utilization efficiency (the first law efficiency) $Z$ when $B = 1$ and the gas turbine efficiency $\eta_{gt}$ when $B = 0$.

The cogeneration efficiencies are influenced by both gas parameters and steam parameters; the details can be found in Fig. 11a and b. The design temperature ratio and saturated steam parameters have more evident influence compared with that of design pressure ratio, but not evident enough to change the basic shape of the curves. Raising steam parameters has opposite effects on

![Fig. 10. Typical performance of gas turbine cogeneration.](image)
exergy efficiency $\eta_e$ and energy utilization efficiency $z$, since the power and heat are regarded as the same for energy utilization efficiency. Improving steam parameters leads to less steam production (heat); the result cannot be other but lower energy utilization ratio. On the contrary, exergy efficiency uses exergy to evaluate power and heat. The higher the steam parameters are, the more the exergy is. It is worthwhile even with less steam generation. It is similar to the economic exergy coefficient $\theta$ if strictly evaluating the steam production based on the economic consideration. However, a unified steam price is adopted for different steam parameters in this paper for the sake of simplicity. Therefore, the influence of steam parameters on the economic exergy coefficient is small.

Both gas turbine design parameters and steam parameters have insignificant influences on the steam output $G_s-N$, as shown in Fig. 12a and b.

The variation of approach temperature $\Delta T_a$ has a considerable importance to the safe running of economizer in HRSG. The influence of gas turbine design parameters and steam parameters on $\Delta T_a$ are shown in Fig. 13a and b. Gas turbine design parameters have only an insignificant influence. The value of $\Delta T_a$ under no load varies within a narrow range of about 2 K, while the saturated steam parameters have a more significant influence. A minor increase will lead to a faster drop of $\Delta T_a$ along with relative power output decrease. When higher steam parameters are adopted, the possibility of a negative $\Delta T_a$ under off-design operation is worth more attention.

The variation of the relative ratio of power to heat $\bar{r} = r/r_0$ is shown in Fig. 14a and b. Gas turbine design parameters have only a negligible influence on it.

The influence of ambient temperature on the performance of cogeneration is shown in Fig. 15a and b. It seems hard to image that the energy utilization efficiency increases when ambient temperature rises. In fact, ambient temperature enhancement will lead to the decrease of both pressure ratio and mass flow rate in compressor and the increase of gas turbine exhaust temperature, and the latter will cause the decline of the HRSG exhaust gas temperature. The drop of
both mass flow rate and exhaust gas temperature in HRSG will bring up a basically stable steam generation. On the other hand, ambient temperature increase will result in a remarkable drop of gas turbine power output and less fuel consumption. So, if heat and power are regarded as the same (as done with the energy utilization efficiency) and heat generation takes the main part of the
total output, the increase of ambient temperature will only cause energy utilization efficiency improvement. On the contrary, when gas turbine efficiency is chosen to be the criterion, heat is regarded as no contribution to the output; hence the efficiency will drop when the ambient temperature rises. If the exergy efficiency or the economic exergy coefficient is chosen as the criterion, the value ratio of heat to power is about 1:3 or 1:6, respectively; the exergy efficiency or the economic exergy coefficient will vary within the range of the above two ends.
It is also worth pointing out that the approach temperature difference drops when ambient temperature decreases, which deserves proper consideration for safe operation.

5. Conclusions

A typical analytical solution of constant rotating speed single shaft gas turbine part-load performances as well as its cogeneration part-load performances is derived based on the typical analytical component—mainly compressor, turbine and heat recovery steam generator—performances. The derivation of appropriate component analytical performances is the key to successfully obtain typical analytical performances of gas turbine and its cogeneration. The main criteria to select component performance analytical forms and the recommended results of the analytical expressions are given.

For the gas turbine installation and its cogeneration plant, it is found that when all parameters are represented by the ratio to design values, their variation in part-load can be expressed in a narrow belt; especially the gas turbine efficiency variation can almost be summarized as a single curve which is consistent with the practical data.

The part-load performance of gas turbine cogeneration has not yet been comprehensively studied in open literature, and its variation rules are summarized in this paper for industrial process heat supply with saturated steam. It is found that the relative steam flow rate and work/heat ratio are almost not influenced by the gas turbine design values. In addition, it is worth emphasizing that the approach temperature difference is likely to be negative in low-load condition with high steam parameter and low ambient temperature; however, many HRSGs cannot operate safely with negative approach temperature difference. Some other part-load performances are summarized also.

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References


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