Analytical model for predicting nonlinear reversed cyclic behaviour of reinforced concrete structural walls

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Abstract

This paper summarizes the results of an investigation on postpeak modelling and nonlinear performance prediction of reinforced concrete (RC) structural walls. The RC walls are designed for seismic loads using the capacity design method. We begin with a review of the proposed inelastic multilayer flexibility-based finite element with multilayer interfaces. An analytical structural engineering model for simulating the nonlinear hysteretic behaviour is then presented. All essential characteristics of the hysteretic behaviour of the RC walls, including strength degradation, stiffness degradation, pinching and slippage, bond slip effect, inelastic shear deformation mechanisms and confinement effects, are explicitly modelled analytically and experimentally. To establish the validity of the proposed model, the correlation between analysis and experimental results for load–displacement hysteretic responses and dissipation energy capacity are studied for cyclic loading. The analysis is, generally, in good agreement with experimental results and show that the model proposed in this paper can be reliably used for performance predictions of RC structures.

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1. Introduction

The development of nonlinear, the finite element methods with increasing computational capacity, has improved the reliability of seismic analyses of complex structures. The stresses induced in structures under seismic loads exceed yield capacities and generate large inelastic deformations in critical regions. Since the seismic response of structures depends on the hysteretic behaviour of these regions, reliable models of such behaviour need to be developed.

Thus, the use of nonlinear structural analysis requires computationally efficient models for performing analyses with sufficient accuracy. These models must describe essential geometrical and material characteristics as well as the basic mechanisms that control the hysteretic behaviour of reinforced concrete structures.

The application of capacity design principles in structural walls lead to an occurrence of plastic deformations in predetermined locations and to development of mechanisms for energy dissipation. This leads to a hierarchy of strength within the walls and to relationships between local and global ductility demand.

The intent of the research presented in this paper is the development of a simplified structural engineering model to study the inelastic performance predictions of RC structural walls as equivalent frame elements and for seismic vulnerability assessment of reinforced concrete existing frame-wall buildings.

2. Finite element model

The finite element model presented in this study is an inelastic one-dimensional finite element for seismic two-dimensional frame analysis. It has two nodes and three degrees of freedom at each node [1,2].

The nodal displacement \( \{u^e\} \) is expressed by the vector: \( \{u^e\} = \{u_i, v_i, \theta_i, u_j, v_j, \theta_j\}^T \) and the corresponding external nodal forces \( F^e \) by the vector \( \{F^e\} = \{N_i, T_i, M_i, N_j, T_j, M_j\}^T \).
\[ N_i \text{ and } N_j \text{ are the axial forces at the node } i \text{ and } j \text{ respectively.} \]
\[ u_i \text{ and } u_j \text{ are the corresponding nodal axial displacements.} \]
\[ T_i \text{ and } T_J \text{ are the shear forces at the node } i \text{ and } j \text{ respectively.} \]
\[ v_i \text{ and } v_J \text{ are the corresponding nodal vertical displacements.} \]
\[ M_i \text{ and } M_J \text{ are the bending moments at the node } i \text{ and } j. \]
\[ \theta_i \text{ and } \theta_J \text{ are the corresponding nodal chord rotations at the beam ends (Fig. 1).} \]

The finite element formulation is based on the force method of structural analysis. The element is decomposed into a multilayer beam element with two connection fibre hinges used as interface fibre subelements at the beam ends (see Section 4).

The multilayer beam element and the connection hinges are connected in series. Thus, the element tangent stiffness matrix in a local coordinates system can be obtained from the direct assembly of these elements as shown below:

\[
\begin{align*}
[K^e] &= [R]^T \left( \left[ F_{\text{lex}}^{\text{beam \ (bnd)}} \right] + \left[ F_{\text{lex}}^{\text{beam \ (shr)}} \right] \right) \\nonumber \\
& \quad + \left[ F_{\text{lex}}^{\text{H1}} \right] + \left[ F_{\text{lex}}^{\text{H2}} \right]^{-1} [R] \\
\left[ F_{\text{lex}}^{\text{beam \ (bnd)}} \right] \text{ and } \left[ F_{\text{lex}}^{\text{beam \ (shr)}} \right] \text{ are the flexural and shear flexibility (3 × 3) matrices of the beam respectively. } \left[ F_{\text{lex}}^{\text{H1}} \right] \text{ and } \left[ F_{\text{lex}}^{\text{H2}} \right] \text{ are the flexibility matrices at node } i \text{ and } j \text{ of the interface subelements.} \\
[\xi] &\text{ is the normalized variable. } \eta_n = x_{n-1}/L \text{ and } \eta_n = x_n/L \text{ are the left side and the right side coordinates of a segment respectively. } \xi_n = (\eta_n + \eta_{n-1})/2 \text{ is the segment centre where the segment behaviour is monitored such that:} \end{align*}
\]

\[
\begin{align*}
[D (\xi)] &= [b (\xi)] [1] [N (\xi), M (\xi)]^T \\
[T (\xi)] &= [C] [Q] \\
[D (\xi)] \text{ is the internal axial and bending moment in a segment and } \{T (\xi)\} \text{ is the shear force in a segment.} \\
\left[ f_{\text{bnd}} (\xi_n) \right] \text{ and } \left[ f_{\text{shr}} (\xi_n) \right] \text{ are the bending flexibility matrix and the shear flexibility matrix of a segment respectively. They are defined as follows:} \\
\end{align*}
\]
with:

\[
[f_{\text{sh}}(\xi_n)]^{-1} = \sum_{k=1}^{n_{\text{layers}}} G_k(\xi_n) S_k'(\xi_n) \quad (12)
\]

\[
S_k'(\xi_n) \leq A_k(\xi_n) \quad (13)
\]

\[
[f_{\text{bnd}}(\xi_n)]^{-1} = \begin{bmatrix} \text{term}_{11} & \text{term}_{12} \\ \text{term}_{21} & \text{term}_{22} \end{bmatrix} \quad (14)
\]

where:

\[
\text{term}_{11} = \sum_{k=1}^{n_{\text{layers}}} E_k(\xi_n) A_k(\xi_n) \quad (15)
\]

\[
\text{term}_{12} = \sum_{k=1}^{n_{\text{layers}}} E_k(\xi_n) y_k(\xi_n) A_k(\xi_n) \quad (16)
\]

\[
\text{term}_{22} = \sum_{k=1}^{n_{\text{layers}}} E_k(\xi_n) y_k^2(\xi_n) A_k(\xi_n) \quad (17)
\]

\([f_{\text{sh}}(\xi_n)]^{-1}\) and \([f_{\text{bnd}}(\xi_n)]^{-1}\) are the shear and flexural stiffness matrices, \(E_k, G_k, A_k, S_k'\) and \(y_k\) represent the Young modulus, shear modulus, layer area, shear area and the layer coordinate. \(d(\xi_n) = [\varepsilon_{NN}(\xi_n), \kappa(\xi_n)]^T\) is the axial strain along the reference axis and the segment curvature vector. \(\{\gamma_{13}(\xi_n)\}\) is the segment shear deformation.

4. Fiber connection hinges

These are two multilayer finite interfaces with zero length. Each multilayer connection hinge has two degrees of freedom. These elements can be specified at the beam ends to model deformations that occur in beam-to-column, column-to-footing or wall-to-footing connections.

There is widespread experimental evidence that bond-slip plays an important role in increasing the flexibility of reinforced concrete structures [6,7]. This is especially true for structural walls due to bond-slip in the rebars anchored in the foundations. During cyclic loading, the vertical reinforcements in the wall base yields due to the large bending moment. This effect combined with the diagonal cracks produced by shear stresses, leads to slippage of reinforcing bars. This manifests itself as bar pullout at the wall-footing interface and results in concentrated rotations known as fixed-end rotations at the wall base. Modelling this effect is necessary for a realistic description of the cyclic and ultimate behaviour of RC structural walls. Rubiano-Benavides [8] has proposed the use of rotational springs at the element ends to account for the added flexibility due to bond-slip.

In the current study, the connection hinges are used as interface bond-slip sub-elements to model the following:

1. The fixed-end rotations due to bond-slip of vertical reinforcement.
2. The effect of opening and closing of flexural cracks, and
3. The gap opening effects at the connection region.

The fibre bond-slip subelements are lumped plasticity finite element models because they were concentrated at the beam ends. Moreover, these finite elements account for coupling between bending moment and axial forces, with distributed plasticity over each bond-slip element.

The corresponding flexibility matrices at nodes \(i\) and \(j\) are obtained by giving \(\xi = 0\) and \(\xi = 1\) respectively in the following expression:

\[
[f^H_{\text{flex}}] = [b(\xi)]^T [f^H] [b(\xi)]
\]

with:

\[
[f^H]^{-1} = \sum_{h=1}^{n_{\text{layers}}} k_h^H A_h^H \quad (18)
\]

\([f^H]^{-1}\) is the stiffness matrix of an interface sub-element. \(k_h^H\) is a layer stiffness. \(A_h^H\) is the layer area. \(y_h^H\) is the layer location. The off-diagonal terms that contain the flexibility matrix (19) represent the coupling between the axial force and the bending moment at the connection hinges.

5. Material models and constitutive laws

The nonlinear response of the multilayer finite elements is entirely derived from the nonlinear constitutive laws of each layer. The constitutive laws of the layers are defined using uniaxial stress–strain \((\sigma–\varepsilon)\) relationships and shear stress–strain \((\tau–\gamma)\) relationships for the beam element. For the connection hinges, the layers are defined using uniaxial stress–displacement \((\sigma–d)\) relationships.

5.1. Cyclic behaviour of a concrete layer

In this study, the plastic fracturing model is used for concrete [9]. The inelastic behaviour of concrete layers is attributed to microcracking and plastic slip. Thus, fracturing (damage) concepts are combined with plasticity theory to derive the constitutive relations. The proposed model can simulate observed characteristics of concrete material behaviour such as and concrete cracking and crushing, damage accumulation (fracturing) and stiffness degradation under cyclic loads.

The behaviour of concrete layers is modelled using the plastic fracturing model. The model is defined in term of a stress–strain curve \((\sigma, \varepsilon)\). The model is then decomposed into a number of simplified parallel models (20–25) (Fig. 2).

The material constitutive law can be defined in ‘\(n\)’ points \((\sigma_m, \varepsilon_m)\) on stress–strain curve with \(1 \leq m \leq n\). If \((\sigma_m, \varepsilon_m)\) are positive input values, they represent the compressive part of the law. If \((\sigma_m, \varepsilon_m)\) are negative, they represent the tension behaviour of concrete with strain-softening (tension-stiffening effect) (Fig. 2). The decomposition into simplified hysteretic models is given by the following expressions (Fig. 3):

Yielding points:

\[
\varepsilon_{by}(m) = \varepsilon_m, \quad \text{for } 1 \leq m \leq n \quad (20)
\]

\[
(\sigma_{by})(m) = \sigma_m - \sigma_{m+1} \frac{\varepsilon_m}{\varepsilon_{m+1}}, \quad \text{for } m < n \quad (21)
\]

\[
(\sigma_{by})(m=n) = \sigma_m, \quad \text{for } m = n. \quad (22)
\]
Fig. 2. Cyclic behaviour of a simplified concrete model and its corresponding events. 1: initial damage, 2: crack opening, 3: crack closing, 4: unloading with stiffness degradation, 5: failure, 6: crack opening from a damaged state, 7: crack closing for a damaged material, 8: progressive damage and softening.

Fig. 3. General cyclic behaviour for a concrete layer $(\sigma_k, \varepsilon_k)$.

Ultimate points:

\[
\begin{align*}
(\varepsilon_{bu})_m &= \varepsilon_{m+1}, \quad \text{for } m < n \\
(\sigma_{bu})_m &= 0, \quad \text{for } m < n \\
\varepsilon_{bu} &= \text{(Very large)}.
\end{align*}
\]

The hysteretic rule of the model is described as follows:

Cracking state:

\[
\sigma(\varepsilon) = 0, \quad \text{for } \varepsilon \leq 0.
\]

Elastic state:

\[
\sigma(\varepsilon) = \frac{\sigma_{by}}{\varepsilon_{by}} \varepsilon = E_{by} \varepsilon, \quad \text{for } 0 < \varepsilon \leq \varepsilon_{by}.
\]

Partially damaged material state:

\[
\sigma(\varepsilon) = -\frac{\sigma_{by}}{\varepsilon_{bu} - \varepsilon_{by}} (\varepsilon - \varepsilon_{bu}), \quad \text{for } \varepsilon_{by} < \varepsilon \leq \varepsilon_{bu}.
\]

Completely damaged material (failure state):

\[
\sigma(\varepsilon) = 0, \quad \text{for } \varepsilon > \varepsilon_{bu}.
\]

Fig. 4. Cyclic behaviour of a simplified steel model and its corresponding events. 1: yielding in tension, 2: yielding in compression, 3: elastic unloading in compression, 4: elastic unloading in tension.

Re-loading and unloading state:

\[
\sigma(\varepsilon) = \left(\frac{\sigma_c}{\varepsilon_c - \varepsilon_p}\right) (\varepsilon - \varepsilon_p), \quad \text{for } \varepsilon_p \leq \varepsilon < \varepsilon_c
\]

with:

\[
\varepsilon_p = \varepsilon_c - \left((E_{by} \sigma_c) + \alpha_c (\varepsilon_c - E_{by} \sigma_c)\right)
\]

\[
\sigma_c = \sigma_{by} \left(\frac{\varepsilon_{by} - \varepsilon_p}{\varepsilon_{by} - \varepsilon_{by}}\right)
\]

\[
\varepsilon_p \text{ is the residual deformations. } \alpha_c \text{ is the unloading factor (degradation factor) such that } 0 \leq \alpha_c \leq 1. \sigma_{by} \text{ and } \varepsilon_{by} \text{ represent the stress and the strain at the yield point respectively of the simplified model. } E_{by} \text{ is the elastic modulus. } \sigma_{bu} \text{ and } \varepsilon_{bu} \text{ are the ultimate stress and deformation respectively. } \sigma_c \text{ and } \varepsilon_c \text{ are the yielding stress and strain respectively at damaged state (Fig. 2).}
\]

5.2. Cyclic behaviour of a steel layer

The response of the reinforcing bars is derived by a superposition of a number of elastic–perfectly-plastic models in parallel (Fig. 4). This takes account of a nonlinear kinematic positive strain-hardening.

The steel layer behaviour is modelled as a steel stress–strain curve $(\sigma_\varepsilon)$. The steel material models are decomposed into a number of simplified elastic perfectly plastic models in parallel (Fig. 5) in the same way as for concrete. The decomposition of a stress–strain curve $(\sigma_m, \varepsilon_m)$ into simplified models is given as follows:

\[
\sigma_{sy} = E_s \varepsilon_{sy}, \quad \text{with } \varepsilon_{sy} = \varepsilon_m
\]

\[
E_s = \frac{\sigma_m - \sigma_{m-1}}{\varepsilon_m - \varepsilon_{m-1}} - \frac{\sigma_{m+1} - \sigma_m}{\varepsilon_{m+1} - \varepsilon_m}, \quad \text{for } 1 < m < n
\]

\[
E_s = \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2}, \quad \text{for } m = 1
\]

\[
E_s = \frac{\sigma_n - \sigma_{n-1}}{\varepsilon_n - \varepsilon_{n-1}}, \quad \text{for } m = n.
\]
The elastic perfectly plastic model is described as follows:

Linear elastic state:

$$\sigma (\varepsilon) = E_s \varepsilon, \quad \text{if } \varepsilon \leq \sigma_{sy}/E_s.$$  \hfill (37)

Perfectly plastic state:

$$\sigma (\varepsilon) = \sigma_{sy}, \quad \text{if } \varepsilon > \sigma_{sy}/E_s$$  \hfill (38)

$\sigma_{sy}$ and $\varepsilon_{sy}$ represent the stress and the strain at the yield point respectively of the elastic plastic model. $E_s$ is the elastic modulus.

5.3. Cyclic behaviour model for shear and bond slip

We have adopted a three-linear ($\sigma$–$d$) model for modeling the connection sub-element behaviour. The three-linear model is composed in three parallel components, two elastic–plastic components and one elastic component. The elastic component represents the softening or hardening effect (Fig. 6). The model accounts for accumulated plastic displacements, stiffness and strength degradation and pinching effect (Figs. 7 and 8). The model can be calibrated using analytical or empirical considerations.

It has been stated that shear effects may be of critical importance in many types of concrete structures and especially for structural walls [10–17].

In general, during the crack formation process, the surface of cracks is usually rough. This irregularity in the crack path influences the loading states that can be transferred across the cracked surfaces. These rough surfaces are capable of generating some shear resistance due to friction [18]. For simplicity, we have treated the shear effects that occur in beam element using the same three-linear model. For each layer, this model is defined using the shear stress–strain ($\tau$–$\gamma$) relationship.

The three-linear model is decomposed into two parallel elastic–plastic components ($m = 1$ and $m = 2$) and one elastic component ($m = 3$). The elastic component can represent either positive or negative strain hardening. The model considers the stiffness and strength degradation due to accumulated...
plastic displacement and pinching. Dissymmetrical behaviour in tension and compression can be specified.

The decomposition of the three-linear law into elastic–plastic components is given by the following expressions:

In tension:

\[
(\gamma_+^t)_m = \left(\gamma_+^t\right)_{m-1} + \frac{\tau T_m - \tau T_{m-1}}{G_m} \tag{39}
\]

\[
(\tau_+^t)_m = \tau T_m - \left(\tau_+^t\right)_{m-1} - G_{m+1}\left(\gamma_+^t\right)_m. \tag{40}
\]

In compression:

\[
(\gamma_-^c)_m = \left(\gamma_-^c\right)_{m-1} + \frac{\tau C_m - \tau C_{m-1}}{G_m} \tag{41}
\]

\[
(\tau_-^c)_m = \tau C_m - \left(\tau_-^c\right)_{m-1} - G_{m+1}\left(\gamma_-^c\right)_m. \tag{42}
\]

\[G_{m^{\text{el-pl}}} = G_m - G_{m+1} \tag{43}\]

The total layer strain of a layer \(k\) is expressed as:

\[\varepsilon_k^t(\xi_m) = \varepsilon_k^{t-1}(\xi_m) + \Delta \varepsilon_k^t(\xi_m). \tag{44}\]

The axial strain increment of a layer \(k\) is given as:

\[\Delta \varepsilon_k^t(\xi_m) = [1\gamma_1]\left[f_{\text{mu}}(\xi_m)\right]^{-1}\left[b(\xi_m)\right]\left(F_e^{-1}\right)^{y-1} \times [R]\left[\Delta u^r\right]^s. \tag{45}\]

The total shear strain of a layer \(k\) is given by:

\[\gamma_k^s(\xi_m) = \gamma_k^{s-1}(\xi_m) + \Delta \gamma_k^s(\xi_m). \tag{46}\]

The shear deformation increment of a layer \(k\):

\[\Delta \gamma_k^s(\xi_m) = [f_{\text{mu}}(\xi_m)]^{-1}\left[C\right]\left(F_e^{-1}\right)^{y-1} [R]\left[\Delta u^r\right]^s \tag{47}\]

\{\Delta u^r\}^s is the nodal displacement increment of an element at the \('s\)' iteration.

(2) For connection hinge at node \(i\): \((\xi_L = 0)\)

\[d_k^i(\xi_L) = d_k^{i-1}(\xi_L) + \Delta d_k^i(\xi_L) \tag{48}\]

\[\Delta d_k^i(\xi_L) = [f^H]^{-1}\left[b(\xi_L)\right]\left(F_e^{-1}\right)^{y-1} [R]\left[\Delta u^r\right]^s \tag{49}\]

\(d_k^i(\xi_L)\) is the axial displacement increment of a fiber connection hinge ‘k’ at the left end;

(3) For connection hinge at node \(j\): \((\xi_R = 1)\)

\[d_k^j(\xi_R) = d_k^{j-1}(\xi_R) + \Delta d_k^j(\xi_R) \tag{50}\]

\[\Delta d_k^j(\xi_R) = [f^H]^{-1}\left[b(\xi_R)\right]\left(F_e^{-1}\right)^{y-1} [R]\left[\Delta u^r\right]^s \tag{51}\]

\(d_k^j(\xi_R)\) is the axial displacement increment of a fiber connection hinge ‘k’ at the right end.

From the updated strains and connection displacements, the new stresses and moduli in each layer, depending on the considered material constitutive laws and models, can be easily determined. Then the finite elements and structural stiffnesses can be updated.

7. Test specimens and test setup

The tests used for validation were performed at the institute of structural engineering (IBK) of the Swiss Federal Institute of Technology (ETH) in Zürich, Switzerland. As part of the research project titled ‘Reinforced concrete structures under cyclic dynamic and cyclic static actions’, six reinforced concrete structural walls were tested under cyclic static action. The walls are scaled at 1:2, simulating the lower half of walls of a six-storey reference building [22] (Fig. 9).

The test specimens used in this study are the four reinforced concrete structural walls WSH2, WSH3, WSH4 and WSH6 (Figs. 10 and 11). They are designed using the capacity design method [23] according to NZS3101 [24] assuming...
different ductility classes defined in Eurocode8 [25] expect WSH4 which is designed according to Eurocode2.

The test setup reproduced the same wall conditions for stress resultants in the plastic region of the specimen as in the structural walls of the prototype building under seismic actions (Fig. 9) [22].

The specimen dimensions were 200 m (horizontal wall length $L_w$) $\times$ 0.15 m (wall width $b_w$). The wall footings are rigidly connected to the strong floor (Fig. 13) [22].

The top of the walls is horizontally displaced back and forth in a plane perpendicular to the walls by a hydraulic actuator located at 4.56 m (4.52 m: WSH6) above the wall footings (Figs. 10 and 12).

The first two cycles were force-controlled up to 75% of the calculated ideal bending strength to determine the idealized yield displacement, $\Delta_y$, and the displacement ductility, $\mu_\Delta = 1$. The subsequent cycles until failure were displacement-controlled such that the displacement ductility was increased in steps of unit beginning with $\mu_\Delta = 2$. For each ductility step, two loading cycles were employed (Fig. 13).

An axial force, $P$, due to gravity load was applied and held constant during the tests (Table 1). The axial force is constant along the height of the wall and is applied using two external post-tensioning cables (Fig. 10). During the test, the force in the cables is kept constant by two flat-jacks located on the top of the wall. This combination of horizontal and vertical forces...
Table 1
Main design parameters of RC structural walls

<table>
<thead>
<tr>
<th>Wall</th>
<th>a_{g}</th>
<th>Soil</th>
<th>q</th>
<th>\mu_{\Delta}</th>
<th>P</th>
<th>\rho_{t} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WSH2</td>
<td>0.16</td>
<td>Stiff</td>
<td>3</td>
<td>3</td>
<td>630</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Restricted ductility</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WSH3</td>
<td>0.20</td>
<td>Soft</td>
<td>4</td>
<td>5</td>
<td>630</td>
<td>0.82</td>
</tr>
<tr>
<td>WSH4</td>
<td>0.10</td>
<td>Soft</td>
<td>2</td>
<td>2</td>
<td>630</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Conventional ductility</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WSH6</td>
<td>0.16</td>
<td>Medium-stiff</td>
<td>5</td>
<td>5</td>
<td>1420</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Full ductility</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(a_{g}\): Peak ground acceleration (g), \(q\): Behaviour factor [25,27], \(\mu_{\Delta}\): Design ductility factor [27,33], \(P\): Axial (Post-tensioning) force (kN) \((0.051A_{g}f_{c})\), \(A_{g}\): Gross area of section, \(f_{c}\): design compressive strength of unconfined concrete, \(\rho_{t}\): Total reinforcement ratio.

The structural engineering model and numerical simulation

The structural engineering model is a cantilever system. The model is discretized into 10 multilayer beam elements (Figs. 14–17), and one interface bond-slip subelement at the wall-to-footing connection to account for the fixed-end rotation at the wall base.

The wall sections, in the plastic hinge region called critical region, are discretized into 150 concrete layers to:
1. well-reproduce the spread of damages and material nonlinearities in the critical region and then over the wall height,
2. to reproduce the shifting of the neutral axis during cyclic loading,
3. to avoid numerical instabilities due to compression softening effect [26]. In the critical region, the concrete layer compression strains and curvatures may increase quickly, and result in a rapid degradation material stiffness (negative material moduli). Then, the postpeak element and structural response may fail to converge and snap-back may occur.

The stress–strain curves associated with the steel layers of vertical boundary and web reinforcements are based on experimental data given in Appendix A [27].
The flexural stress–strain curves of concrete in compression are based on Hogonostad’s model [28] for the unconfined concrete. For confined concrete the Saatcioglu and Ravzi analytical model [29] is adopted (Table 2). This model accounts for the strain gradient effect and the influence of the confinement pressure on the short and long sides in the boundary regions. A descending branch was considered for simulating the compression softening.

The horizontal web reinforcement consisted of bars of $\phi 6$ mm at a vertical spacing of 150 mm. In the boundary regions, 6 vertical bars had a larger diameter ($\phi$ up to 12 mm) than the vertical bars in the web region. With the exception of wall WSH4, these bars were stabilized against buckling by additional closed ties and S-shaped ties with a diameter of 6 mm or 4.2 mm at a vertical spacing $S_v$ of 50 mm or 75 mm. These bars act also as concrete confinement at the compression zone (Fig. 18).

The unconfined concrete strength $f_{cc0}$ is based on an average of 45 MPa. The yield strength of transverse reinforcement, $f_{yt}$, is taken equal to the proof stress at 0.2% non-proportional elongation $R_{p0.2}$. The yield strength $f_{yt}$ in the Saatcioglu and Ravzi model for transverse reinforcement with a diameter $\phi 6$ mm is selected to be equal to 498.0 MPa for WSH2, 494.8 MPa for WSH3 and 531.9 MPa for the wall specimen WSH6. On the other hand, $f_{yt}$ is selected equal to 532.7 MPa for WSH2 and 564.4 MPa for both WSH3 and WSH6, for transverse reinforcement with $\phi 4.2$ mm.

Calculations performed with concrete in tension have pointed out that it does not play an important role under ultimate loads during reversed cyclic loading and does not significantly affect the nonlinear response of structural wall analyses under seismic loads (Fig. 18).

The parameters of the shear constitutive laws and of the interface bond-slip subelement are derived from experimental measurements.

The parameters of the shear stress–strain relation are based on a bilinear or three-linear characterization of the shear force $F_k$–lateral displacement, $\delta$, due to the distortion envelope curve measured at 1.56 m above the wall footing (Figs. 12 and 19) (see Appendix B). The rest of the walls are assumed to be elastic in shear. This is the direct consequence of the capacity design method.

The shear force–lateral displacement relationship due to distortion relation must be carefully calibrated [30–32]. The main limitation of the application of the model in this study is that the shear stiffness is derived directly from the global behaviour and it does not represent the local shear stiffness of the structure. Thus, it depends closely on the loading scheme, such as the magnitude of inelastic load reversals, the number of postyield load cycles (Celebi 1973) [13] and the axial vertical load (Atalay 1975) [14].

In the same manner, the parameters of the interface bond-slip stress–displacement relationship are based on a bilinear or three-linear characterization of the bending moment $M$ — Fixed-end rotation $\theta_H$ at the walls base envelope curve (Fig. 19) (see Appendix B). These parameters are measured at the same level of the critical region, 1.56 m (Fig. 12).

9. Results

The analytical and experimental studies lead to the following conclusions:

(1) A satisfactory agreement between numerical and experimental results is observed. The hysteretic curves (Figs. 20–23) drawn to the same scale are the most significant results. They show the influence of the test and analytical parameters on the deformation behaviour of the walls:

(2) The model can simulate with sufficient accuracy the loss of strength exhibited by some of structural walls. The pinching
effects shown in the global behaviour are essentially caused by both shear and slippage at the wall base;

(3) The model shows that it can account for the influence of the selected design parameters such as (1) the displacement ductility, $\mu_{\Delta}$, (2) the mechanical properties ($R_m$, $R_{p0.2}$, $R_m/R_{p0.2}$); (4) the local ductility ($A_{gt}$) of the reinforcing steel; (5) the vertical reinforcement ratio, $\rho_v$, (5) the confinement effects ($S_e$, $S_w$, $f_y$), and (6) the gravity load, $P$. For example:

(3.1) with increasing displacement ductility (Table 1), specimen WSH3 shows, in comparison with WSH2, a much higher deformability (+32%) and much better dissipated energy capacity (+14%) (Figs. 20 and 21). This was achieved by increasing the vertical reinforcement ratio and the improvement of the mechanical properties of the reinforcing steel ($A_{gt}$ is increased by 3% in the boundary region but of 33.88% in the web region).

(3.2) The increase in the axial force, $P$, and the improvement of the vertical spacing of the horizontal reinforcement in the web region lead to very good strength and deformation capabilities for the WSH6 specimen in relation to WSH4 specimen (Figs. 22 and 23).
The wall WSH3 shows an excellent behaviour in comparison with the other walls. This was obtained by a higher ratio $R_m/R_p0.2$, a lower axial force $P$, and a suitable vertical spacing of horizontal reinforcement in the edge regions. The deformability of WSH3 specimen is larger than WSH6, WSH2 and WSH4 by 16.5%, 31.7% and 33.3% respectively.

The good deformation capacity and the good energy dissipation capacity of the WSH3 specimen (Fig. 25) are essentially due to the large inelastic deformation mechanisms developed particularly in the critical region of the structure such as the contribution of the flexural, shear (Table 3) and bond-slip deformation mechanisms (Table 4).

The lateral displacement due to both bond slip mechanism and shear effect, compared to the total lateral top-displacement, $\Delta w_1$, confirms the flexural dominant behaviour of the walls (Tables 3 and 4).

In particular for WSH3 wall, the shear contribution calculated in the critical region represents 35% of the total displacement $\Delta w_4$ (Fig. 24) at the failure time, whereas the bond-slip mechanism contribution does not exceed 15%.

(6) The dissipated energy in the WSH6 specimen is greater than that in WSH4 specimen even though both specimens have the same energy dissipation capacity ($E_d/E_i$). Also the presence of a higher axial force in WSH6 leads to a more pronounced pinching behaviour.

Fig. 26 provides a comparison of the contributions, in terms of dissipated energy. Thus, the energy dissipation is different along the wall height such that the dissipated energy is 91.83 kJ between the wall base up to 1.56 m level (critical region), and
117.368 kJ in the central region between 1.56 m and 3.40 m, and 62.136 kJ in the upper region of the wall. For WSH3 wall, both shear and bond-slip mechanism contributions to total energy dissipation (Fig. 26) seem to be directly proportional to their contributions in both the total lateral displacement at the top and at the critical region of the wall. Hence, the flexural failure mode is predominant.

(7) Fig. 27 shows both the curvature pattern along the wall height of 3.4 m and the crack pattern at the critical region of the WSH3 wall, calculated at time KS61 and KS63 of loading cycle. The curvatures distribution is very closely related to the formation of cracks and steel plasticity distribution. The multilayer beam finite element model permits the extraction of the curvature values that are very useful for seismic structural design and analysis [22,31]. It also shows also the concentration of inelastic behaviour at the critical region and the plastic hinge development.

10. Conclusion

A multilayer model made of an inelastic beam element and interface bond-slip subelements is presented. It is successfully used to predict the hysteretic behaviour and nonlinear performance of reinforced concrete structural walls under seismic loads. The intent of the proposed model is to simulate the inelastic response of frame-wall buildings.

Different approaches are used to describe deformation mechanisms such as flexural, shear and bond-slip effects. These control the hysteretic behaviour through existing analytical models and empirical relationships.

Good agreement between analytical predictions and experimental results is observed. In general, the results demonstrate that the proposed model can be reliably used for inelastic seismic performance prediction of reinforced concrete structural walls and for code calibration.

The model can be used to provide optimal structural design solutions in order to have sufficient energy dissipation capacity and resistance to seismic actions. This is achieved trough considering design parameters such the reinforcement mechanical properties, confinement effects and gravity load. It can be used also for developing capacity curves for seismic vulnerability assessment of existing RC buildings. The model is also a flexible platform for developing analytical models since subelements can be added into the finite element model. Finally, new hysteretic models can be employed.

Future work includes validating the model for dynamic performance predictions of reinforced concrete walls through correlation studies on specimens subjected to dynamic loads.
Fig. 27. Curvatures at time KS61 and KS63 for WSH3 (left) and observed cracks for WSH3 at main loading point KS63 along the height of the WSH3 specimen (right).

Appendix A

See Table A.1.

Appendix B

See Table B.1.

Table A.1
Steel mechanical properties

<table>
<thead>
<tr>
<th>Wall</th>
<th>$\phi_e$ (mm)</th>
<th>$\phi_{wv}$ (mm)</th>
<th>$e$ (mm)</th>
<th>$S_{wv}$ (mm)</th>
<th>$\rho_t$ (%)</th>
<th>Bar</th>
<th>$R_{p0.2}$ (MPa)</th>
<th>$R_m$ (MPa)</th>
<th>$R_m/R_{p0.2}$</th>
<th>$A_{gi}$ (%)</th>
<th>$E_t$ (GPa)</th>
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</thead>
<tbody>
<tr>
<td>WSH2</td>
<td>10</td>
<td>75</td>
<td>0.54</td>
<td>4</td>
<td>580.0</td>
<td>4</td>
<td>743.2</td>
<td>1.28</td>
<td>6.85</td>
<td>191.7</td>
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<tr>
<td></td>
<td>6</td>
<td>150</td>
<td>10b</td>
<td>6</td>
<td>577.8</td>
<td>9</td>
<td>715.8</td>
<td>1.24</td>
<td>7.06</td>
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</tr>
<tr>
<td>WSH3</td>
<td>12</td>
<td>75</td>
<td>0.82</td>
<td>6b</td>
<td>579.1</td>
<td>6c</td>
<td>673.8</td>
<td>1.16</td>
<td>6.92</td>
<td>208.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>150</td>
<td>9</td>
<td>2c</td>
<td>574.1</td>
<td>705.7</td>
<td>673.8</td>
<td>1.16</td>
<td>6.92</td>
<td>208.7</td>
<td></td>
</tr>
<tr>
<td>WSH4</td>
<td>12</td>
<td>50</td>
<td>0.82</td>
<td>6</td>
<td>579.1</td>
<td>705.7</td>
<td>673.8</td>
<td>1.16</td>
<td>6.92</td>
<td>208.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>150</td>
<td>6c</td>
<td>2c</td>
<td>574.1</td>
<td>705.7</td>
<td>673.8</td>
<td>1.16</td>
<td>6.92</td>
<td>208.7</td>
<td></td>
</tr>
</tbody>
</table>

$\phi_e$–$\phi_{wv}$ are edge and web vertical reinforcement diameters, $e$–$S_{wv}$ are edge and web vertical spacing of horizontal reinforcement, $\rho_t$ total reinforcement ratio.

Table B.1
Model parameters for shear and bond-slip mechanisms in critical region ($F$ (kN) and $M$ (kN m))

<table>
<thead>
<tr>
<th>Model Wall</th>
<th>Type</th>
<th>Shear force–lateral displacement relation ($F$–$\delta$)</th>
<th>Moment–fixed end rotation at the base ($M$–$\theta_H$)</th>
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</thead>
<tbody>
<tr>
<td>WSH2</td>
<td>Trilin</td>
<td>$\frac{IK_1}{IK_2}$ (–)</td>
<td>$\frac{IK_3}{IK_2}$ (–)</td>
</tr>
<tr>
<td>WSH3</td>
<td>Trilin</td>
<td>656.184</td>
<td>0.02234</td>
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<tr>
<td>WSH4</td>
<td>Bilin</td>
<td>463.636</td>
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<td>WSH6</td>
<td>Trilin</td>
<td>478.354</td>
<td>0.02216</td>
</tr>
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<td>WSH7</td>
<td>Trilin</td>
<td>1017.76</td>
<td>0.01460</td>
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</table>

$\frac{IK_1}{IK_2}$, $\frac{IK_3}{IK_2}$, $\frac{M_1}{M_2}$, $\frac{M_2}{M_2}$.
References