Voltage Multipliers and the Cockcroft-Walton generator

Jason Merritt and Sam Asare

1. Background

Voltage multipliers are circuits – typically consisting of diodes and capacitors, although there are variations using resistors and spark gaps – designed to take in an AC input with a certain amplitude and output a higher voltage at DC. Due to the need for extremely high voltages in high-energy physics and some kinds of safety testing – e.g., lightning safety – voltage multipliers have extremely important scientific and industrial applications, but are also used commonly in commercial products such as CRT televisions and LCD monitors. While the use of transformers in combination with a full-wave rectifier and capacitors could also output high, fairly stable voltages, they are larger, heavier, and more expensive to include in circuits than capacitors and diodes, limiting their commercial viability, in addition to their inherent safety risks. On the other hand, voltage multipliers are hardly ideal, and suffer both from being able to deliver only small amounts of current and from large ripple effects. In our project we examined experimentally the behavior of several voltage multiplier circuits.

2. Voltage Doublers

![Bridge doubler diagram]

For a bridge doubler with an AC input of amplitude A, the top terminal's voltage is equal to the bottom terminal's voltage + 2A. In the simplest case, the bottom terminal is connected to ground, so the circuit's output is simply 2A. On positive swings, the top diode conducts and delivers current to the top of the top capacitor, and on negative swings, the bottom diode conducts and delivers current to the region between the two capacitors. With the bottom half of the bottom capacitor held at ground, the
bottom capacitor charges (ideally) to a voltage A, and with the bottom half the of the top capacitor at about A, the top capacitor itself charges to a voltage A, leaving the top terminal at a total voltage of 2A above ground. While the top terminal's voltage may be raised by providing a higher DC voltage to the bottom terminal, this is not particularly effective for reaching higher voltages because it demands one already have access to a DC voltage source of X – 2A, where X is the desired voltage. In our experiment, we used the transformers used in some of the earlier labs, which produces a sine wave with an amplitude of roughly 12.5V. Connecting the bottom terminal to ground, the output voltage was found to be about 23.2V – slightly under the ideal 25V output – with a ripple voltage of 2.5V, nearly 10% of the signal itself.

![Greinacher doubler](image)

Another kind of voltage doubler – using the same components, but a different setup – is the Greinacher doubler. The circuit is again simplest to consider with the bottom terminal held at ground. Here, the capacitor on the left is charged through the left diode during negative swings, then on positive swings delivers some of its charge to the right capacitor; over the course of successive cycles the right capacitor is charged to nearly 2A above ground. The Greinacher circuit is interesting because it can be easily adapted to multiply the output voltage to higher values, as shown below:

3. Voltage Quadrupler

![Greinacher quadrupler](image)
The Greinacher quadrupler works in essentially the same way as the doubler, but in two parts; the bottom right capacitor charges to 2A with respect to ground, as the top right capacitor charges to 2A with respect to the bottom right capacitor, leaving the top terminal at 4A with respect to ground. Using the same 12.5V transformer and the lower terminal connected to ground, the output voltage was here found to be 40.4V with just 1V ripple. While this is a significantly higher output than the voltage doubler, it reaches just over 80% of its ideal output of 50V. In this case, the low frequency of the AC signal – due to being hooked up to an outlet running at just 60 Hz – has a much more significant impact on the behavior of the circuit, which operates much better with more charging cycles per second, as will be shown for voltage multipliers more generally later. For similar reasons, the ability of the circuit to supply current is disappointingly low; with a 1k resistor load to ground, the output of the circuit drops to just 0.7V, meaning it is not capable of delivering so much as 1 mA even to very small loads. For a much larger 100k load the voltage drops to 19.2V – still below half of its unloaded output – and even for a 1M load the voltage drops by 10% to 36.1V. Despite the circuit’s poor behavior, the fact that the Greinacher doubler was easily adapted to become a quadrupler indicates the likelihood of expanding on this design to create circuits that could multiply up to much higher voltages. Indeed, by making a small modification to the design and redrawing it in a more suggestive form, one retrieves the design for what is more commonly known as a Cockcroft-Walton generator.

4. Cockcroft-Walton generators
The Cockcroft-Walton generator design is much simpler, allowing the DC output voltage to increase – ideally, in steps, to arbitrary large voltages – by twice the amplitude of the AC input signal simply by attaching another "stage" consisting of two diodes and a capacitor, and works in virtually the same way as the Greinacher circuits (CW generators are sometimes also referred to as Greinacher multipliers). In our project, we examined in-depth the CW generator's behavior from just 1 stage all the way up to 10 stages. Our highest measured voltage output from this circuit was 264V at DC, all from a handful of components and an AC input signal with less than 15V amplitude.

### 4.1 Unloaded behavior

For an 11V AC square wave signal at 5 MHz with no load, the Cockcroft-Walton generator performs admirably, outputting nearly \((2n)\times11\) volts at DC for \(n\) stages, as shown below:

\[
\text{DC voltage output (no load)} \\
\text{AC input voltage: 11V}
\]

<table>
<thead>
<tr>
<th>Stages</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
</tr>
<tr>
<td>4</td>
<td>88</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
</tr>
<tr>
<td>6</td>
<td>136</td>
</tr>
<tr>
<td>7</td>
<td>160</td>
</tr>
<tr>
<td>8</td>
<td>184</td>
</tr>
<tr>
<td>9</td>
<td>206</td>
</tr>
<tr>
<td>10</td>
<td>225</td>
</tr>
</tbody>
</table>
In fact, for higher stages the output voltage is frequently higher than the expected value by as much as 8V. This is not an intrinsic property of the CW generator, but rather an effect due to the high frequency and circuit itself, which act on the AC signal from the function generator with nontrivial distortions visible on the oscilloscope that have the effect of slightly boosting the average amplitude of the AC signal itself for high numbers of stages; these slight distortions are then themselves magnified up to 20 times as they go up the chain.

4.1.1 Waveform dependence

The Cockcroft-Walton generator's efficiency to some extent depends on the input waveform. The output voltage is lowest for triangle waves, and highest for square waves, with sine waves in the middle; for small numbers of stages, these differences were small – particularly between the sine and triangle waves – but could result in differences of up to 10V in output between the square and sine wave inputs for higher stages. The Cockcroft-Walton generator favors square waves over sine waves – and sine waves over triangle waves – because the AC signal spending more time at the highest voltages allows the capacitors to charge faster.

4.1.2 Frequency dependence

While the Cockcroft-Walton generator's output voltage is nominally frequency-independent, the fact that it works not at all at DC combined with the impedances of the capacitors implies strong frequency dependence. In fact, the output voltage of the CW generator was found to approach its maximum value logarithmically on a large range of frequencies for the lower stages.
This model evidently breaks down at two ends: extremely low and extremely high frequencies. For very low frequencies the predicted behavior breaks down entirely as the circuit has trouble delivering charge up the chain; additional stages eventually result in a lower output voltage for small frequencies:

![Stage voltages at 10 Hz](image)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
</tr>
</tbody>
</table>

The other region where this behavior breaks down is evident on the graph; for frequencies above roughly 100 kHz, the voltage appears to suddenly start increasing faster. This, however, is another phenomena not intrinsic to the CW generator itself. Instead, it is a result of how the function generator generates square waves and that at very high frequencies it tends to “spike” above the supposed maximum amplitude at the beginning of switch. As shown below, this spike lasts proportionally longer during each cycle for very high frequencies, raising the “amplitude” of the input signal:

For very high numbers of stages, the logarithmic model works much more poorly, as shown for 10 stages on the next page:
At this point, the CW generator does not even begin to closely track the expected output value until frequencies of around 10 kHz, and the oscillations and distortions caused to the input AC are prominent enough that, as shown on the graph, increasing the frequency may actually decrease the output voltage. In this case, we found the generator's voltage to increase until reaching its maximum at roughly 3.8 MHz, then falling as the function generator approached its maximum frequency of just over 5 MHz.

4.2 Behavior under load

4.2.1 Voltage droop and current

CW generators with 1 through 10 stages were used with loads of 1M, 100k, 10k, and 1k resistors to determine how much the voltage droops under load and the current CW generators are capable of supplying. CW generators are, in general, capable of delivering only very small currents, which limits their usefulness in applications requiring high currents. In fact, while adding more stages can result in extremely large voltages – enough to shock one of the authors seven times while attaching loads! – it significantly limits the amount of current the generator is capable of delivering, meaning that, as shown below, the output voltage may be smaller for CW generators with more stages if the load is sufficiently small. All data values were taken for at very high frequencies – about 5 MHz – to avoid non-ideal behavior caused by lower frequencies.
As is clear from the tables, the CW generators were generally incapable of delivering more than 10 mA, with a maximum recorded output of 17 mA at 2 stages. Interestingly, although for the rest of the experiment we used 0.33 µF capacitors, here we tested other values and found that larger capacitors were capable of delivering slightly higher currents. For instance, a two-stage CW generator made with 560 µF capacitors was able to deliver 18.5 mA rather than 17.

4.2.2 Ripple voltage

Using a scope probe, the output waveforms of the CW generator was examined for 1 through 5 stages under resistor loads of 1k, 10k, 100k and 1M ohm resistance. As before, the CW generator's behavior is less and less ideal as additional stages are added, and in this case the ripple was larger the smaller the load is – i.e., the ripple is larger the more current the CW generator is being asked to deliver.
The ripple voltage can be a serious problem for CW generators with many stages; for a 5 stage CW generator with a 1k load, we found a ripple voltage of 5.3V – nearly half the supposedly “DC” 10.9V output. In addition, the ripple had an extremely unusual waveform for lower loads. Although the frequency of the main oscillation was measured to be about 5 MHz – the same frequency as the input signal – there were higher frequency oscillations, partially matching up with the distortions in the input signal. The ripple waveform for 3 stages with a 1k load is shown below:

4.3 Improvements

While high ripple voltage, low current, and even higher unloaded voltage drooping at constant frequency is a chronic problem for Cockcroft-Walton generators, there are a number of established ways to improve a CW generator's performance. The simplest is simply to mirror the design to create a full-wave CW generator. Shown below is a 2-stage full-wave CW generator.

\[ V_{\text{out}} = 4V_{\text{in}} \]

This “doubles” the input frequency in the sense that there twice as many charging cycles per second, which has the effective of lowering voltage droop as more stages are added, decreasing ripple voltage, and raising the maximum current output, all for the cost of just 50% more components.