Optimal food price stabilisation policy*

Christophe Gouel†

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Abstract

This paper proposes a framework for designing optimal food price stabilisation policies in self-sufficient developing countries. It uses a rational expectations storage model with risk-averse consumers and incomplete markets. Government stabilises food prices by carrying public stock and by applying state-contingent subsidies/taxes to production. The policy rules are designed to maximise intertemporal welfare. The optimal policy under commitment crowds out all private stockholding activity by removing the profit opportunity from speculation. It increases both consumer welfare and short-run producer profits. The countercyclical subsidy to production contributes little to welfare gains, most of which come from stabilisation achieved through public storage.

Keywords: food price stabilisation, incomplete markets, risk-aversion, welfare gains.

JEL classification: D52, Q11, Q18.

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†INRA, Économie Publique, AgroParisTech, 16 rue Claude Bernard, 75005 Paris, France, CEPII and École Polytechnique (christophe.gouel@inra.grignon.fr)
1 Introduction

Most developing countries pursue some form of food price stabilisation policy. For example, China and India operate very large stabilisation programmes (Dorosh, 2008). In China, government controls most international grain trade and carries large stocks (in the 2000s Chinese stocks of cereals—corn, rice, wheat—represented 40% of world stocks). India operates the world’s largest public food distribution system, covering about 600 million people: government builds stocks from procurements at minimum support prices and sells subsidised rations to the poorest people. The Egyptian government subsidises cooking oil, sugar, bread and flour (Löfgren and El-Said, 2001). Bread is also subsidised in Iran, involving an outlay of 6.5% of government expenses in 2001 (Amid, 2007). These policies have been largely analysed in the food policy literature with general lessons drawn from these experiences. However, the findings are generally qualitative, and few quantitative analyses of food policies have been conducted.

This paper defines a framework for studying food stabilisation policies in self-sufficient developing countries. We build a model representing the main features of food prices. The model includes a market imperfection justifying public intervention: market incompleteness associated with consumer risk aversion. The model features the behaviours of a competitive storer, a producer exposed to productivity shocks, and a risk-averse consumer. Prices are stabilised through an optimal policy under commitment using as instruments a countercyclical subsidy/tax on planned production and a public stock. This work builds on two existing approaches to commodity price stabilisation: the literature on competitive storage and the works of Newbery and Stiglitz.

Much attention has been paid to the study of commodity price stabilisation schemes (for a survey, see Wright, 2001), with the purpose mainly of analysing agricultural policies, such as deficiency payments or floor-price programmes (Miranda and Helmberger, 1988, Wright and Williams, 1988a, Glauber et al., 1989, Gardner and López, 1996). These works impose exogenous policy rules in the rational expectations storage model. Although very stylised, this model reproduces the most important features of commodity markets (Cañiero et al., forthcoming). A setting common to most studies based on this model is perfect market equilibrium, which implies that there is no rationale for public intervention, and stabilising markets beyond what is done by private agents is a waste of public money.

One exception is the work of Gardner (1979) who considers that periods of high food prices generate external costs that justify public intervention. To increase welfare, he designs a discretionary optimal stockpiling policy. He also discusses price-band policies and analyses the uncertainties that could hinder the implementation of such policies. Unlike Gardner (1979), the present paper is more explicit about the source of market failure in considering market incompleteness, which is in line with the abundant literature on the cost of commodity price instability against which consumers are assumed to be unable to insure (Waugh, 1944, Turnovsky et al., 1980, Helms, 1985b, Wright and Williams, 1988b). It also makes a methodological contribution since, in addition to solving

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1Its traditional form is presented and analysed in depth by Wright and Williams (1982a).
the model numerically, it follows the modern literature on optimal dynamic policies (Marcet and Marimon, 1999) in allowing the derivation of first-order conditions for an optimal dynamic policy, which delivers interpretable conditions.

Newbery and Stiglitz’s approach departs from the framework of the infinite-horizon storage model. Newbery (1989) shows that market incompleteness and consumer risk aversion justify government intervention in the form of public storage or distribution of food rations. This study enlarges on his previous work with Stiglitz, which considers the role of public intervention in incomplete commodity markets (Newbery and Stiglitz, 1981, 1982, 1984). Their work exploits two-period models, which cannot account for the stylised facts of food prices. For example, commodity prices are positively serially correlated (Deaton and Laroque, 1992), so storage management must account for the fact that high prices may be followed by high prices, making it suboptimal to dump all the stock in one period, as would be recommended by Newbery’s model. More generally, two-period models allow a single optimal stockpile size to be identified, whereas any applications based on a realistic framework require an optimal storage rule that stipulates how stock should be accumulated or sold given the prevailing market conditions. Such a storage strategy is the result of an infinite-horizon problem, as in the present paper.

Our analysis provides three main findings. First, on the transitional dynamics following policy implementation, provided that the budget share of food and risk aversion are sufficiently high, there is an optimal food price stabilisation policy that improves both consumers’ and producers’ welfare. The optimal policy consists of a public storage rule and a subsidy/tax on planned production, which works like a set-aside programme. The management of public stock is such that it removes any profit opportunity from speculation and crowds out all private storage. Second, producers enjoy short-run gains because the launch of the policy is accompanied by a phase of transitional stock build-up which pushes up prices. In the long-run, producers may lose because stabilisation entails lower prices on average. Third, the countercyclical subsidisation of production contributes little to welfare gains, most of which come from stabilisation through public storage.

Section 2 defines the stylised facts on food prices and evaluates the potential gains to be expected from a stabilisation policy by calculating the welfare gains from perfect stabilisation. Section 3 describes the storage model without public policy. Section 4 presents the model under optimal policy. We define a social welfare function that serves as policy objective, and then derive and analyse the first-order conditions of the optimal policy problem. In Section 5, we calibrate the model and analyse the numerical results, discussing the various decision rules, then separately analysing transitional and asymptotic behaviours. Section 6 concludes.

2 Welfare gains from an ideal stabilisation

Before we consider the effect of a stabilisation policy, this section evaluates the consumer’s gain from perfect stabilisation, i.e., costless stabilisation at a given price. It allows us to quantify the
potential welfare gains. This section draws on the works of Turnovsky et al. (1980), Helms (1985a), and Wright and Williams (1988b).

Because stabilisation gains depend heavily on the initial price distribution, some facts about staple prices are presented in Table 1. The first moments of the price distributions of maize, rice and wheat show similar properties. Detrended cereal prices show a high and positive one-year autocorrelation, between 0.4 and 0.61, meaning that periods of high or low prices tend to come in a row. The coefficient of variation is around 20%. Prices are positively skewed, so the tails of the price distribution are longer for high prices than low prices, and most of the values are below the mean price—a pattern often related to storage. Visually, agricultural prices display long periods of doldrums followed by booms and busts. The model proposed in the next section tries to account for these facts.

\begin{table}[h]
\centering
\caption{Main cereals price facts, 1900–2003}
\begin{tabular}{lccc}
\hline
 & Maize & Rice & Wheat \\
\hline
One-year autocorrelation & 0.40 & 0.61 & 0.57 \\
Coefficient of variation & 0.20 & 0.20 & 0.18 \\
Skewness & 0.68 & 0.98 & 0.98 \\
\hline
\end{tabular}
\end{table}

\textit{Note:} Statistics obtained after HP-filtering (smoothing parameter of 400) of the logarithm of price indexes deflated by the manufactures unit value index.

\textit{Source:} Underlying data are annual commodity price indexes from Pfaffenzeller et al. (2007).

Modern analysis of the welfare effect of price stabilisation started with Waugh (1944). Using a Marshallian surplus analysis, Waugh demonstrated that price stabilisation makes consumers worse off. This counterintuitive proposition stems from the negative slope in the consumer demand curve, which allows consumers to benefit from low prices by consuming more, and to limit the nuisance of high prices by consuming less. Marshallian consumer surplus analysis, however, rests on the hypothesis of an income elasticity equal to the coefficient of relative risk aversion (Turnovsky et al., 1980). In the absence of this hypothesis, the welfare change from stabilisation should be evaluated by the equivalent variation, $EV$, implicitly defined by

$$E \left[ v (P, Y + EV) \right] = E \left[ v (\tilde{P}, Y) \right],$$

with $v (\cdot, \cdot)$, an indirect utility function; $P$, the price before stabilisation; $\tilde{P}$, the stabilised price; $Y$, the income; and $E (\cdot)$, the expectation operator. The price after stabilisation follows a probability distribution with lower variance than the price before stabilisation. Using subscripts to denote partial derivatives, the equivalent variation can be approximated (see the appendix for the derivation) at the second order by

$$EV \approx \left[ v_P (\tilde{P}, Y) \Delta \tilde{P} + v_{PP} (\tilde{P}, Y) \Delta \sigma^2_P / 2 \right] / v_Y (\tilde{P}, Y)$$

$$\approx D (\tilde{P}, Y) \cdot \left\{ -\Delta \tilde{P} + \left[ \gamma (\eta - \rho) - \alpha \right] \cdot \Delta \sigma^2_P / (2 \tilde{P}) \right\},$$
where $\alpha < 0$ and $\eta$ stand for the price and income elasticities of demand; $D(\bar{P}, Y)$ is demand at the mean price, $\bar{P}$; $\Delta \bar{P}$ and $\Delta \sigma_{\bar{P}}^2$ are the changes in mean price and variance; and $\gamma$ and $\rho$ are the commodity budget share and the relative risk aversion parameter.

Using Roy’s identity, the second term on the right-hand side of equation (2) can be reformulated as $(-\nu_{PP}/\nu_P) \cdot (D\Delta \sigma_{\bar{P}}^2/2)$, which isolates a term similar to a coefficient of absolute risk aversion, but defined here with respect to price risk. This coefficient is shown in (3) to depend on relative risk aversion, commodity budget share, and price and income elasticity.

Waugh’s result is apparent in equation (3). For stabilisation at the mean price (i.e., $\Delta \bar{P} = 0$) of a risk-neutral consumer with income inelastic demand ($\rho = \eta = 0$), the equivalent variation is equal to $-\alpha \Delta \sigma_{\bar{P}}^2 \cdot D(\bar{P}, Y) / (2\bar{P})$, which is always negative. Risk aversion can compensate for this effect and make stabilisation beneficial only if the budget share is sufficiently high. It implies that stabilisation at the mean price would be detrimental to consumers from developed countries, since they devote a low share of their budget to staple food.

To extend these qualitative statements, we provide a numerical illustration. The equivalent variation defined in equation (3) is calculated in Table 2 for complete stabilisation of a price distribution with mean 1 and standard deviation 0.2. Demand at mean prices is assumed to be equal to $\gamma Y$. By assuming income to be equal to 100, the equivalent variation can be interpreted as a percentage change in income. Two stabilisation policies are considered: stabilisation at the mean price and stabilisation at 2% below the mean price. The latter situation accounts for the possible depressing effect of storage policies on prices (Wright and Williams, 1982a).

**Table 2. Welfare change for consumers from perfect stabilisation of food prices** (as a percentage of income). The mean of the price distribution is equal to 1 and standard deviation is equal to 0.2.

<table>
<thead>
<tr>
<th>$(\eta, \alpha)$</th>
<th>$\rho$:</th>
<th>0</th>
<th>0.15</th>
<th>0.3</th>
<th>0</th>
<th>0.15</th>
<th>0.3</th>
<th>0</th>
<th>0.15</th>
<th>0.3</th>
<th>0</th>
<th>0.15</th>
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<tr>
<td>$\gamma$:</td>
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<tr>
<td>Stabilisation at mean price</td>
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<tr>
<td>(0.00, -0.1)</td>
<td>-0.00</td>
<td>-0.03</td>
<td>-0.06</td>
<td>-0.00</td>
<td>0.11</td>
<td>0.48</td>
<td>-0.00</td>
<td>0.24</td>
<td>1.02</td>
<td>-0.00</td>
<td>0.38</td>
<td>1.56</td>
<td></td>
</tr>
<tr>
<td>(0.25, -0.1)</td>
<td>-0.00</td>
<td>-0.04</td>
<td>-0.11</td>
<td>-0.00</td>
<td>0.09</td>
<td>0.44</td>
<td>-0.00</td>
<td>0.23</td>
<td>0.98</td>
<td>-0.00</td>
<td>0.36</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>(0.25, -0.4)</td>
<td>-0.01</td>
<td>-0.13</td>
<td>-0.29</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.26</td>
<td>-0.01</td>
<td>0.14</td>
<td>0.79</td>
<td>-0.01</td>
<td>0.27</td>
<td>1.34</td>
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<tr>
<td>(0.50, -0.4)</td>
<td>-0.01</td>
<td>-0.14</td>
<td>-0.33</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.21</td>
<td>-0.01</td>
<td>0.13</td>
<td>0.75</td>
<td>-0.01</td>
<td>0.26</td>
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<td>(0.50, -0.7)</td>
<td>-0.01</td>
<td>-0.23</td>
<td>-0.51</td>
<td>-0.01</td>
<td>-0.10</td>
<td>0.03</td>
<td>-0.01</td>
<td>0.04</td>
<td>0.57</td>
<td>-0.01</td>
<td>0.17</td>
<td>1.11</td>
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<tr>
<td>Stabilisation at 2% below mean price</td>
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<tr>
<td>(0.00, -0.1)</td>
<td>0.02</td>
<td>0.27</td>
<td>0.54</td>
<td>0.02</td>
<td>0.41</td>
<td>1.08</td>
<td>0.02</td>
<td>0.54</td>
<td>1.62</td>
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<tr>
<td>(0.25, -0.1)</td>
<td>0.02</td>
<td>0.26</td>
<td>0.50</td>
<td>0.02</td>
<td>0.39</td>
<td>1.04</td>
<td>0.02</td>
<td>0.53</td>
<td>1.58</td>
<td>0.02</td>
<td>0.66</td>
<td>2.12</td>
<td></td>
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<tr>
<td>(0.25, -0.4)</td>
<td>0.01</td>
<td>0.17</td>
<td>0.32</td>
<td>0.01</td>
<td>0.30</td>
<td>0.86</td>
<td>0.01</td>
<td>0.44</td>
<td>1.40</td>
<td>0.01</td>
<td>0.57</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>(0.50, -0.4)</td>
<td>0.01</td>
<td>0.16</td>
<td>0.27</td>
<td>0.01</td>
<td>0.29</td>
<td>0.81</td>
<td>0.01</td>
<td>0.43</td>
<td>1.35</td>
<td>0.01</td>
<td>0.56</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>(0.50, -0.7)</td>
<td>0.01</td>
<td>0.07</td>
<td>0.09</td>
<td>0.01</td>
<td>0.20</td>
<td>0.63</td>
<td>0.01</td>
<td>0.34</td>
<td>1.17</td>
<td>0.01</td>
<td>0.47</td>
<td>1.71</td>
<td></td>
</tr>
</tbody>
</table>

*Notes:* The first column displays income elasticity and own-price elasticity. The first two rows display relative risk aversion and budget share.

Stabilisation at mean prices is always detrimental to consumers spending 1% of their budget on the volatile commodity, but the welfare loss is negligible given the low budget share for the consumer. It is also detrimental to a risk-neutral consumer. The importance of the loss depends on the budget
share and the elasticities, especially the price elasticity. Income elasticity has a limited effect on all the results. In the above approximation, it is supposed to have the same effect as the risk aversion parameter, but, given its limited range of possible variations, it has little effect on welfare. If we consider a relative risk aversion of 9 and a budget share of 30%, the welfare gains can reach 1.56% of income.

When prices are stabilised at 2% below their mean, the mean price change is responsible for a large share of overall gains. In particular, consumers with low risk aversion or low budget share earn most of their gains from the change in the mean price.

This analysis motivates the fact that in the following we consider a model calibrated to the situation of a developing countries where consumers are likely to suffer from price instability of food because of the high budget share they devote to it.

3 The model without public intervention

Analysis of food price stabilisation policy should take account of the origin of food price volatility and reproduce its main features, as characterised in Table 1. The model used in most of the work on commodity price stabilisation schemes, such as Miranda and Helmberger (1988) and Wright and Williams (1988a), is the single-country competitive storage model. Cafiero et al. (forthcoming) recently proved that this model performs well for explaining international commodity price behaviour.

Our framework extends this model to account for consumer risk aversion, market incompleteness, and government stabilisation policy. Time is discrete. This partial equilibrium model features an annual market for a storable commodity with a competitive storer, a producer whose output is subject to multiplicative shocks, and a final demand.

Despite the fact that international prices are an important driver of the domestic price in many countries, our focus is on a closed economy and we assume that domestic productivity shocks are the only source of dynamics. Agricultural markets are the object of many protectionist policies (Anderson, 2010), including self-sufficiency (e.g., cereals in China and India), making the autarkic case a not uncommon situation.

3.1 Consumers

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic specification: $D(P_t, Y) = dP_t^\alpha Y^\eta$, where $d$ is a normalisation parameter. Assuming there are only

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2In an estimation of food consumption elasticities, Seale and Regmi (2006) find income elasticities for breads and cereals between 0.04 and 0.62 for 114 countries.
two goods and the second good is the numeraire, the integration of this demand function gives the following instantaneous indirect utility function (Hausman, 1981)

\[ \hat{v}(P_t, Y) = \frac{Y^{1-\eta}}{1-\eta} - \frac{P_t^{1+\alpha}}{1+\alpha}. \] (4)

This utility function has a relative risk aversion equal to the income elasticity of demand. To distinguish income elasticity from risk aversion, we follow Helms (1985a) and apply a monotone transformation to the indirect utility function,

\[ v(P_t, Y) = \frac{\hat{v}(P_t, Y)^{1+\theta}}{1+\theta}. \] (5)

This specification is still consistent with the isoelastic demand function, but its coefficient of relative risk aversion is

\[ \rho(P_t, Y) = \eta - \theta \frac{Y^{1-\eta}}{\hat{v}(P_t, Y)}, \] (6)

with \( \theta \) indexing the degree of risk aversion.\(^3\)

The representative consumer is assumed to adopt hand-to-mouth behaviour. He consumes current income and does not save to smooth out fluctuations. This assumption simplifies the dynamics of the problem, since consumer’s “cash on hand” does not have to be included as a state variable; however, it overestimates the effect of public policy by neglecting the possibility of self-insurance added by saving.\(^4\)

Given the absence of saving, the consumer does not solve an intertemporal problem. At each period, the consumer is concerned only with current-period demand, which is not affected by the degree of risk aversion. So speculators face the same profit opportunities if the economy is populated by risk-averse or risk-neutral consumers. This absence of effect of risk aversion on demand creates the need for public intervention, since private storers do not account for it.

### 3.2 Storers

There is a single representative speculative storer, which is risk neutral and acts competitively. Its activity is to transfer a commodity from one period to the next. Storing the quantity \( S_t^s \) from period \( t \) to period \( t+1 \) entails a purchasing cost, \( P_t S_t^s \), and a storage cost, \( k S_t^s \), with \( k \) the unit physical cost of storage. The benefits in period \( t \) are the proceeds from the sale of previous stocks: \( P_t S_{t-1}^s \).

\(^3\)Following the same method, other demand curves could have been contemplated. Few, however, have an indirect utility function with a known form. In such case, the numerical algorithm of Vartia (1983) can be used to find a numerical approximation of the indirect utility function.

\(^4\)Introducing saving would create important difficulties, such as the need to consider borrowing constraint, which creates strong nonlinearities (Deaton, 1991), whereas the model proposed below is already strongly nonlinear.
The storer follows a storage rule that maximises its expected profit as stated by

$$\max_{\{S_{t+i} \geq 0\}_{i=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ P_{t+i} S_{t+i-1}^s - (P_{t+i} + k) S_{t+i}^s \right] \right\},$$

with \( \beta = 1/(1 + r) \), the discount factor. The storer’s problem can be expressed in a recursive form using the following Bellman equation:

$$V^S(S_{t-1}^s, P_t) = \max_{S_t \geq 0} \left\{ P_t S_{t-1}^s - (P_t + k) S_t^s + \beta E_t \left[ V^S(S_t^s, P_{t+1}) \right] \right\}.$$  

This equation has two state variables: the price, whose dynamics is considered by the storer to be exogenous, and the stock carried over from the previous year. Using the first-order condition on \( S_t^s \) and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition\(^5\)

$$S_t^s \geq 0 \perp \beta E_t (P_{t+1}) - P_t - k \leq 0,$$

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. So this is a situation of a stabilising speculation, the storer buys when prices are low and when he rationally expects that they will be higher later.

### 3.3 Producers

A representative producer makes his productive choice one period before bringing output to market. He puts in production in period \( t \) a level \( H_t \) for period \( t+1 \), but a multiplicative disturbance affects final production (e.g., a weather disturbance). The producer chooses the production level by solving the following maximisation of expected profit:

$$\max_{\{H_{t+i} \}_{i=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ P_{t+i} \epsilon_{t+i+1} H_{t+i-1} - \Psi (H_{t+i}) \right] \right\},$$

where \( \Psi (H_t) \) is the cost of planning the production \( H_t \) and \( H_t \epsilon_{t+1} \) is the realised level. \( \epsilon_{t+1} \) is the realisation of an i.i.d. stochastic process of mean 1 exogenous to the producer, which follows a translated beta distribution. Like the storer’s problem, this problem can be reformulated into a recursive form, and its solution gives the following Euler equation:

$$\beta E_t (P_{t+1} \epsilon_{t+1}) = \Psi' (H_t).$$

\(^5\)Complementarity conditions in what follows are written using the “perp” notation \( (\perp) \). This means that the expressions on either side of the sign are orthogonal. If one equation holds with strict inequality, the other must have an equality.
This equation has a straightforward interpretation: it is the equality between the marginal cost of production and the expected discounted marginal benefit of one unit of planned production. The production cost function is assumed to be convex, as increasing production requires increasing the use of less fertile lands, and to follow an isoelastic form

$$\Psi (H) = h \frac{H^{1+\mu}}{1+\mu},$$

where $h$ is a scale parameter and $\mu$ is the inverse of supply elasticity.

### 3.4 Recursive equilibrium

At the beginning of each period, three predetermined variables define the state of the model: $S_{t-1}$, $H_{t-1}$ and $\epsilon_t$. They can be combined in one state variable, availability, the sum of production and private carry-over:

$$A_t = S_{t-1}^s + H_{t-1}\epsilon_t.$$

(13)

Market equilibrium can be written as

$$A_t = D(P_t) + S_t^s.$$

(14)

From the above, we can define the recursive equilibrium of the problem without public policies:

**Definition.** In the absence of stabilisation policy, a recursive equilibrium is a set of functions, $S^s (A)$, $H (A)$ and $P (A)$, defining storage, production and price over the state $A$ and the transition equation (13) such that (i) storer solves (7), (ii) producer solves (10), and (iii) the market clears.

### 4 Optimal policy approach

We assume that government is able to commit at the first period to following a policy rule that maximises expected intertemporal welfare (the so-called Ramsey policy). We design the optimal policy by following the modern literature on optimal policy in dynamic economies (Marcet and Marimon, 1999). As the policy is conceived to correct for market incompleteness, i.e., the inability of consumers to insure against food price volatility, the first-best policy would be to mimic the role of the futures market by providing state-contingent cash payments to consumers. In the real world, few policies work in this way, but some can be related to this logic. Food and cash-for-work schemes, for example, are labour-intensive work programmes offering in kind or cash payments (Barrett, 2002). They are designed to combat food insecurity and to target those suffering adverse shocks. Other types of policies more frequent in the developing world are food price stabilisation policies. Because of their practical importance, we focus on these second-best policies, which target price behaviour.

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6As income is assumed to be constant, in what follows demand function is only expressed as a function of price.
Here, the stabilisation instruments are subsidy/tax on planned production and public stocks, where government is assumed to be able to manage storage facilities at the same costs as private storers.

The policy starts at period 0 and is unanticipated by the agents. The parameters of the stabilisation policy are determined by maximising a social welfare function that aggregates consumer utility, other agents’ surpluses and fiscal cost. Initial availability is taken as being at its deterministic steady-state level and initial private stocks are assumed to be null.

### 4.1 Social welfare function

Working with a partial equilibrium model imposes some constraints. For example, it is not possible to take the expected sum of discounted consumer’s utility as the objective of the policy authority as is often done in optimal policy problems. Instead, all effects on the welfare of other agents must be included in the objective. In a general equilibrium model, there is no need for this since the consumers’ income includes all earnings. The approach adopted here also differs from most partial equilibrium analyses due to its consideration of risk aversion. In partial equilibrium models, policy design is based on maximising the sum of all agents’ surpluses. This does not take account of risk aversion, though, since the expected consumer surplus is not a measure of the welfare of risk-averse agents (Helms, 1985b, Stennek, 1999).

Instead of using the sum of agents’ surpluses, we have to define the preferences of the government. We assume that government tries to maximise a social welfare function that weights linearly the welfare of each agent. We want to leave aside all distributional considerations. For this purpose, the determination of weights has to ensure that transfers between agents do not affect welfare. The weights should remove the transfers and leave in the social welfare function only efficiency gains from stabilisation and the cost of the policy. At period $t$, social welfare is given by:

$$ W_t = v(P_t, Y) + w_H [P_t H_{t-1} + \zeta_t H_t - \Psi (H_t)] + w_S [P_t S_{t-1} - (P_t + k) S_t] - w_C Cost_t, $$

(15)

where $w_H$, $w_S$ and $w_C$ are the weights given in social welfare to producers’ income, storers’ realised profit and fiscal cost; $\zeta_t$ is a state-contingent subsidy/tax on planned production, received by producers at planting time; and $Cost_t$ is the fiscal cost of the policy. Since producers and storers are both assumed to be risk neutral, if government has no distributional bias, there is no reason to weight their profits differently, so their weights can be considered equal, $w_S = w_H$. Defined in this way, the social welfare function shows that the optimal policy will consist in a risk-sharing arrangement between risk-averse consumers and risk-neutral agents, government in particular.

To rule out transfers between agents, we need to know how the consumer values a monetary transfer in utility terms. For a given price $P$, a small monetary transfer $\tau$ to the consumer increases utility by approximately (at the first order) $v_Y (P, Y) \tau$. Thus, we could use the marginal utility with respect to income as the weight. But, given that the price is stochastic, we have to consider the marginal utility of income over all the price distribution and not just for one price. The computation is based
on the ergodic distribution of price without public policies:

\[ w_H = E[v_Y (P, Y)] . \]  \hspace{1cm} (16)

This is the unconditional expectation of marginal utility over income.\textsuperscript{7}

Although also a monetary value, fiscal cost does not necessarily have the same weight as agents' profit. Unless raised by a lump-sum tax, income raised for public expenses entails distortionary costs due to revenue collection. We ignore these costs and assume \( w_C = w_H \). This assumption can be removed, but turns out to be convenient as described below.

The cost of the policy is the cost of carrying public stock plus the cost of subsidising planned production. The cost of public storage is similar to the profit from private storage. It is equal to the difference between the purchasing plus the storage costs of new stock and the revenue derived from selling previous stock, \( S_{t-1}^C \):

\[ \text{Cost}_t = (P_t + k) S_t^C - P_t S_{t-1}^C + \zeta_t H_t . \]  \hspace{1cm} (17)

Because of public stock, availability is now defined by

\[ A_t = S_t^s + S_{t-1}^C + H_{t-1} , \]  \hspace{1cm} (18)

Using \( w \) as the unique weight of monetary terms in social welfare, we can write social welfare as

\[ W_t = \Psi (H_t) - (P_t + k) (S_t^s + S_t^C) . \]  \hspace{1cm} (19)

Here the common money measure is used to sum the predetermined terms, \( H_{t-1} , S_t^s , S_{t-1}^C \), to obtain current availability, \( A_t \). This means that it does not matter for the welfare evaluation who holds the availability; whether it is in the producers' hands, in private stock or in public stock is a matter of indifference. As a result, production, private and public stock need not be considered as separate state variables. They are combined in one state variable, which is very convenient for the numerical solution since complexity increases exponentially with the number of state variables.\textsuperscript{8}

\textsuperscript{7}This weighting scheme takes as reference the situation without intervention, so it is defined in an equivalent variation approach. In a compensating variation logic, the weight would be different and would be equal to the unconditional expectation of marginal values of utility over income on the ergodic distribution of price after stabilisation. Numerically, they do not differ much, but the compensating variation approach poses the problem that the welfare weight is endogenous to the optimal policy determination.

\textsuperscript{8}Other social welfare functions could have been contemplated (McLaren, 1997, Slesnick, 1998, Im, 2001). Agricultural and food policies are often designed to serve particular interests such as those of the urban poor in developing countries or of farmers in developed countries, alternative weighting schemes could reflect such distributional biases. The weighting scheme chosen here, however, has the advantage to focus on efficiency effects; it obeys the compensation principle and leads to easily interpretable equations.
4.2 Optimal policy problem

The government maximises the expected sum of the discounted instantaneous social welfare function

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ v(P_t, Y) + w [P_t A_t - \Psi(H_t) - (P_t + k) (S_t^s + S_t^c)] \right\}, \tag{20} \]

subject to the constraints imposed by private agents’ behaviour and market equilibrium. In the previous equation, it is assumed that the consumer’s rate of time preference is equal to the discount parameter, \( \beta \), which corresponds to a discount by the real interest rate. This equality does not generally occur, but this assumption avoids the notational clutter of having to use two parameters to discount utility and monetary terms separately.

Among the constraints imposed by private agents’ behaviour, the first-order condition for the storers’ behaviour (9) is a complementarity equation and cannot enter directly as constraint in a maximisation problem. To restate this complementarity equation, we introduce a slack variable, \( \phi \), with its associated complementarity slackness conditions

\[ \phi_t = P_t + k - \beta E_t (P_{t+1}), \tag{21} \]

\[ S_t^s \phi_t = 0, \tag{22} \]

where \( S_t^s \geq 0 \) and \( \phi_t \geq 0 \). \( \phi \) refers to the marginal loss from storage. If positive, arbitraging by storage entails a loss in expectation, and so there is no private storage. If it is null, prices are arbitraged between the two successive periods.

The first-order condition for producers now includes the subsidy/tax and is

\[ \beta E_t (P_{t+1} \epsilon_{t+1}) + \zeta_t = \Psi'(H_t). \tag{23} \]

The market equilibrium equation includes the public stock:

\[ A_t = D(P_t) + S_t^s + S_t^c. \tag{24} \]

By using equations (18) and (21)–(24) as constraints to the maximisation problem, introducing the corresponding present-time Lagrange multipliers, and applying the law of iterated expectations, the
government choice depends on both the natural state variable, of private agents. The need to satisfy private expectations makes the problem history-dependent. Indeed in a rational expectations equilibrium, government confirms the earlier expectations actions. From the first-order conditions, and after some manipulation we get the following system of complementarity conditions, which define the dynamics of the optimal policy under commitment

\[
\begin{align*}
S_t^g : S_t^g &\geq 0 \quad \downarrow \quad -wP_t - \chi_t - w k + \beta E_t (w P_{t+1} + \chi_{t+1}) + \delta_t \phi_t \leq 0, \\
H_t : \beta E_t (\chi_{t+1} \epsilon_{t+1}) - w \zeta_t - \nu_t \Psi'' (H_t) &= 0, \\
P_t : v_P (P_t, Y) + w D (P_t) - \lambda_t + X_t - \chi_t D' (P_t) &= 0, \\
S_t^c : S_t^c &\geq 0 \quad \downarrow \quad -wP_t - \chi_t - w k + \beta E_t (w P_{t+1} + \chi_{t+1}) \leq 0, \\
\phi_t : \phi_t &\geq 0 \quad \downarrow \quad \lambda_t + \delta_t S_t^g \leq 0, \\
\lambda_t : \phi_t &= P_t + k - \beta E_t (P_{t+1}), \\
\delta_t : S_t^g \phi_t &= 0, \\
\zeta_t : \nu_t &= 0, \\
\nu_t : \beta E_t (P_{t+1} \epsilon_{t+1}) + \zeta_t &= \Psi'' (H_t), \\
\chi_t : A_t &= D (P_t) + S_t^g + S_t^c,
\end{align*}
\]

with transition equations (18) and

\[X_t = \lambda_{t-1} + \nu_{t-1} \epsilon_t.\]

Corresponding to the two forward-looking equations (21) and (23), two Lagrange multipliers, \(\lambda\) and \(\nu\), appear as lagged variables in the first-order conditions. As expressed in (36), they are summed together in the state variable \(X\). The introduction of these lagged variables as new state variables gives the problem its recursive form (Kydland and Prescott, 1980, Marcet and Marimon, 1999). Government, in its decisions, has to account for private agents’ forecasts of its own future actions. Indeed in a rational expectations equilibrium, government confirms the earlier expectations of private agents. The need to satisfy private expectations makes the problem history-dependent. Government choice depends on both the natural state variable, \(A\), and the history of the state.
The role of the lagged multipliers is to summarise this trajectory by measuring the social cost of confirming past expectations in current behaviour.

4.3 Discussion of first-order conditions

The first-order conditions can be interpreted as follows. The first-order condition on public storage (29) is an arbitrage equation in the utility metric. While similar to the arbitrage equation of private storage without policy (9), it differs in one important aspect: public storage arbitrages the total marginal values of the commodity across two consecutive periods, while private storage arbitrages only on the private marginal values. The total marginal value of the commodity is the sum of its social marginal value, summarised by the Lagrange multiplier on market clearing equation, $\chi$, and its private marginal value (i.e., price) converted in the utility metric, $wP$.

Equation (33) states that the Lagrange multiplier on the producers Euler equation is zero. It means two things, which are related. First, the condition on producers’ behaviour does not constrain welfare maximisation and, second, the forward-looking behaviour of producers does not imply any time-inconsistencies for the optimal policy. More specifically, a technical distinction between a policy with a credible precommitment and a discretionary policy would be that in the former case the policy maker can manipulate expectations, but not in the latter. Here, the policy maker can use an instrument equivalent to manipulating the producers’ expectations since he can choose the level of the subsidy, which, in terms of the producers’ behaviour, has the same incentive effect as changing expectations.

Using $\nu_t = 0$, the first-order condition on production (27) can be written as

$$\zeta_t = \beta \mathbb{E}_t (\chi_{t+1} + 1) / w,$$

which has a straightforward interpretation. The subsidy is equal to the discounted expected marginal social value in the money metric of one unit of planned production. This implies that production is no longer planned with respect to prices only, as in equation (11), but is based on total marginal values.

Private storers’ behaviour is hard to identify in the first-order conditions. So we rely on intuitions confirmed in the numerical analysis carried in the next section. Private and public stocks have perfectly symmetric roles in the social welfare function, market equilibrium and definition of availability, since, in these equations, they are always summed. As policy instruments they have the same effects on price dynamics, even though private storage imposes an additional constraint: private storers are profit-seeking and behave according to equations (21)–(22). Since the two instruments are equivalent, but private storage imposes constraints, the maximisation should privilege public storage, and as long as public storage leads to higher stock level than private storage it should crowd out this latter.\footnote{There is one situation where there is no crowding out: the situation of a risk-neutral consumer. In this case,
In the absence of private storage, price behaviour can be characterised through equation (29), by reformulating it as

\[ P_t = \max \left[ D^{-1}(A_t), \beta E_t \left( P_{t+1} + \chi_{t+1}/w \right) - \chi_t/w - k \right]. \]  

(38)

This equation defines a two-regime behaviour for the price function: for low availability there is no storage, price is defined by the inverse of the demand function at current availability (first term). For higher availability, public storage links current price to the future marginal value of the commodity.

The issue of commitment comes from the expectations of private agents. Producers' expectations are indirectly under the control of the subsidy/tax on planned production. And since, under an optimal policy, speculative storers do not stock anything, their expectations are irrelevant for public decision. Hence, the problem can be simplified. Without private stock, equation (21) does not impose any constraint and \( \lambda = X = 0 \). The state variable \( X \) is introduced to account for the effect of lagged multipliers on current decision. These multipliers being null, the problem has one state variable: availability. The crowding out of private storage implies that there is no commitment problem. The public decision would be identical under commitment and under discretion.

This absence of commitment is specific to the combination of the instruments implemented. The government takes control of all intertemporal decisions, so it does not face any commitment problem. If only one instead of two instruments of stabilisation is considered, the distinction between commitment and discretion re-emerges. Stabilisation through public storage without production-related policy would crowd out private storage, but producers' expectations of future public actions would create a time-inconsistency problem. The same applies to a policy using only the subsidy/tax on planned production: storers' expectations affect the time-consistency of the policy. Since in analysing numerical results it is important to identify the contribution of each policy instrument to stabilisation, in the following section, we simulate the model with each instrument separately.

From the previous system of equations, an optimal public storage policy is obtained by removing equation (33) and all appearances of \( \zeta \). An optimal policy of a subsidy/tax to production is the solution of the previous system without equation (29) and terms in \( S^c \).

Equation (28) gives some insights into the behaviour of the social marginal value of the commodity. Using \( \lambda = X = 0 \) and Roy’s identity, this equation gives

\[ \chi_t = \frac{P_t}{\alpha} \left[ w - v_Y (P_t, Y) \right]. \]  

(39)

The sign of \( \chi_t \) can be discussed by comparing \( w \) and \( v_Y \). \( w \) is the average value of marginal utility over income for the price distribution without stabilisation. If the stabilisation is not too important (and the stabilisation policy cost will prevent it), the marginal utility over income should still fluctuate around \( w \) and the sign of \( \chi_t \) should alternate. For a relative risk aversion superior to income

there is no rationale for public intervention and the optimal policy is to stockpile exactly as would be done by the private storer alone. Here, equations (21)–(22) do not impose constraints since the optimal stock level is the profit maximising one (Scheinkman and Schechtman, 1983).
elasticity, \( v_Y \) increases with price.\(^\text{10}\) So the social marginal value of the commodity is positive for low prices, and correspondingly high availability, and negative for high prices, and correspondingly low availability. This implies that the optimal policy should try to move the commodity from periods of high availability, when the social marginal value is negative, to periods of low availability, when it is positive. This discussion sheds some light on the behaviour of the subsidy/tax on production as defined by (37). When the next-period availability is expected to be high, which implies that a large stock is carried over, producers are taxed in order to limit supply in a period with negative social marginal value. On the contrary, for low stock levels, the next-period availability is expected to be low and production is subsidised.

5 Numerical results

The model without stabilisation policy, and a fortiori the model with optimal policy, cannot be solved analytically. Decision rules must be approximated numerically. The problems are solved under Matlab using a projection method with a collocation approach. The solver is similar to Fackler (2005), except that the equilibrium equations are solved by the mixed complementarity problems solver \textsc{PATH} (Dirkse and Ferris, 1995). A grid on the state variables is chosen, on which decision rules are approximated by splines and the number of grid nodes is chosen such that the use of these decision rules entails on average less than a $1 error every $1,000 of decision (measured by the Euler equation error).

5.1 Calibration

Parameterisation is required in order to quantify the welfare effects of introducing a food stabilisation policy. Table 3 presents the parameters used to calibrate the model. The parameters are set such that, at the model’s deterministic steady state, price, production, consumption and availability are equal to 1. Since steady-state consumption and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share, \( 1/\gamma \). An annual interest rate of 5% is used for discounting.

Seale and Regmi (2006) estimate elasticities for food consumption across 144 countries. From their research, we choose cereal elasticities typical of low-income countries: \(-0.4\) for price elasticity and 0.5 for income elasticity. For poor households, the budget share devoted to one commodity can be important. For example, rice expenditures represent roughly 20–35% of total expenditure for the bottom quintile in Bangladesh, India, Indonesia and Philippines (for a share of food expenditures around 62–70%, Asian Development Bank, 2008). For the top quintile, it is about 5–10% of total expenditure. Since the main focus of a food security policy is the poorest populations, the representative consumer is assumed to spend, at the steady state, 15% of its income on the staple.

\(^{10}\)By differentiating Roy’s identity with respect to income and by using the definition of income elasticity and relative risk aversion, we have \( v_Y P = v_Y Y D (\eta/\rho - 1) \).
Table 3. Parameterisation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Economic interpretation</th>
<th>Assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Annual discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Income elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Own-price demand elasticity</td>
<td>$-0.4$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Commodity budget share</td>
<td>0.15</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Inverse of supply elasticity</td>
<td>10</td>
</tr>
<tr>
<td>$Y$</td>
<td>Income</td>
<td>6.67</td>
</tr>
<tr>
<td>$d$</td>
<td>Normalisation parameter of demand function</td>
<td>0.39</td>
</tr>
<tr>
<td>$h$</td>
<td>Normalisation parameter of production cost function</td>
<td>0.95</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Parameter defining risk aversion</td>
<td>$-6.13$</td>
</tr>
<tr>
<td>$k$</td>
<td>Physical storage cost</td>
<td>0.02</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Probability distribution of yield</td>
<td>$B(2, 2) \cdot 0.5 + 0.75$</td>
</tr>
</tbody>
</table>

There is less evidence on which to base the level of risk aversion since the literature on risk aversion parameters does not converge on a consensus.\(^{11}\) In our case, the parameter has to be sufficiently high to justify a stabilisation policy. From equation (3), this implies that we need $\rho$ such that $\gamma (\eta - \rho) - \alpha < 0$, that is, with our parameters, $\rho > 3.17$. We assume at the steady state a relative risk aversion parameter of 4, implying $\theta = -6.13$. For this calibration, the third derivative of indirect utility with respect to price, $v_{PPP}$, is negative. Given that utility decreases with price, $v_{PPP} < 0$ implies an aversion to downside price risk (Eeckhoudt and Schlesinger, 2006). For a given mean and standard deviation, the consumer prefers a price distribution with higher third moment, that is, shifting the volatility toward low rather than high prices. It matters for the design of stabilisation policies, because storage tends to skew prices upwards.

We follow Lence and Hayes (2002) and assume a per-unit storage cost of 2% of the steady-state price (i.e., $k = 0.02$). Supply elasticity is set at 0.1, so $\mu = 10$. This value is confirmed by Roberts and Schlenker (2010) who analyse a supply and demand model of world commodities, and estimate global supply elasticity for the caloric equivalent of corn, rice, soybean and wheat between 0.08 and 0.13. FAPRI’s elasticities database\(^{12}\) confirms this order of magnitude. For example, for developing countries they propose supply elasticity for wheat between 0.07 and 0.43. The productive shocks, $\epsilon$, are assumed to follow a beta distribution of shape parameters, 2 and 2, which makes it unimodal at 0.5 and symmetric. The distribution is centred and rescaled to vary between 0.75 and 1.25, which implies a coefficient of variation of 11.2%.

\(^{11}\) For example, Pålsson (1996) analyses risk taking by Swedish households and finds a relative risk aversion of the order 10 to 15 for all real assets, and 2 to 4 when restricted to financial assets. Estimating a DSGE model van Binsbergen et al. (2010) find a value of 79. Chetty (2006) shows that to be consistent with observed labour supply, relative risk aversion must be low, with an upper bound at 2 and a central estimate at 1. Part of this variability may be explained by a large heterogeneity in individual risk aversion (Guiso and Paiella, 2008), which can be related to changes in the environment (e.g., imperfections in financial markets may discourage risk-taking behaviours).

5.2 Decision rules

5.2.1 Storage rules

Storage rules are presented in Figure 1 (left panel). Without public policy, private storers (solid line) do not stock anything for low availability (the threshold is close to the steady-state availability level, 1), and increase the level of stock with market availability above the threshold. When normal consumption is satisfied, any additional quantity in the market tends to lower prices. The speculator takes opportunity of these lower prices to accumulate stock that can be sold in periods of lower availability. The middle panel shows the behaviour of the marginal loss from storage, $\phi$. It is positive for low availability, which explains the absence of storage: profit is negative. As soon as stock is positive, marginal loss is null. Private storers operate in expectations at zero profit.

Figure 1. Storage, marginal loss from storage and production rules

Stabilisation through an optimal public storage rule (dashed line) results in levels of public stock that are always superior to the levels reached by private storage without policy. In consequence, public stocks discourage any speculative storage, and the marginal loss from storage is always positive. The crowding-out of private storage means that the degree of involvement of government in storage has to go beyond the difference between the private storage rule without intervention and the optimal rule. Because public storage removes profit opportunity, public intervention means government is responsible for both speculative storage and additional storage motivated by consumer risk aversion. This complete crowding-out is absent in the optimal storage rules in Wright and Williams (1982b) and Williams and Wright (1991, Ch. 15), which analyse the management of strategic petroleum reserves. Two features explain the coexistence of both public and private stocks in these works: in the first study, private storers are assumed to receive a convenience yield from the holding of stocks, implying that they hold stock even if the apparent return is negative; in the second study, they suppose that public stock is not held at the same location as private stock, which gives justification for the existence of private storers that are closer to the market. For these reasons, and because private storers hold stocks to smooth the natural seasonality of agriculture production, it
is reasonable to think an optimal public storage policy would not in practice completely crowd out private storage. But there will be very little scope for private storage to obey a speculative motive in the presence of welfare-maximising public storage.

Since there is a complete crowding out, the stabilisation brought by public storage could also be decentralised through a state-contingent subsidy to private storage. The subsidy would consist in covering the marginal loss from storage induced by the higher stock level to bring the expected private storers’ profit to zero; hence the subsidy would follow $\phi$ as represented in Figure 1.

### 5.2.2 Production rules

Like the storage rule, the production rule presents a kink at the stockout limit (Figure 1, right panel). When storage is null, whatever the current availability, the planned production level is the same. For positive stock, the next-period expected price decreases with availability and so too does planned production.

The effect of public policy is to make production more elastic to current availability. Production is higher for low availability and lower for high availability. This is achieved through a subsidy to planned production for low availability and a tax for high availability (see Figure 2, left panel). For low availability levels, the storage policy is ineffective, since there will be no stockpiling. But it is possible to increase future availability by producing more. This is the point of subsidising planned production. It has to be high (it can reach 15% of steady-state price) to compensate for the higher costs (there are decreasing returns to scale) and the depressing effect of supplementary production on price.

![Figure 2. Subsidy/tax to planned production, price rules, and social marginal value of the commodity](image)

A state-contingent subsidy policy makes sense only for a storable commodity. Without storage, the rational expectations solution to the model without public intervention is a constant planned production level (Muth, 1961). Storage creates variation in the expected price and therefore justifies
the variation in production with growers producing more when availability is expected to be low. The subsidy/tax magnifies this effect beyond private behaviour. In essence, this instrument plays a complementary role to storage.

It may seem strange to tax producers in abundant years when they are suffering low prices, but taxation is the incentive part of the policy; it is possible to compensate producers for any loss through non-distortive transfer. It is possible also to design policy such that producers are not taxed but are paid to take land out of production. These types of policies were common in the US as part of supply management until the 1996 FAIR Act (Gardner, 2002, Ch. 7). The US Secretary of Agriculture decided on the percentage of land that farmers should set aside each year, in order to be eligible for the price support programme. This policy was seen as complementing storage policies, since it limited production when stocks were already high.

The supply control through subsidies and taxes to planned production may, however, be more difficult to implement in a developing country where a direct control of producers’ behaviour is much more difficult, given the administrative system required to verify acreages. An alternative instrument, which could partly mimic the role of a supply control policy, is a state-contingent fertiliser subsidy, since fertiliser subsidies have been common in many African countries (Morris et al., 2007).

5.2.3 Price and social marginal value of the commodity

The two regimes, without and with storage, are also apparent in the price rules (Figure 2, middle panel). In a stockout situation, price is determined solely by current consumer demand and can increase a lot for small availability. For higher availability, demand for storage adds to consumer demand, which makes total demand more elastic and limits the decrease in the price with availability. Under an optimal policy, for positive stocks, the price is higher because demand for storage is higher.

The behaviour of the social marginal value, $\chi$, is represented in Figure 2 (right panel). It is positive for low availability, meaning it is socially justifiable to try to transfer more resources to low availability periods, hence the higher level of stock. The social marginal value is negative for high availability, which explains taxation of production in these situations in order to prevent excess supply.

5.3 Dynamics under optimal stabilisation policy

When stabilisation policy is first introduced, the new asymptotic distribution is not reached immediately. Since the policy involves a higher mean stock level than without intervention, it begins with a phase of stock build-in. The transition is illustrated in Figure 3 with the dynamics of mean total stock, mean government outlays and mean prices when the policy starts with the system on the asymptotic distribution without public intervention.
Figure 3. Transitional dynamics. Before period 0, the system is on the asymptotic distribution of the model without public intervention. The unexpected policy is announced and starts in period 0. (Obtained by Monte Carlo simulation as the average over 100,000 simulated paths.)

In the first years, the purchases made to accumulate stocks push the mean price above its long-run mean. After a few years, the mean price drops below its value without public intervention. Stabilisation affects the mean price through the curvature of the inverse demand function (Wright, 1979). Since we apply constant elasticity of demand, the inverse demand function is convex. Storage also makes the inverse of the total demand function convex, as shown in Figure 2. Thus, the price at mean consumption is lower than the mean price. So stabilisation of the consumption results in a lower mean price.

Mean government outlays are very high in the first period, because stocks must be accumulated and there is no stock to sell. The first year cost reaches 12% of the steady-state commodity budget share and 1.7% of income. Public storage outlays decrease in later periods and, on the asymptotic distribution, the public stock is financially self-balanced: the proceeds from sales cover purchase and storage costs. The outlay from the subsidy/tax on production is always less than the amount from public stock and, after a few periods, it is negative. This means that production is more taxed than subsidised. Indeed, in the long run mean availability reaches 1.13 corresponding to a tax on planned production in Figure 2.

The dynamic welfare effects of the policy are presented in Table 4. In addition to the effects of an optimal policy, the welfare results from an optimal public storage policy on its own, and an optimal subsidy/tax to planned production are depicted. Consumer gains are \textit{ex ante} per-period equivalent variation calculated using

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ v\left( P_t, Y + EV \right) - v\left( \tilde{P}_t, Y \right) \right] \right\} = 0. \quad (40)$$

Producers’ gains are the annualised changes in expected profit from equation (10). Government outlays are the annualised expected sum of the discounted costs from equation (17). Changes in
storers’ surpluses are ignored since storers operate at zero profit on average and we have assumed no stock in the first period. The welfare results are all conditional expectations that depend on initial availability, which is assumed to be equal to its deterministic steady state level. The sensitivity to this starting point is assessed in Section 5.6. To put the magnitude of the welfare results in perspective, they can be compared to income or to the steady-state commodity budget share. Total gains from the optimal policy using both instruments represent 0.05% of income and 0.32% of the commodity budget share. The low level of these gains is not surprising given that the gains resulting from a reduction of risk are of second order. The finding parallels Lucas’s (1987) insight that individuals would be unlikely to sacrifice more than 0.1% of their lifetime consumption to live in a world with no macroeconomic volatility.

Table 4. Welfare results on transitional dynamics (as a percentage of the steady-state commodity budget share.)

<table>
<thead>
<tr>
<th></th>
<th>Public storage</th>
<th>Subsidy/tax to production</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers gains</td>
<td>0.39</td>
<td>0.26</td>
<td>0.36</td>
</tr>
<tr>
<td>Producers gains</td>
<td>0.42</td>
<td>1.02</td>
<td>0.35</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0.53</td>
<td>1.20</td>
<td>0.39</td>
</tr>
<tr>
<td>Total gains</td>
<td>0.28</td>
<td>0.08</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Stabilisation policies result in transfers between agents (see Newbery and Stiglitz, 1981, Ch. 6 and 9, for a discussion of efficiency and transfer from price stabilisation). Consumers gains include two components: a transfer term corresponding to the change in mean expenditures, and an efficiency term corresponding to the risk reduction and the change in mean consumption. The mean expenditure change is a transfer with producers, and private and public storers. Public storage entails both storage costs and changes in mean price, i.e., a transfer with producers and consumers. The subsidy/tax on production is a transfer between government and producers. Overall, the transfers sum to zero and do not affect social welfare, which is composed only of efficiency gains from consumers, and changes in production and storage costs.

For a policy of public storage, stockpiling pushes prices up in the early periods and in later periods pushes them below their mean value without stabilisation. For producers, this implies long-term losses from lower prices. However, since later periods are discounted, overall producers gain from a stockpiling policy. These producers’ gains stem mainly from transfers from government through the effect of public storage on mean price.

Producers’ gains are higher under a pure subsidy/tax policy. This policy is a combination of subsidy in periods of low availability and tax in periods of abundance. But, without public storage, subsidies are more frequent than taxes, and producers benefit greatly from this instrument. Under this policy, situations of low availability are more frequent than when both instruments apply. It is precisely

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13These welfare results stem from expected infinite sums of discounted values (e.g., equation (10)). To calculate them, we follow the dynamic programming trick of transforming the infinite sums in recursive problems that can be solved easily by value function iteration.

14This result does not depend on the value of the discount parameter. It holds for any reasonable value of $\beta$. 

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when availability is low that the producers are subsidised to produce more, which explains why producers gain more when output subsidy is the only policy.

Having accounted for all transfers, public storage contributes the most to overall welfare. A policy of planned production subsidy only complements public storage.

### 5.4 The long-run effects of stabilisation

Since it is not possible to borrow the commodity for the future (the non-negativity constraint on storage), increased stabilisation as a result of public storage requires an increase in mean stock levels, as stock dynamics show (Figure 3). Following this transitory phase of stock building availability increases due to the higher stock levels as is apparent from the asymptotic distribution of availability (Figure 4).\(^\text{15}\) Because public storage significantly shifts the distribution of availability toward higher values, it reduces the risk of shortfalls. Subsidising planned production acts differently on availability. It reduces the probability of low and high availability by making supply more elastic, although the effect appears marginal.

![Figure 4. Density of availability.](image)

The curves are obtained by kernel density estimation over 1,000,000 simulated points.

Confirming previous results, Table 5 shows that all stabilisation policies involving public storage decrease the mean price. This decrease brings long-run losses for producers, because stabilisation decreases mean profit (Turnovsky, 1976, shows that this is always true for a constant-elasticity demand where elasticity is below unity).

Public stockpiling increases price autocorrelation, since a higher level of stock allows a better

\(^{15}\)Long-run results are calculated over 1,000,000 sample observations from the asymptotic distribution.
smoothing of shocks over several periods. A production subsidy has the opposite effect: it makes supply more elastic to price expectations and tends to decrease price autocorrelation.

Most of the stabilisation is achieved through public storage, which on its own decreases the coefficient of variation from 0.19 to 0.14. Introducing in addition state-contingent subsidy decreases it to 0.13. Storage stabilises by moving the commodity from periods of low prices to periods of high prices. This is apparent in the fact that the probability of the occurrence of low and high prices decreases. Very low prices are perfectly alleviated by storage. The possibility of stockouts means that high prices are not all alleviated, and the skewness of prices increases with the level of storage.

The quantiles of prices show how storage policies operate. Because the public storage rule is more aggressive than the private storage rule, low price periods are less frequent since government uses these opportunities to build up stocks. More than half of the distribution ends up at higher prices, but the higher stock level allows the occurrence of high prices to reduce.

### 5.5 Nature of the risk-sharing arrangement

The stabilisation policies increase total welfare by transferring risk from risk-averse consumers to risk-neutral agents. Table 6 illustrates this risk sharing by presenting the standard deviation of each component of the social welfare function (15) on the asymptotic distribution: consumers’ utility, producers’ and storers’ profit and government outlays.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Public storage</th>
<th>Subsidy/tax to production</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers utility</td>
<td>0.090</td>
<td>0.064</td>
<td>0.082</td>
<td>0.059</td>
</tr>
<tr>
<td>Producers profit</td>
<td>0.105</td>
<td>0.098</td>
<td>0.191</td>
<td>0.136</td>
</tr>
<tr>
<td>Storers profit</td>
<td>0.067</td>
<td>0.0</td>
<td>0.073</td>
<td>0</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0</td>
<td>0.087</td>
<td>0.130</td>
<td>0.093</td>
</tr>
</tbody>
</table>

As price volatility decreases with public intervention, the standard deviation of consumers’ utility
decreases for all policies. The effects on producers' profit are more contrasted. A public storage policy decreases the volatility of their profit, but the other policies increase it. The subsidy/tax has a procyclical behaviour with respect to producers' profit. In periods of low availability, producers' profit is high because the high prices more than compensate for the poor harvests; this is also the period when they receive subsidies to increase production. And conversely, they are taxed to limit production when their profit is the lowest, when availability is high. This increase in profit volatility creates no welfare loss in our model since producers are assumed to be risk neutral. This assumption is useful in limiting the number of state variables, but it is not realistic. Since a policy of subsidy/tax alone almost doubles the volatility of producers' profit, this policy would not be followed as such. Even, in association with public storage, this instrument increases the volatility of profit by 30%. It does not imply, however, that a supply control policy is useless with risk-averse producers. Its precise form as stemming from the optimal policy design would just be different.

With regard to risk sharing, public storage appears the most balanced policy. It decreases the volatility perceived by both consumers and producers, and transfers the risk to government budget. The public storage policy found here would not be optimal with risk-averse producers, but it would probably increase social welfare, and the welfare of both consumers and producers.

5.6 Sensitivity to initial availability

Above we have considered the effects of stabilisation policies starting when availability is equal to its deterministic steady-state value. Given the importance of transitional dynamics effects, we analyse in this section the sensitivity to initial availability by considering the welfare gains when the policies starts with availabilities of 0.8, 1 (the benchmark) and 1.2. As before, private stocks are assumed to be null, which rules out transfers to private storers.

For all policies total gains are not much affected by initial availability; it is the distribution of gains that are more influenced by the starting point (see Table 7). This means that the starting point influences more the transfers among agents than the efficiency gains. For a public storage policy, because storage entails an initial stock building phase, starting from higher availability reduces the costs associated with this phase by providing excess quantities to be stockpiled.

For a subsidy/tax to planned production, the distributive effects are important with changes in signs of welfare gains depending on the starting point. For an initial availability of 1.2, consumers lose from the policy. For this high availability, there are few efficiency gains to expect from the policy in the short-run, since private storers already provide stabilisation. Producers are initially taxed to induce them to produce less because the social marginal value of the commodity is negative for this availability level. This taxation entails higher prices, leading to welfare losses of consumers. For low availability on the contrary, producers are subsidised in the short-run, which explains their

\footnote{With risk-averse producers, it would not have been possible to sum production and stocks into one state variable, availability. The increase in dimension would have made the problem more difficult to solve and the results more difficult to interpret.}
high welfare gains. They increase their production, which decreases mean price and price volatility for consumers, explaining their welfare gains. This policy has a high fiscal cost, representing 1.7% of the commodity budget share, because of the high initial subsidies.

From the above, we can conclude that the instrument of subvention/tax to production leads to results which are dominated by transfer gains, efficiency gains being very limited in contrast.

## 6 Conclusion

In this paper we proposed a framework for food price stabilisation policy design. This framework takes some inspiration from the method used in the modern literature on optimal monetary policy (Clarida et al., 1999): (i) we start with a simple model, the rational expectations storage model, that is able to account for the stylised facts of commodity prices; (ii) because markets are incomplete, consumers are unable to insure themselves, which could justify public intervention; (iii) corresponding to the market failure, a public policy maximising the welfare is designed.

This work brings together two approaches. That proposed by Newbery and Stiglitz, which uses very stylised models to propose stabilisation policies in imperfect markets, but does not result in implementable policies due to the restrictive framework, and the tradition of competitive storage modelling (e.g., Miranda and Helmberger, 1988, Wright and Williams, 1988a) in which public policies are discussed within a framework that mimics the main features of commodity markets, but neglects market imperfections. The resulting model, although still highly stylised, provides a theoretical benchmark against which food security propositions can be assessed.

We show that an optimal policy under commitment composed of state-contingent public storage and state-contingent subsidy/tax on planned production improves consumers’ and producers’ welfare and crowds out all private storage activity by removing any profit opportunity from speculation. Most of the welfare gains are from public storage. A countercyclical production policy, which provides incentives for increased production in times of scarcity and decreased production in times of abundance, makes little difference and can have undesirable consequences such as increasing the volatility of producers’ profit.

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### Table 7. Sensitivity of welfare results to initial availability

<table>
<thead>
<tr>
<th></th>
<th>Public storage</th>
<th></th>
<th>Subsidy/tax to production</th>
<th></th>
<th>Both instruments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_0$</td>
<td>$1^a$</td>
<td>1.2</td>
<td>$A_0$</td>
<td>$1^a$</td>
<td>1.2</td>
</tr>
<tr>
<td>Consumers gains</td>
<td>0.42</td>
<td>0.39</td>
<td>0.39</td>
<td>0.35</td>
<td>0.26</td>
<td>−0.07</td>
</tr>
<tr>
<td>Producers gains</td>
<td>0.36</td>
<td>0.42</td>
<td>0.49</td>
<td>1.43</td>
<td>1.02</td>
<td>−0.24</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0.52</td>
<td>0.53</td>
<td>0.58</td>
<td>1.70</td>
<td>1.20</td>
<td>−0.40</td>
</tr>
<tr>
<td>Total gains</td>
<td>0.26</td>
<td>0.28</td>
<td>0.29</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

#### Notes:
- Initial private stocks are assumed to be null, production is the only component of initial availability.
- $^a$ Benchmark.
By choice, we kept the model as simple as possible. Future work could extend this study along a number of important dimensions. It could account for more realistic features, such as the effect of international trade or substitution among food staples (Bellemare et al., 2010), or, and perhaps more importantly, the welfare framework could be improved. The one proposed in this paper is a first attempt to provide some justification for stabilisation policies. For example, our modelling of food demand is very simple and does not acknowledge the fact that food consumption cannot decrease too much without consequences for health or even death. Accounting for the specific role of food would reinforce the need to prevent the occurrence of high food prices. This could be done with the help of the survival function proposed by Ravallion (1987) or the microeconomic framework developed in Chavas (2000). Assigning to food a peculiar role as a consumption good should increase the welfare gains from stabilisation beyond the low values found here.

Also, the optimal policy with state-contingent public storage and subsidy to planned production implies heavy government involvement in the functioning of markets and the application of highly nonlinear policy rules. Historical experience of the involvement of states in grain markets shows mixed results and several instances of large inefficiencies (Dorosh, 2008). It would be useful to consider the design of simpler policies that would decentralise a part of the policy implementation to the private sector.

In this paper, for simplicity, the analysis was restricted to a partial equilibrium framework. Because food price instability is a concern only for people who spend a large part of their budget on food, partial equilibrium reasoning might not account for important general equilibrium effects. For example, in developing countries, the poorest people are likely also to be subsistence farmers, and the high variability in food prices means that they devote limited acreage to cash crops and self-insure by producing food crops (Fafchamps, 1992). More stable food prices would reduce their need for insurance and allow them to specialise more.

Finally, we do not allow for heterogeneity among consumers. If people likely to suffer from high prices represent only a limited share of the total population, it may not be optimal to engage in large-scale public stock programmes. A better scheme might include policies specifically targeting people in need. They might take the form of targeted cash transfer programmes (Skoufias et al., 2001) or the distribution of subsidised rations. The public stock backing a targeted policy would not necessarily crowd out private storage since it would not be designed to affect the whole market.

**Appendix**

A second-order approximation of the right-hand side of equation (1) around the mean price without stabilisation yields

\[ v(\bar{P}, Y) = v(\bar{P}, Y) + (\bar{P} - \bar{P}) v_P(\bar{P}, Y) + (\bar{P} - \bar{P})^2 v_{PP}(\bar{P}, Y) / 2. \] (41)
And around \((\bar{P}, Y)\), the left-hand side yields

\[
v(P, Y + EV) = v(\bar{P}, Y) + (P - \bar{P}) v_P (\bar{P}, Y) + EV v_Y (\bar{P}, Y) + \left[ (P - \bar{P})^2 v_{PP} (\bar{P}, Y) + EV^2 v_{YY} (\bar{P}, Y) + EV (P - \bar{P}) v_{PY} (\bar{P}, Y) \right] / 2. \tag{42}
\]

Taking the expectations of both sides, and for simplicity omitting the variables inside the partial derivatives of utility, we have

\[
\Delta \bar{P} v_P + E \left[ (\bar{P} - \bar{P})^2 \right] v_{PP}/2 = EV v_Y + \left[ \sigma_P^2 v_{PP} + EV^2 v_{YY} \right] / 2, \tag{43}
\]

where \(\sigma_P^2\) is price variance without stabilisation. Using

\[
E \left[ (\bar{P} - \bar{P})^2 \right] = E \left[ (\bar{P} - \bar{P})^2 + (\bar{P} - \bar{P})^2 \right], \tag{44}
\]

where \(\bar{P}\) is the mean price after stabilisation, we have

\[
EV v_Y + EV^2 v_{YY}/2 = \Delta \bar{P} v_P + \left[ (\Delta \bar{P})^2 + \sigma_P^2 - \sigma_P^2 \right] v_{PP}/2. \tag{45}
\]

Terms \(EV^2\) and \((\Delta \bar{P})^2\) are of order \(\sigma_P^4\); they can be neglected for small \(\sigma_P\), and equation (2) follows. Using Roy’s identity, we have

\[
EV = -D \Delta \bar{P} + \Delta \sigma_P^2 v_{PP}/(2v_Y). \tag{46}
\]

Differentiating Roy’s identity gives the expression of \(v_{PP}\),

\[
v_{PP} = D^2 v_{YY} + D \frac{\partial D}{\partial Y} v_Y - \frac{\partial D}{\partial P} v_Y, \tag{47}
\]

from which equation (3) follows.

**References**


