

Determination of Realistic Uncertainty Bounds for the Stewart Platform with Payload Dynamics

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Abstract — Robust Control of Stewart Platform has been successfully demonstrated by various authors [8-12]. The work done so far is based on the assumption that the bounds on uncertainty are known and the chosen reachability gains are greater than these bounds. This assumption can only be justified if those bounds could be quantified, which is not the case in the existing approaches. The problem gets severe when the controller has to compete against the variations in payload, especially when the payload is asymmetric. This paper addresses such uncertainties. The novelty of the paper lies in the extension of existing nonlinear model to include asymmetric payloads and quantification of uncertainties arising from the use of a variety of asymmetric payloads. The actual Moments of Inertia (*MOI*) of the Stewart Platform with asymmetric payload are computed and used to find worst case uncertainty bounds. The control performance of the proposed algorithm is verified by computer simulations. These simulations show that the system follows the desired trajectory and errors converge to equilibrium points efficiently.

terms are almost zero and can be neglected), while for asymmetric load, off-diagonal terms in *MOI* matrix have significant values and hence, could not be ignored.

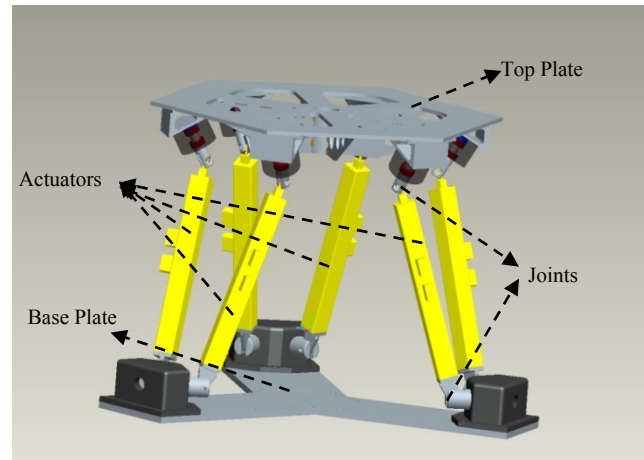


Figure 1: The Stewart platform

I. INTRODUCTION

The Parallel link manipulators attracted many researchers in the current decade due to its precision, rigidity and high load-to-weight ratio. A Stewart platform [1] is a parallel robot that provides six-degree-of-freedom (6DOF) i.e roll, pitch, yaw, surge sway and heave. Its practical usage is for disturbance isolation, precise machining and flight simulators.

This platform has six variable-length electro-mechanical actuators connecting a top plate to a base plate with spherical joints. Angular and translation motion of the top plate with respect to the base plate is produced by reducing or extending the actuator lengths. The proper coordination of the actuators length enables the top plate to follow the desired trajectory with high accuracy. Thus the six inputs to the Stewart platform in term of torque are calculated by controller. The outputs of the Stewart platform are the upper plate angular and translation positions (in surge, sway, heave, roll, pitch and yaw) sensed by highly precise sensors or estimated by the motors encoders. The structure of the platform is shown in Figure 1 and the schematic is depicted in Figure 2.

In practical situations, the Stewart platform may serve as a base of different payloads such as surgery tables, satellite antenna etc, mounted on a ship in turbulent sea. These payloads may be symmetric or asymmetric with different weights, so the mass and moment of inertia (*MOI*) of payload will always be uncertain. For better performance and accuracy, dynamics of payload should also be included in the dynamical equation of the Stewart platform. For symmetric loads *MOI* are diagonal matrix (off-diagonal

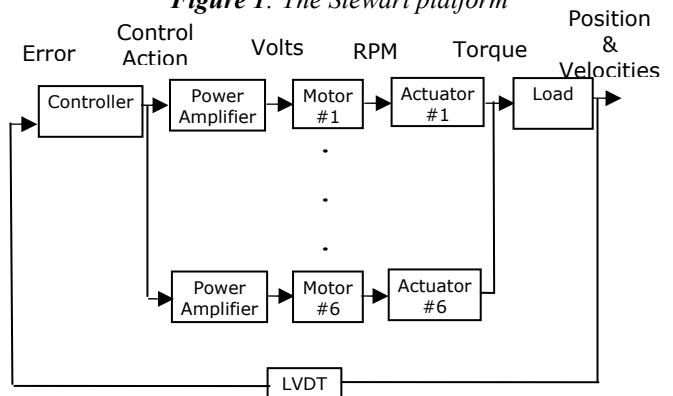


Figure 2: Block diagram of the Stewart platform

Nguyen *et al* [2] and Lebret *et al* [3] used Euler-Lagrange method for the dynamical equation of Stewart platform; they assumed only the Stewart platform dynamics and considered *MOI* as diagonal matrix. They didn't consider the payload dynamics and ignored off-diagonal terms in the inertia matrix. Dasgupta *et al* [4] solved the dynamics by Newton-Euler method and included leg dynamics so *MOI* was no more diagonal, but leg inertia was very small as compared to payload inertia, so it would not cater for the uncertainties in *MOI* due to payload variation. Liu *et al* [5] analyzed the dynamic model of Gough-Stewart Platform using Kane's equations; they also used *MOI* as a diagonal matrix. Ting *et al* [6] presented the dynamical model of the Stewart platform CNC machine using Euler-Lagrange method, in their analysis *MOI* is again a diagonal matrix. Guo *et al* [7] used Newton-Euler method with Lagrange formulation including

the dynamics of legs for the Stewart platform; they also used *MOI* as diagonal matrix for the derivation of dynamic equation. Generally sufficient accuracy cannot be attained except when the inertia of the payload is also considered. In particular, for the case where the platform was used as base of the payload, the effect of payload is no longer negligible and thus accurate modeling is required to include payload dynamics for control performance. If the off-diagonal terms were ignored then the controller should be more robust so that it could cater for the uncertainties due to asymmetric nature of the payload, but at the cost of losing performance.

The payload taken for the Stewart platform is satellite antenna with moving parabolic dish. This antenna can perform *2DOF* motion, i.e. rotation about x-axis called elevation and rotation about z-axis called azimuth, thus payload can become symmetric as well as asymmetric with movement of the dish. Table 1 shows the characteristics of satellite antenna.

Elevation Angle(E)	-5° to 90°
Azimuth Angle(A)	0° to 360°
Dish diameter	2400 millimeter (mm)
Hex Column height	1245 mm
Hex Column side	210 mm
Total weight	300 kg

Table 1: Characteristics of satellite antenna

Considering the angle with the x-axis, the dish can move from -5° to 90° in elevation. As shown in Figure 3 the payload is symmetric when the dish is at 90° elevation. The *MOI* matrices for the symmetric load at different azimuth angles are shown in Table 2. The slight asymmetric results are due to balancing weights of the dish.

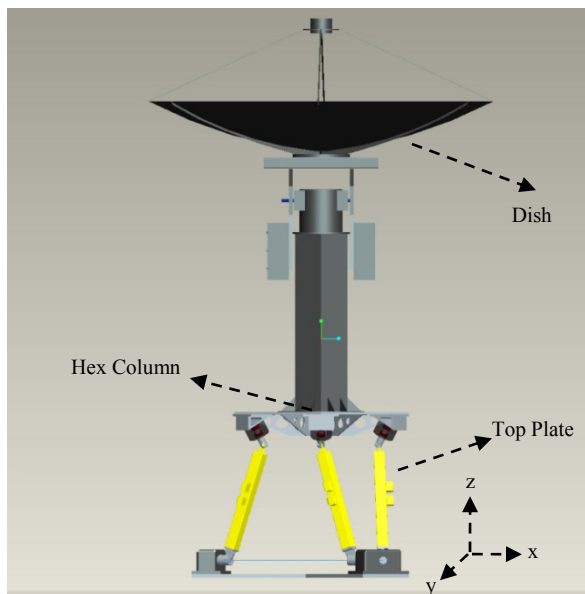


Figure 3: The Stewart platform with symmetric load (90° of elevation).

A(°)	<i>I</i>_{XX}	<i>I</i>_{XY}	<i>I</i>_{XZ}
	<i>I</i>_{YX}	<i>I</i>_{YY}	<i>I</i>_{YZ}
	<i>I</i>_{ZX}	<i>I</i>_{ZY}	<i>I</i>_{ZZ}
0°	238.00E+06	5.07E+03	6.79E+03
	5.07E+03	241.72E+06	-6.79E+03
	6.79E+03	-6.79E+03	138.97E+06
45°	240.00E+06	-1.58E+03	4.90E+03
	-1.58E+03	240.00E+06	-4.55E+03
	4.90E+03	-4.55E+03	138.97E+06
90°	241.00E+06	-5.07E+03	3.48E+03
	-5.07E+03	238.56E+06	-6.43E+03
	3.48E+03	-6.43E+03	138.97E+06

Table 2: *MOI* with symmetric load (Kg*mm²) with constant elevation and varying azimuth angle.



Figure 4: The Stewart platform with asymmetric load (45° degree of elevation).

However for other angles of elevation the antenna structure becomes asymmetric (Figure 4). *MOI* matrix of asymmetric load is shown in Table 3.

E(°)	<i>I</i>_{XX}	<i>I</i>_{XY}	<i>I</i>_{XZ}
	<i>I</i>_{YX}	<i>I</i>_{YY}	<i>I</i>_{YZ}
	<i>I</i>_{ZX}	<i>I</i>_{ZY}	<i>I</i>_{ZZ}
0°	236.24E+06	1.54E+06	0.26E+06
	1.54E+06	237.27E+06	0.10E+06
	0.26E+06	0.10E+06	145.27E+06
45°	239.68E+06	1.57E+06	4.91E+06
	1.57E+06	240.13E+06	4.54E+06
	4.91E+06	4.54E+06	138.97E+06

Table 3: *MOI* with asymmetric load (Kg*mm²) with constant azimuth and varying elevation angle

Asymmetry of the payload structure causes off diagonal terms in the *MOI* matrix to become non-zero, motivating the need for the extension of the nonlinear model to include asymmetric structure.

Regarding the control aspect, in recent years many people worked on robust controllers for Stewart platform. Kang *et al* [8] proposed a Lyapunov based approach for designing the robust PD controller for the Stewart platform in presence of uncertainties. Lee and Kim [9] in 1998 presented the model based sliding mode control for the Stewart platform with perturbation included in the model. Kim *et al* [10] in 2000 discussed robust tracking control design for *6DOF* parallel manipulator in presence of time varying uncertainties. A sliding-mode control for Stewart platform, which can drive motion tracking error to zero asymptotically, has been proposed [11] in 2004 by Huang and Fu. Iqbal and Bhatti [12, 13] in 2006 suggested a simple way to calculate the control law using sliding-mode technique. So far, the characterization of uncertainty against asymmetry and variation of movable payload is not found by the authors in the existing literature. The cited works consider Stewart platform without payloads and assumed uncertainty bounds and proceed for controller design. However in the current work the authors have attempted to address this gap.

The novelty in this paper is the extension of nonlinear model of Stewart platform using Euler-Lagrange method with symmetric and asymmetric payloads and calculation of uncertainty bounds for sliding mode control. The rest of this paper is structured as follow; dynamics (including model extension to contain off-diagonal Moments of Inertia) are explained in section II. Section III deals with direct sliding-mode controller design, and used a thin layer for chattering-free control action. Section IV addresses the computation of Moments of Inertia and their consequent use in the determination of uncertainty bounds. The design is validated through full envelope nonlinear simulations in section V. Conclusions are drawn in section VI.

II. DYNAMICAL MODEL WITH PAYLOAD

The dynamic equations of the Stewart platform considering all inertial and Coriolis effects is very challenging to determine, Lebret in [3] developed the dynamic equation using Lagrange method as:

First calculate kinetic and potential energy as a function of q as follows.

$$K = K(q, \dot{q}) = \frac{1}{2} \dot{q}^T M \dot{q} \quad \text{where } M \in \mathbb{R}^{6 \times 6}$$

$$P = P(q)$$

and then develop Lagrange equation as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = J^T(q)\tau$$

Where $q \in \mathbb{R}^{6 \times 1}$ of the form $q = [x \ y \ z \ \alpha \ \beta \ \gamma]^T$, $M \in \mathbb{R}^{6 \times 6}$ is an inertial matrix; $C \in \mathbb{R}^{6 \times 6}$ is Coriolis/centripetal matrix; $G \in \mathbb{R}^{6 \times 1}$ is vector containing gravity torques; $J \in \mathbb{R}^{6 \times 6}$ is Jacobean matrix which changes angular velocities into Cartesian velocities and $u \in \mathbb{R}^{6 \times 1}$ is vector of input signals,

respectively, taking $G = J^T(q)J^T(0)w$ where

$J^T(0) = J^T(q)|_{q=0}$. Let $u = \tau - J^T(0)w$, then we have

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = J^T(q)u \quad (1)$$

When the payload is static and symmetric, the *MOI* in $M(q)$ is constant and diagonal matrix; off-diagonal terms are almost zero and can be neglected. Moreover *MOI* derivative is also zero. Detailed components of inertial, Coriolis and Jacobean matrices for the dynamical equation of Stewart platform with symmetric and static payload can be seen in [3].

For asymmetric and moving payload, off-diagonal terms in *MOI* matrix have significant values and can't be neglected as usually ignored by people working in this area. The asymmetric shape of the antenna around 45° elevation gives rise to significant off-diagonal terms in the *MOI* matrix. Inertia matrix for this situation can be seen in [14]. It can be observed from the new inertia matrix that it has some extra off-diagonal *MOI* terms simply adding into previous inertial matrix, so the changed inertial matrix can be split into two matrices such as

$$M = M_{\text{nominal}} + M_{\text{moi-offdiag}} \quad (2)$$

e.g. for instance take 4x4th component of inertia matrix M becomes:

$$M_{44} = I_{xx} \cos^2 \gamma + I_{yy} \sin^2 \gamma + 2I_{xy} \cos \gamma \sin \gamma$$

It can be split into two parts as follows

$$(M_{\text{nominal}})_{44} = I_{xx} \cos^2 \gamma + I_{yy} \sin^2 \gamma$$

$$(M_{\text{moi-offdiag}})_{44} = 2I_{xy} \cos \gamma \sin \gamma$$

Where M_{nominal} and $M_{\text{moi-offdiag}}$ matrices are due to diagonal *MOI* and off-diagonal *MOI* matrices respectively.

The changed Coriolis matrix for asymmetric payload can be seen in [16]. It has some extra off-diagonal terms due to asymmetric nature of payload, which are simply adding into previous Coriolis matrix, so new Coriolis matrix can be split into two matrices such as

$$C = C_{\text{nominal}} + C_{\text{moi-offdiag}} \quad (3)$$

e.g. consider 4x4th component of Coriolis matrix C

$$C_{44} = \frac{1}{2} \left((I_{yy} - I_{xx}) \sin 2\gamma \right) \dot{\gamma} + I_{xy} (\cos 2\gamma) \dot{\gamma}$$

It can also be split into two parts as follows

$$(C_{\text{nominal}})_{44} = \frac{1}{2} \left((I_{yy} - I_{xx}) \sin 2\gamma \right) \dot{\gamma}$$

$$(C_{\text{moi-offdiag}})_{44} = I_{xy} (\cos 2\gamma) \dot{\gamma}$$

Where C_{nominal} and $C_{\text{moi-offdiag}}$ matrices include terms due to diagonal *MOI* matrix and off-diagonal matrix respectively. Finally the dynamical equation of Stewart platform with asymmetric payload can be written as

$$(M(q)_{\text{nom}} + M(q)_{\text{off-diag}})\ddot{q} + (C(q)_{\text{nom}} + C(q)_{\text{off-diag}})\dot{q} = J^T(q)u$$

III. ROBUST SLIDING-MODE DESIGN

In sliding-mode controller design a hyper-plane is defined as a sliding-surface. This design approach comprises of two stages; first is the reaching phase and second is the sliding phase. In the reaching phase, states are driven to a stable manifold by the help of appropriate control law and in the sliding phase states slide to an equilibrium point. One advantage of this design approach is that the effect of non-linear terms which may be construed as a disturbance or uncertainty in the nominal plant has been completely rejected. Another benefit accruing from this situation is that the system is forced to behave as a reduced order system; this guarantees that no overshoot occurs when attempting to regulate the system from an arbitrary initial condition to the designed equilibrium point. Define a hyper-plane in R^6 as the sliding surface for the Stewart platform as

$$s = \Lambda q + \dot{q} \quad (4)$$

Where $q \in R^6$ is state vector. $\Lambda \in R^{6 \times 6}$ is diagonal and positive definite matrix. Equation (4) can also written as

$$\dot{q} = -\Lambda q + s \quad (5)$$

The above system equation is stable if $s=0$. The rate of convergence of system depends upon the Eigen values of matrix Λ . The Lyapunov candidate function [15] is

$$V = \frac{1}{2} s^T M s \quad (6)$$

It is a positive definite function. Its time derivative is as follows

$$\begin{aligned} \dot{V} &= s^T \dot{M} s + \frac{1}{2} s^T \dot{M} s \\ \dot{V} &= s^T (M \Lambda \dot{q} + M \ddot{q}) + \frac{1}{2} s^T \dot{M} s \end{aligned}$$

From equation (1) and by using skew symmetric property of $(\dot{M} - 2C)$ matrix, the above equation becomes:

$$\dot{V} = s^T (M \Lambda \dot{q} + C \Lambda q + J^T u) \quad (7)$$

The M and C matrices are defined in (2) and (3). From equation (7) control law can be deduced as

$$u = J^{-T} (-M \Lambda \dot{q} - C \Lambda q - K \text{sat}(s)) \quad (8)$$

Where vector $K \in R^6$, and $\text{sat}(s)$ is a saturation function and can be defined as follow [15]

$$\text{sat}(s) = \begin{cases} \frac{s(t)}{\|s(t)\|} & \text{if } s > \delta \\ \frac{s(t)}{\|s(t) + \delta\|} & \text{if } s < \delta \end{cases}$$

This provides a very smooth and chatter-free control action.

Let M and C matrices in equation (2) and (3) are perturbed matrices and have bounded uncertainties such as ΔM and ΔC , so these can be written as

$$\tilde{M} = M + \Delta M; \quad \tilde{C} = C + \Delta C$$

The \tilde{M} matrix has some perturbation in nominal terms and also in the off-diagonal terms. Same as \tilde{C} matrix has some perturbation in nominal, off-diagonal and varying terms. So \tilde{M} and \tilde{C} matrices can also be written as:

$$\begin{aligned} \tilde{M} &= M_{\text{nominal}} + \Delta M_{\text{nominal}} + M_{\text{offdiagonal}} + \Delta M_{\text{offdiagonal}} \\ \tilde{C} &= C_{\text{nominal}} + \Delta C_{\text{nominal}} + C_{\text{offdiagonal}} + \Delta C_{\text{offdiagonal}} \end{aligned}$$

Put the control law form equation (8) in the equation (7), we get

$$\dot{V} = s^T (\Delta M \Lambda \dot{q} + \Delta C \Lambda q - K \text{sat}(s))$$

Now if $k_i > \|\Delta M \Lambda \dot{q} + \Delta C \Lambda q\|_i$ (9)

$$\dot{V} = -s^T s$$

Here \dot{V} always be negative definite for non-zero s . The above condition assures that the sliding surface variable reaches zero in finite time and once the trajectories are on sliding surface they remain on the surface, and approaches to the equilibrium points exponentially.

IV. UNCERTAIN BOUNDS COMPUTATION

Uncertainties in the system can be incorporated by two sources; one is the model uncertainties and second is the variation in mass and moment of inertia of different symmetric and asymmetric payloads. In this section we deal with both of these uncertainties.

As we know, position and velocity vectors in the dynamic model will always be bounded due to the mechanical structure limitations, so the system's uncertainty will always remain bounded. Owing to the factors described above, mass and MOI undergo variation which is assumed as below:

$$m = m + \Delta m;$$

$$I_{xx} = I_{xx} + \Delta I_{xx}; \quad I_{yy} = I_{yy} + \Delta I_{yy};$$

$$I_{zz} = I_{zz} + \Delta I_{zz}; \quad I_{xy} = I_{xy} + \Delta I_{xy};$$

$$I_{yz} = I_{yz} + \Delta I_{yz}; \quad I_{xz} = I_{xz} + \Delta I_{xz};$$

The change in these parameters cause change in Inertial and Coriolis matrices:

$$\tilde{M} = M + \Delta M; \quad \tilde{C} = C + \Delta C$$

Now the ΔM can be seen in [14] and ΔC matrix can be seen in [16]. Now we calculate the uncertainty bounds for symmetric and asymmetric payloads separately.

CASE I: Symmetric payload

Uncertainty analysis for symmetric payload is performed, x and y components of moment of inertia are equal, i.e $I_{xx} = I_{yy}$. Let the mass may change from m to $m + \Delta m$ and moment of inertia may change from I_{xx} to $I_{xx} + \Delta I_{xx}$; I_y to $I_{yy} + \Delta I_{yy}$ and $\Delta I_{zz} = 0$. Where $\Delta m > \zeta_1$

and $\Delta I_i > \zeta_2$ such that ζ_1 and ζ_2 are any positive real numbers.

As discussed in equation (9), the stability condition is

$$k_i \geq \left\| \left[\Delta M \Lambda \dot{q} + \Delta C \Lambda q \right]_i \right\|$$

If we put the values of ΔM and ΔC in the above equation we get

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix} = \begin{bmatrix} \Delta m \lambda_1 \dot{x} \\ \Delta m \lambda_2 \dot{y} \\ \Delta m \lambda_3 \dot{z} \\ \Delta I_x \lambda_4 \cos^2 \beta \dot{\alpha} - 2 \Delta I_x \lambda_4 (\cos \beta \sin \beta) \dot{\alpha} \beta \\ \Delta I_y \lambda_5 \dot{\beta} + \Delta I_x \lambda_5 (\cos \beta \sin \beta) \dot{\alpha} \dot{\beta} \\ 0 \end{bmatrix} \quad (10)$$

It is assumed that Δm and ΔI_i are bounded. The state vector \dot{q} is also bounded due to mechanical constraints, and λ_i is the design factor, so the R.H.S of the above equation will always be remains bounded.

Tables 2 depict the significant variation taking place in *MOI* for no-load and symmetric payload condition. From these tables I_{xx} and other *MOI* perturbations are computed and used in the gain computations as given in equation (10).

CASE II: Asymmetric payload

In this case all the *MOI* have perturbation in them, so ΔM and ΔC are kept generic. Again according to equation (9), for

$$k_i \geq \left\| \left[\Delta M \Lambda \dot{q} + \Delta C \Lambda q \right]_i \right\|$$

We get general uncertainty matrix as

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \\ k_6 \end{bmatrix} \geq \begin{bmatrix} m \Delta \lambda_1 \dot{x} \\ m \Delta \lambda_2 \dot{y} \\ m \Delta \lambda_3 \dot{z} \\ \lambda_4 (\Delta M_{44} - \Delta C_{44}) \dot{\alpha} + \lambda_5 (\Delta M_{45} - \Delta C_{45}) \dot{\beta} + \lambda_6 (\Delta M_{46} - \Delta C_{46}) \dot{\gamma} \\ \lambda_4 (\Delta M_{54} - \Delta C_{54}) \dot{\alpha} + \lambda_5 (\Delta M_{55} - \Delta C_{55}) \dot{\beta} + \lambda_6 (\Delta M_{56} - \Delta C_{56}) \dot{\gamma} \\ \lambda_4 (\Delta M_{64} - \Delta C_{64}) \dot{\alpha} + \lambda_5 (\Delta M_{65} - \Delta C_{65}) \dot{\beta} + \lambda_6 (\Delta M_{66} - \Delta C_{66}) \dot{\gamma} \end{bmatrix} \quad (11)$$

Again gains are computed as described in the previous case. Table 3 depicts the significant variation taking place in *MOI* for the payload under different elevation angles. From these tables I_{xx} and other *MOI* perturbations are computed and used in the gain computations as given in equation (11).

V. SIMULATION RESULTS

For simulation, gain vector K of control law can be calculated with the help of uncertainty in the following steps;

- i. Compute the maximum I_{ii} for a given asymmetric load as given in table 3,
- ii. Calculate the variation in ΔI_{ii} for the payload with tolerance of $\pm 10\%$.
- iii. Get max limits of $q \in R^{6 \times 1}$ and $q' \in R^{6 \times 1}$ vectors of mechanical structure as given in table 4.
- iv. Put these values in equations [16] and get the values of ΔM and ΔC matrices.

v. Compute $\Lambda \in R^{6 \times 6}$ diagonal matrix, put fast dynamics for roll and pitch.

vi. Put these values for the calculation of K of the control law.

DOF	DISPLACEMENT (q)	VELOCITY (q')	ACCELERATION (q'')
Surge (X)	± 0.25 m	± 0.50 m/s	+0.8 G
Sway (Y)	± 0.49 m	± 0.76 m/s	+0.8 G
Heave (Z)	± 0.52 m	± 0.76 m/s	+0.8 G
Roll (α)	± 24 deg	± 40 deg/s	± 250 deg/s ²
Pitch (β)	± 22 deg	± 35 deg/s	± 250 deg/s ²
Yaw (γ)	± 29 deg	± 50 deg/s	± 250 deg/s ²

Table 4: MOOG's (6DOF5000E) Specifications

Simulation has been performed in order to examine the effectiveness of the proposed controller design. The platform can perform rotational and translation motion i.e. surge, sway, heave, roll, pitch and yaw.

Simulations were performed in two phases. In the first phase the model used for the sliding mode controller design was the traditional *6DOF* model with no consideration for payload asymmetry or dynamics. Consequently the uncertainty bounds would be much higher as they were supposed to be bigger than the size of uncertainty arising from asymmetry and moving payload. The results are given in Figures 5 and 6.

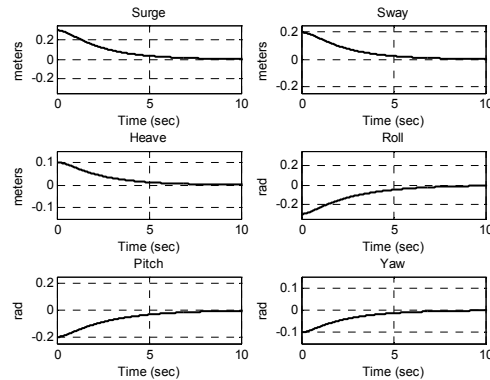


Figure 5: Regulation of 6DOF Platform with model based on diagonal terms only

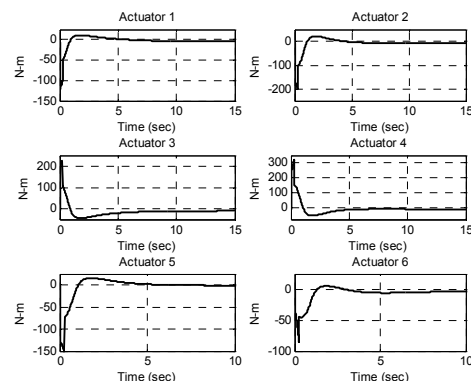


Figure 6: Control Action for regulation of 6DOF Platform based on model having diagonal terms only

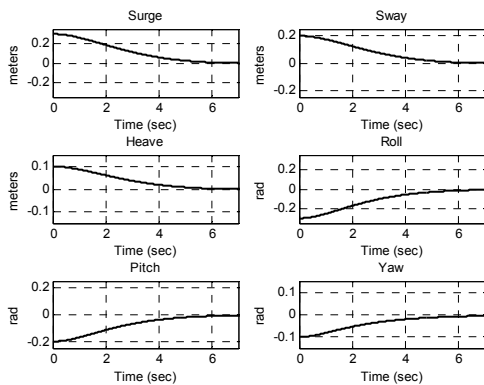


Figure 7: Regulation of 6DOF Platform with model based on off-diagonal and dynamic terms (extended model)

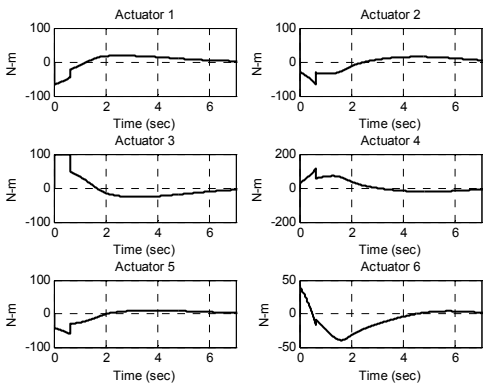


Figure 8: Control action for regulation of 6DOF Platform with model based on off-diagonal and dynamic terms (extended model)

In the second phase the extended 6DOF model as proposed in this paper was used. The extended model contains the asymmetry and moving payload effects. The results are depicted in Figures 7 and 8. Comparison of the two sets of simulations show marked improvement in the case of extended model in the form of reduced largest settling times (ca 9 sec versus ca 6 sec) and reduced largest control actions (approximately 300 N-m versus 100 N-m approx). In summary the controller which includes moving payload dynamics achieve stability earlier and perform better.

VI. CONCLUSION

A realistic way of calculating sliding mode controller gains through the determination of uncertainty bounds resulting from the payload variation, lead to judicious choice of sliding mode gains. This obviates the need of keeping the sliding mode controller gains arbitrarily high, thus reducing controller effort significantly. The paper shows a realistic way of analyzing model uncertainties and controller robustness for a complex engineering system i.e. Stewart platform. The proposed extended model can find its use in industry where stabilized platforms are expected to cope with a variety of payload variations.

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