CCSS mathematics

The chance for change…

And the challenge

example item from new tests

Write four fractions equivalent to the number 5

Common ground or next steps?

- Decades of standards based school accountability systems
- Lessons learned?
- Time for next step?

Evidence, not Politics

- High performing countries like Japan
- Research
- Lessons learned

Problem from elementary to middle school

Jason ran 40 meters in 4.5 seconds.

Three kinds of questions can be answered

Jason ran 40 meters in 4.5 seconds

- How far in a given time
- How long it takes to go a given distance
- How fast is it going
- *Understanding how these three questions are related mathematically is is central to the understanding of proportionality called for by CCSS in 6th and 7th grade, and to prepare for the start of algebra in 8th.*

How do these two fraction items differ?

I. 4/5 is closer to 1 than 5/4. Show why this is true on the number line.

II. Which is closer to 1?

a) 5/4
b) 4/5
c) ¾
d) 7/10

With your partner, discuss ***how these items differ***. What do they demand from students?

Old State Standard

Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:

2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

Old Boxes

- People are the next step
- If people just swap out the old standards and put the new CCSS in the old boxes
  - – into old systems and procedures
  - – into the old relationships
  - – Into old instructional materials formats
  - – Into old assessment tools,
- Then nothing will change, and perhaps nothing will

Mile wide –inch deep

causes

cures

Mile wide –inch deep

cause:

too little time per concept

cure:

more time per topic

= less topics

Two ways to get less topics

1. Delete topics
2. Coherence: A little deeper, mathematics is a lot more coherent
   a) Coherence across concepts
   b) Coherence in the progression across grades

Why do students have to do math problems?

a) to get answers because Homeland Security needs them, pronto

b) I had to, why shouldn't they?

c) so they will listen in class

d) to learn mathematics

Why give students problems to solve?

- To learn mathematics.
- Answers are part of the process, they are not the product.
- The product is the student’s mathematical knowledge and know-how.
- The ‘correctness’ of answers is also part of the process. Yes, an important part.

Wrong Answers

- Are part of the process, too
- What was the student thinking?
- Was it an error of haste or a stubborn misconception?

Three Responses to a Math Problem

1. Answer getting
2. Making sense of the problem situation
3. Making sense of the mathematics you can learn from working on the problem

Answers are a black hole: hard to escape the pull

- Answer getting short circuits mathematics, making mathematical sense
- Very habituated in US teachers versus Japanese teachers
- Devised methods for slowing down, postponing answer getting

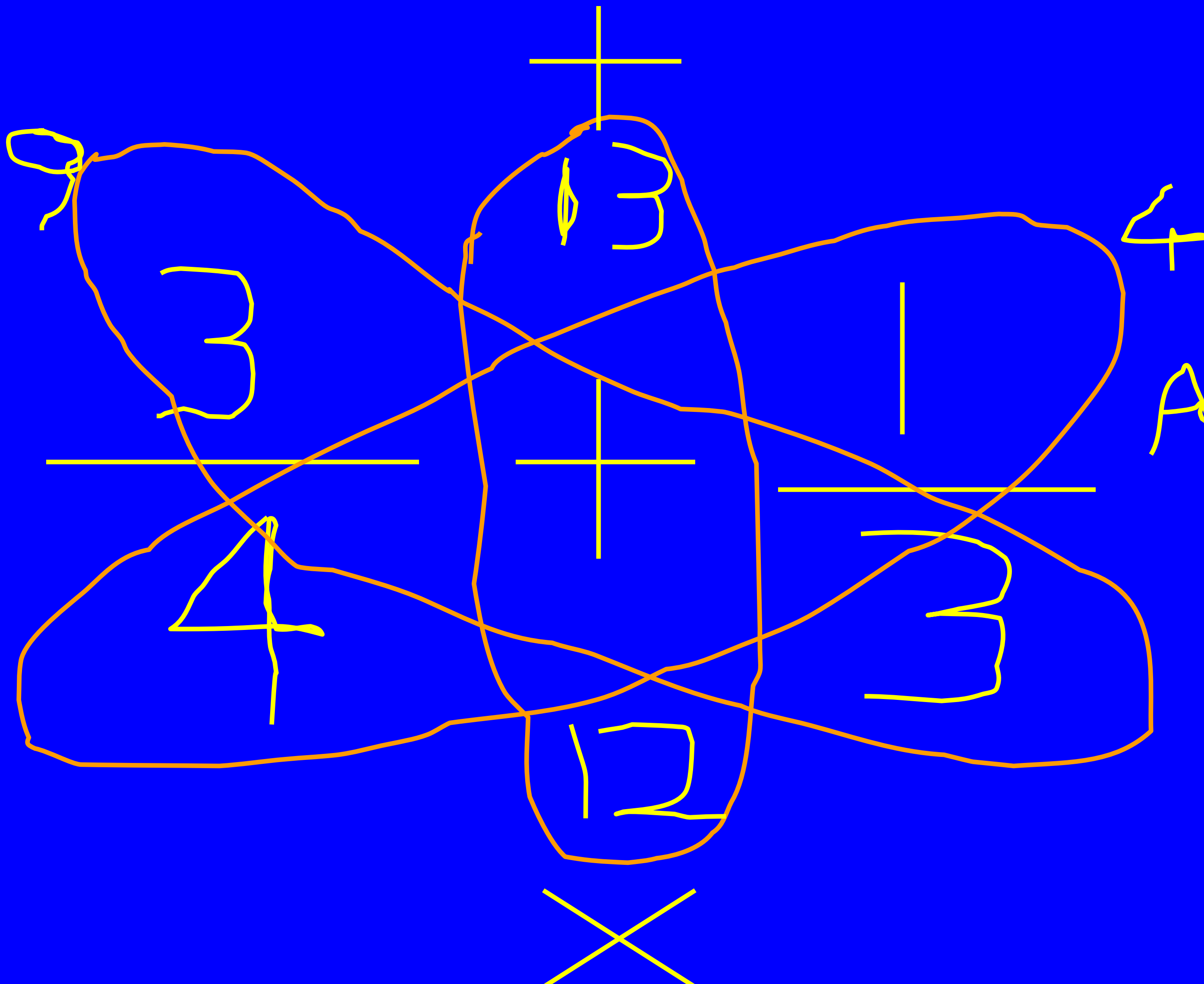
Answer getting vs. learning mathematics

- USA:
- **How can I teach my kids to get the answer to this problem?**

  Use mathematics they already know. Easy, reliable, works with bottom half, good for classroom management.
- Japanese:
- **How can I use this problem to teach the mathematics of this unit?**

Butterfly method

$$\frac{3}{4} + \frac{1}{3}$$



Ans: $\frac{13}{12}$

Use butterflies on this TIMSS item

- $1/2 + 1/3 + 1/4 =$

“set up and cross multiply”

- Set up a proportion and cross multiply
- It’s an equation, so say,

“set up an equation”

Solve it

how? Using basic tools of algebra: multiply both sides by a number, divide both sides by a number.

Three major design principles, based on evidence:

- **Focus**
- **Coherence**
- **Rigor**

Grain size is a major issue

- Mathematics is simplest at the right grain size.
- “Strands” are too big, vague e.g. “number”
- Lessons are too small: too many small pieces scattered over the floor, what if some are missing or broken?
- Units or chapters are about the right size (8-12 per year)
- Districts:
  - STOP managing lessons,
  - START managing units

What mathematics do we want students to walk away with from this chapter?

- Content Focus of professional learning communities should be at the chapter level
- When working with standards, focus on clusters. Standards are ingredients of clusters. Coherence exists at the cluster level across grades
- Each lesson within a chapter or unit has the same objectives….the chapter objectives

The Importance of **Focus**

- TIMSS and other international comparisons suggest that the U.S. curriculum is ‘a mile wide and an inch deep.’

- “On average, the U.S. curriculum omits only 17 percent of the TIMSS grade 4 topics compared with an average omission rate of 40 percent for the 11 comparison countries. The United States covers all but 2 percent of the TIMSS topics through grade 8 compared with a 25 percent non coverage rate in the other countries. **High-scoring Hong Kong’s curriculum omits 48 percent of the TIMSS items through grade 4, and 18 percent through grade 8. Less topic coverage can be associated with higher scores on those topics covered because students have more time to master the content that is taught.”**

- Ginsburg et al., 2005

U.S. standards organization

- **[Grade 1]**
- Number and Operations
  - …
- Measurement and Geometry
  - …
- Algebra and Functions
  - …
- Statistics and Probability
  - …

U.S. standards organization

- **[12]**

- Number and Operations
  - …
- Measurement and Geometry
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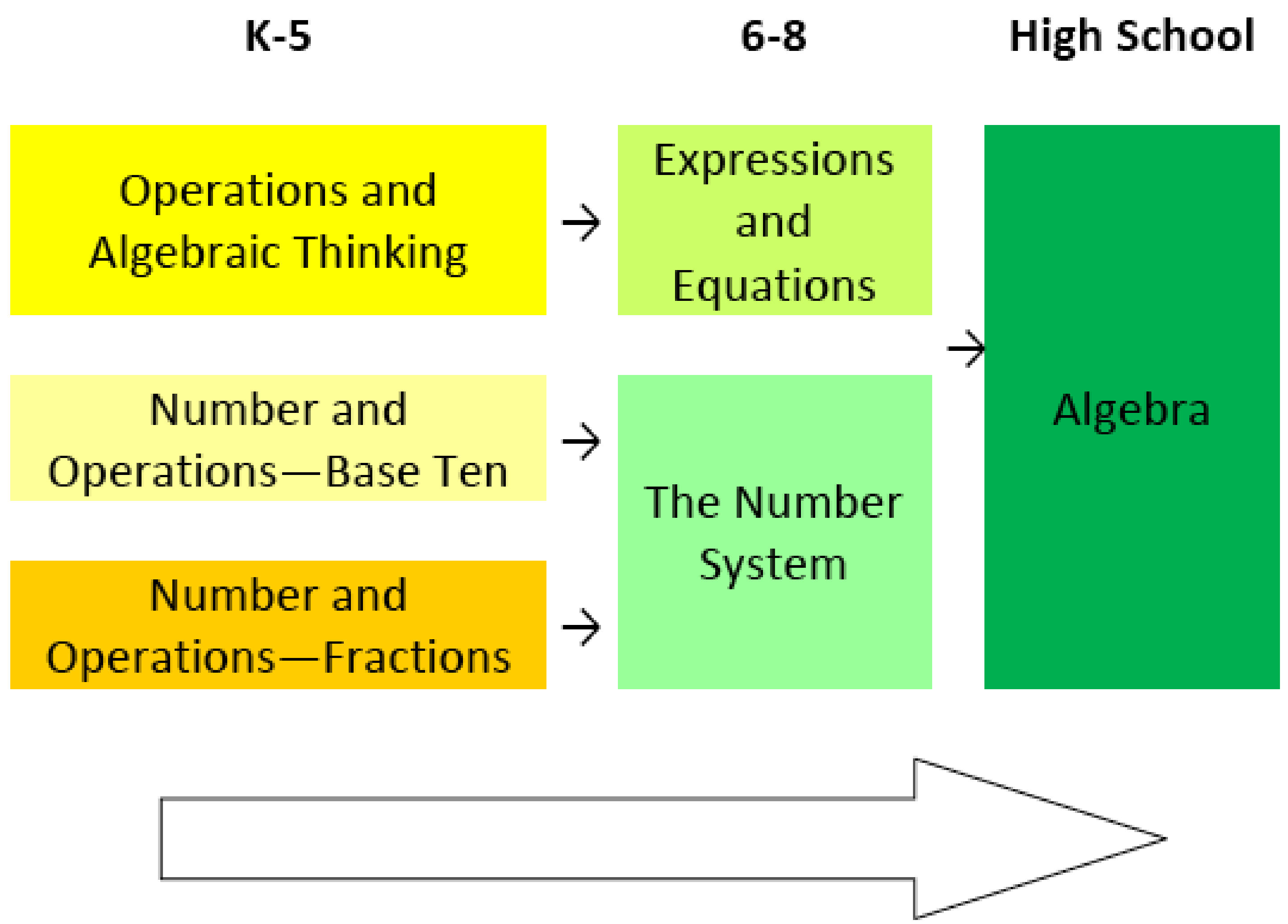
- Algebra and Functions
  - …
- Statistics and Probability
  - …

Focusing attention within Number and Operations



Silence speaks

no explicit requirement in the Standards about simplifying fractions or putting fractions into lowest terms.

instead a progression of concepts and skills building to fraction equivalence.

putting a fraction into lowest terms is a special case of generating equivalent fractions.

Fractions Progression

- Understanding the arithmetic of fractions draws upon four prior progressions that informed the CCSS:
  - equal partitioning and number line
  - unit fractions and operations
  - equivalent fractions

Unit fractions link operations on fractions to whole number arithmetic

- Students' expertise in whole number arithmetic is the most reliable expertise they have in mathematics
- It makes sense to students
- If we can connect difficult topics like fractions and algebraic expressions to whole number arithmetic, these difficult topics can have a solid foundation for students

Units are things you count

- Objects
- Groups of objects
- 1
- 10
- 100
- ¼   unit fractions
- Numbers represented as expressions

Units add up

- 3 pennies + 5 pennies = 8 pennies
- 3 ones + 5 ones = 8 ones
- 3 tens + 5 tens = 8 tens
- 3 inches + 5 inches = 8 inches
- 3 ¼ inches + 5 ¼ inches = 8 ¼ inches
- ¾ + 5/4 = 8/4
- 3(x + 1) + 5(x+1) = 8(x+1)

Grade 3:
unit fractions

1. The length from 0 to1 can be partitioned into 4 equal parts. The size of the part is ¼.
2. Unit fractions like ¼ are numbers on the number line.

Adding and multiplying Unit Fractions

Whatever can be counted can be added, and from there knowledge and expertise in whole number arithmetic can be applied to newly unitized objects.

Grade 4

1. ¼ +1/4 + ¼ = ¾
2. Add fractions with like denominators
3. 3 x ¼ = ¾
4. Multiply whole number times a fraction; n(a/b) =(na)/b

Grade 5

1. Add and subtract fractions with unlike denominators using multiplication by n/n to generate equivalent fractions and common denominators
2. 1/b = 1 divided by b; fractions can express division
3. Multiply and divide fractions

Fraction Equivalence Grade 3:

– Fractions of areas that are the same size, or fractions that are the same point (length from 0) are equivalent
– recognize simple cases: ½ = 2/4 ; 4/6 = 2/3
– Fraction equivalents of whole numbers 3 = 3/1, 4/4 =1
– Compare fractions with same numerator or denominator based on size in visual diagram

Fraction equivalence Grade 4:

- Explain why a fraction a/b = na/nb using visual models; generate equivalent fractions
- Compare fractions with unlike denominators by finding common denominators; explain on visual model based on size in visual diagram

Use equivalent fractions to add in Grade 5:

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

Fraction Item

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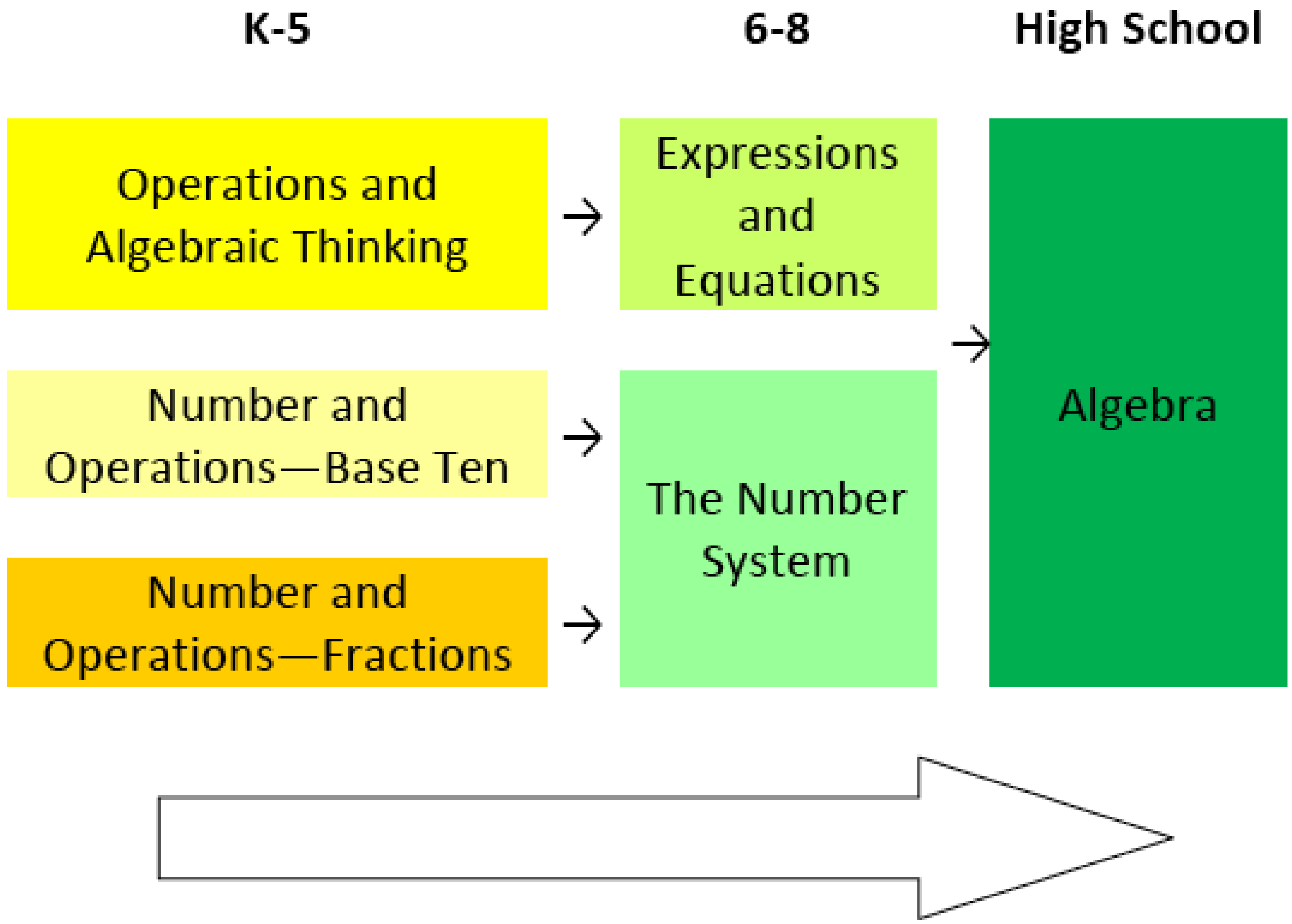
Two major design principles, based on evidence:

- **Focus**
- **Coherence**

Focusing attention within Number and Operations



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Participants: Answer questions a. and b.

Emma ran 30 laps around the gym in 15 minutes. (A lap is once around the track.)

After her run Emma made this calculation:

$$30 \div 15 = 2$$

a. What does the "2" tell you about Emma's run?

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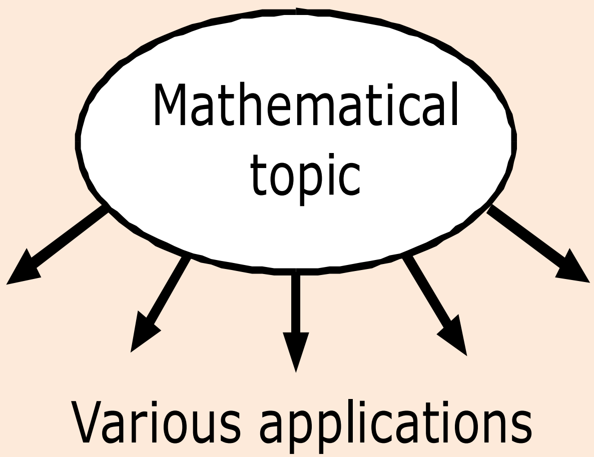
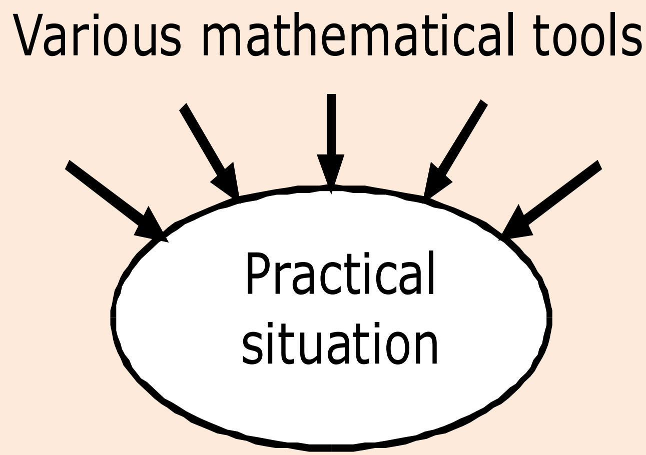
“Concept focused” v “Problem focused”:

Illustrative applications



Active modelling



What does good instruction look like?

- The 8 standards for Mathematical Practice describe student practices. Good instruction bears fruit in what you see students doing. Teachers have different ways of making this happen.

Mathematical Practices Standards

1. Make sense of complex problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

College and Career Readiness Standards for Mathematics

Expertise and Character

- Development of expertise from novice to apprentice to expert
  - Schoolwide enterprise: school leadership
  - Department wide enterprise: department taking responsibility
- The Content of their mathematical Character
  - Develop character

Personalization

The tension: personal (unique) vs. standard (same)

Why Standards? Social Justice

- Main motive for standards
- Get good curriculum to all students
- Start each unit with the variety of thinking and knowledge students bring to it
- Close each unit with on-grade learning in the cluster of standards
- Some students will need extra time and attention beyond classtime

Standards are a peculiar genre

1. We write as though students have learned approximately 100% of what is in preceding standards. This is never even approximately true anywhere in the world.
2. Variety among students in what they bring to each day's lesson is the condition of teaching, not a breakdown in the system. We need to teach accordingly.
3. Tools for teachers…instructional and assessment…should help them manage the variety

Four levels of learning

I. Understand well enough to explain to others
II. Good enough to learn the next related concepts
III. Can get the answers
IV. Noise

Four levels of learning

The truth is triage, but all can prosper

I. Understand well enough to explain to others
   As many as possible, at least 1/3
II. Good enough to learn the next related concepts
   Most of the rest
III. Can get the answers
   At least this much
IV. Noise
   Aim for zero

Efficiency of embedded peer tutoring is necessary

Four levels of learning

different students learn at levels within same topic

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   - An asset to the others, learn deeply by explaining

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   - Profit from tutoring

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Walk into a classroom, you see this

- Teacher says,
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When the content of the lesson is dependent on prior mathematics knowledge

- “I – We – You” design breaks down for many students
- Because it ignores prior knowledge
- I – we – you designs are well suited for content that does not depend much on prior knowledge… new content

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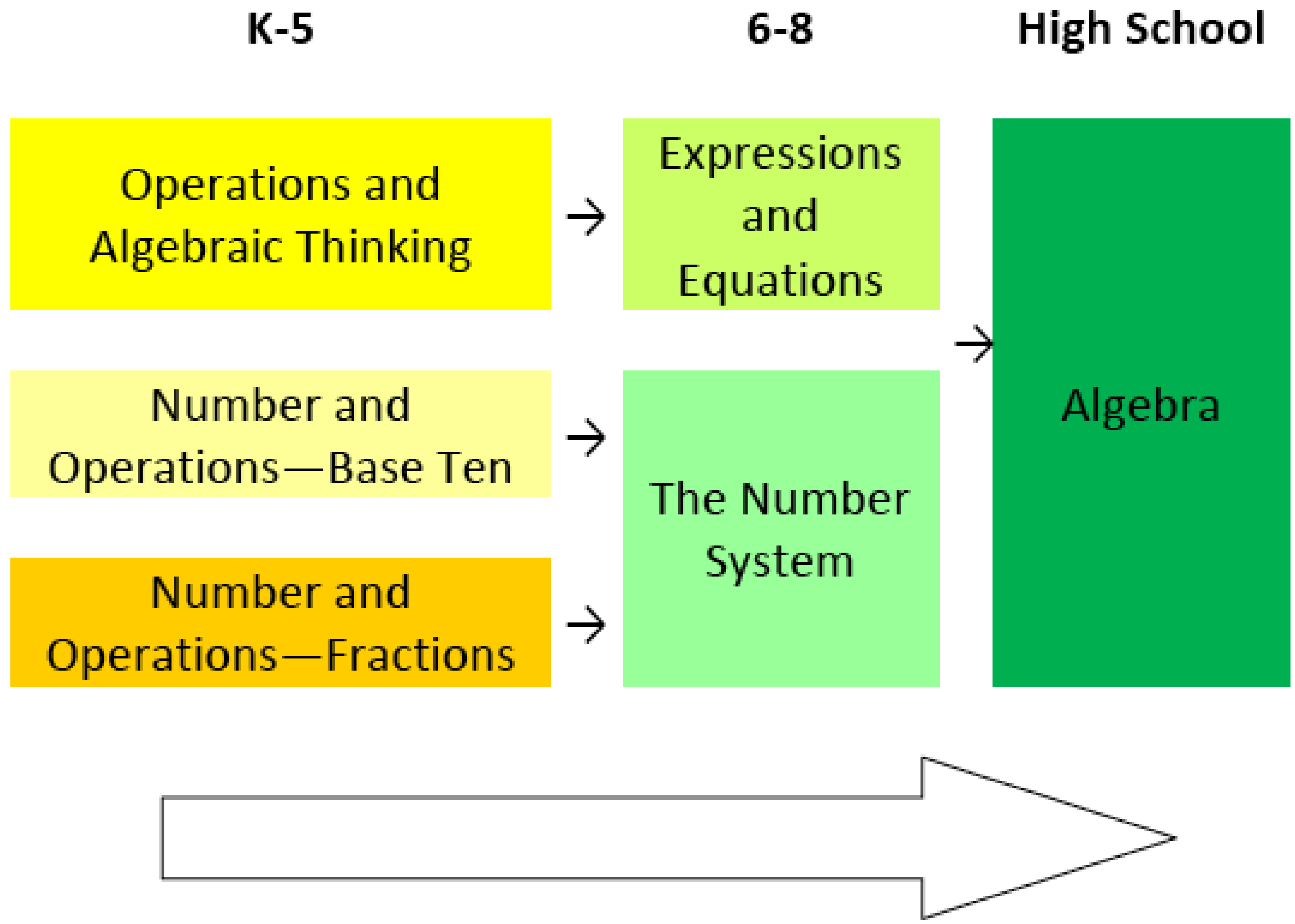

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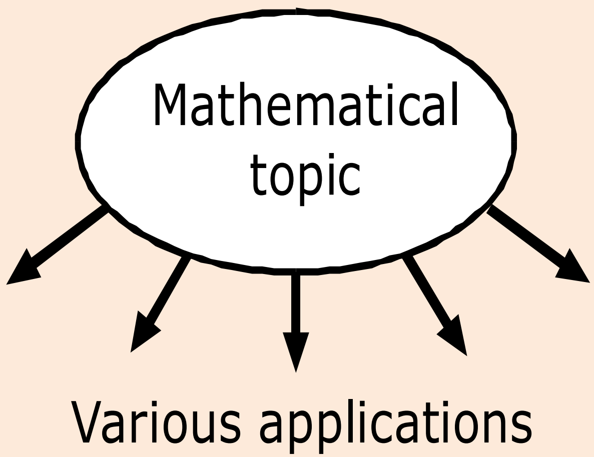

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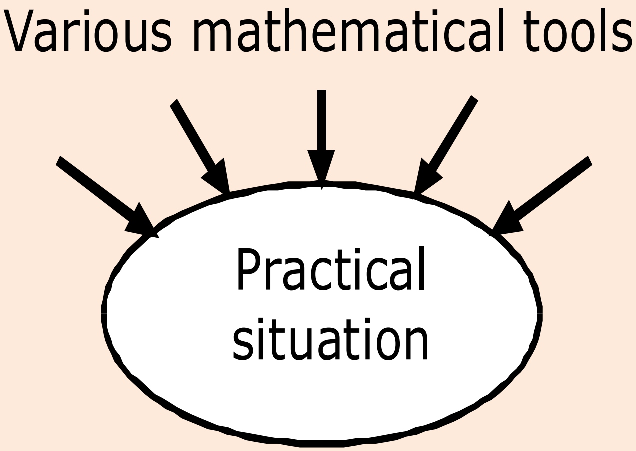

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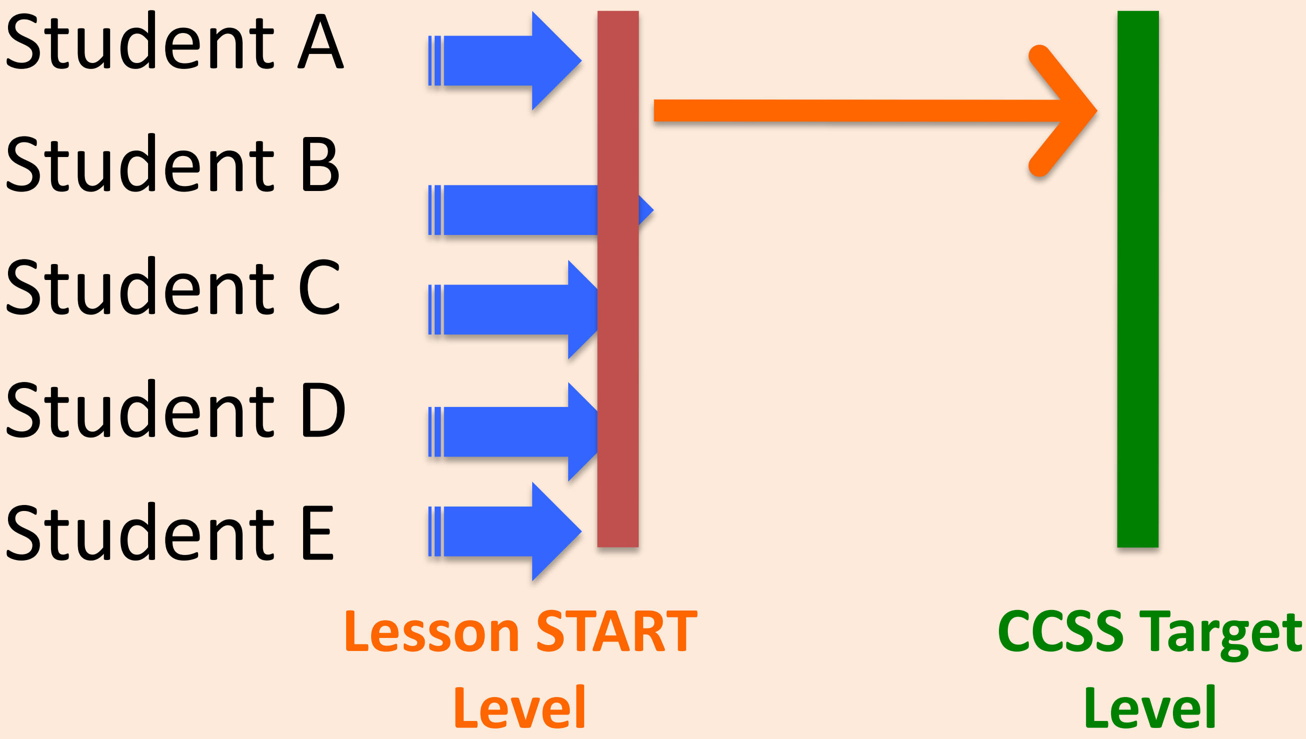
Minimum Variety of prior knowledge in every classroom; I - WE - YOU



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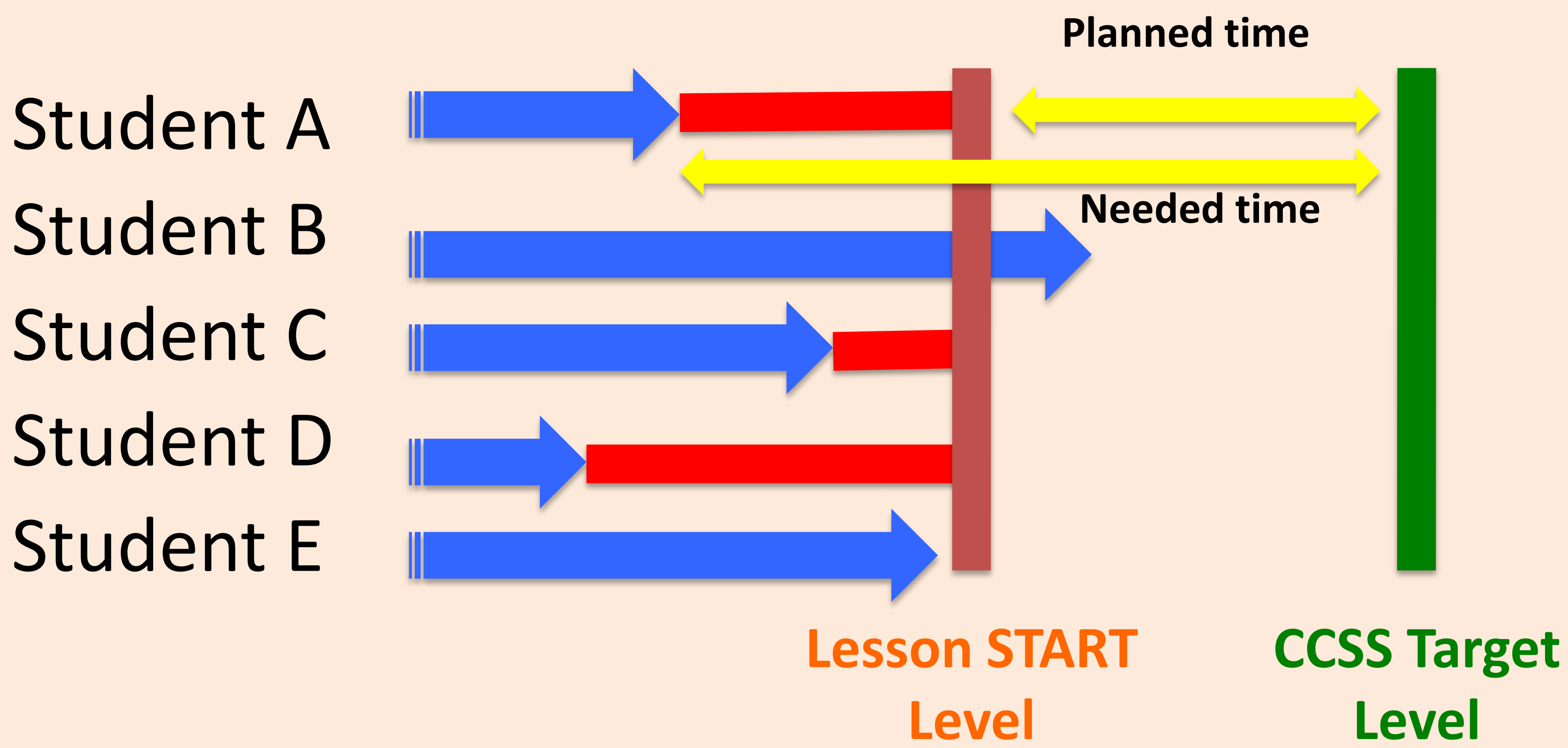

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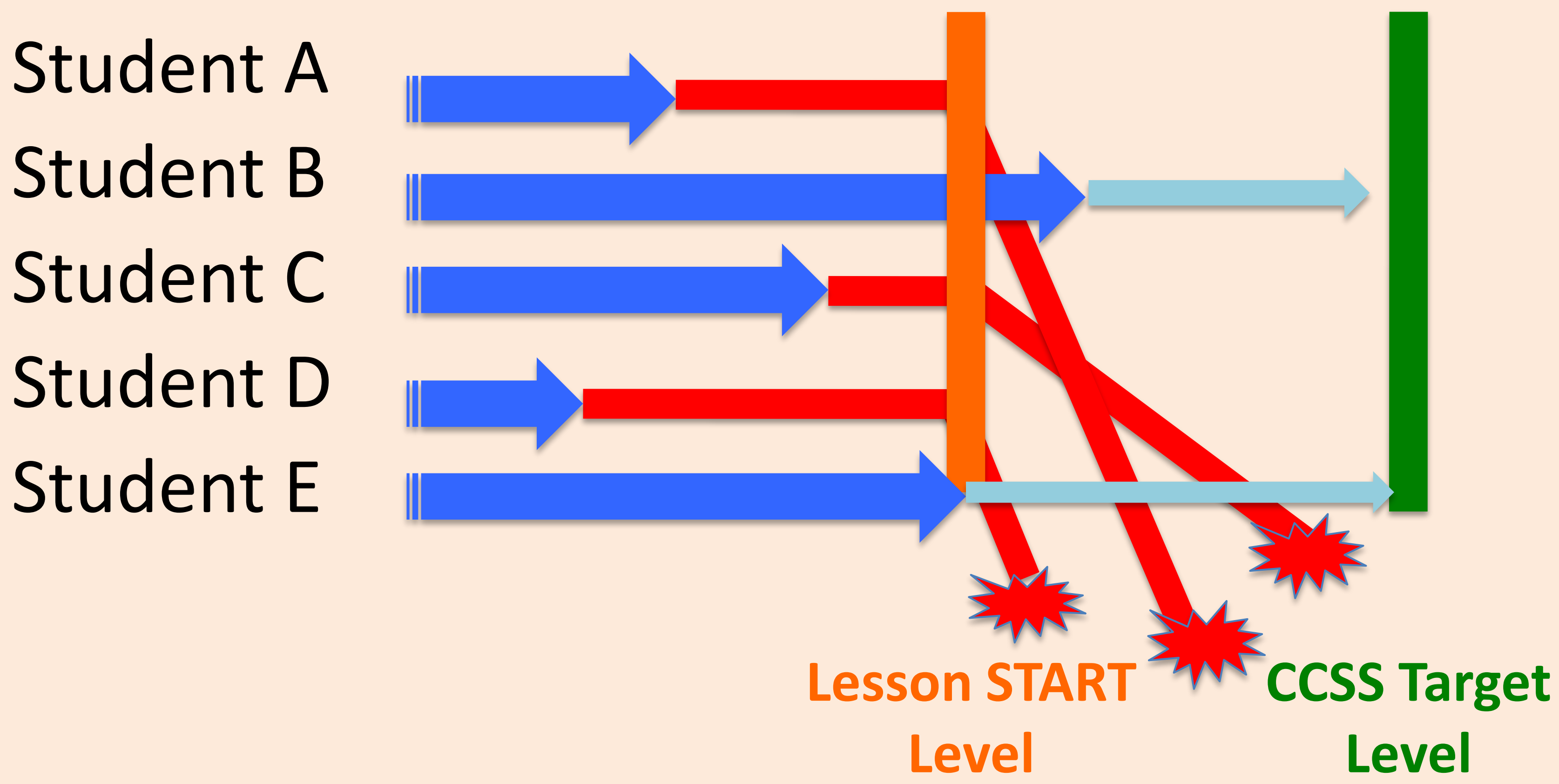

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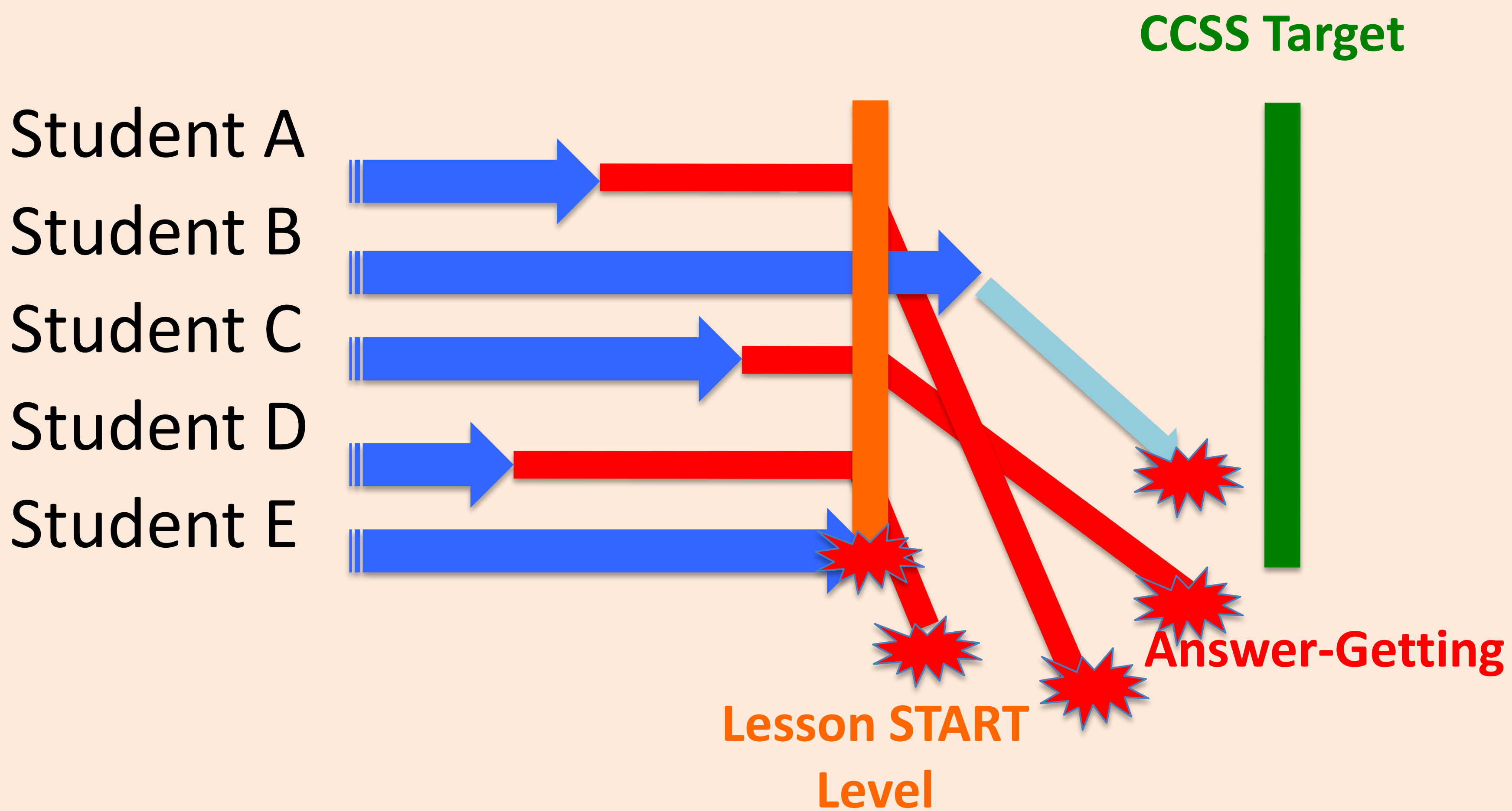
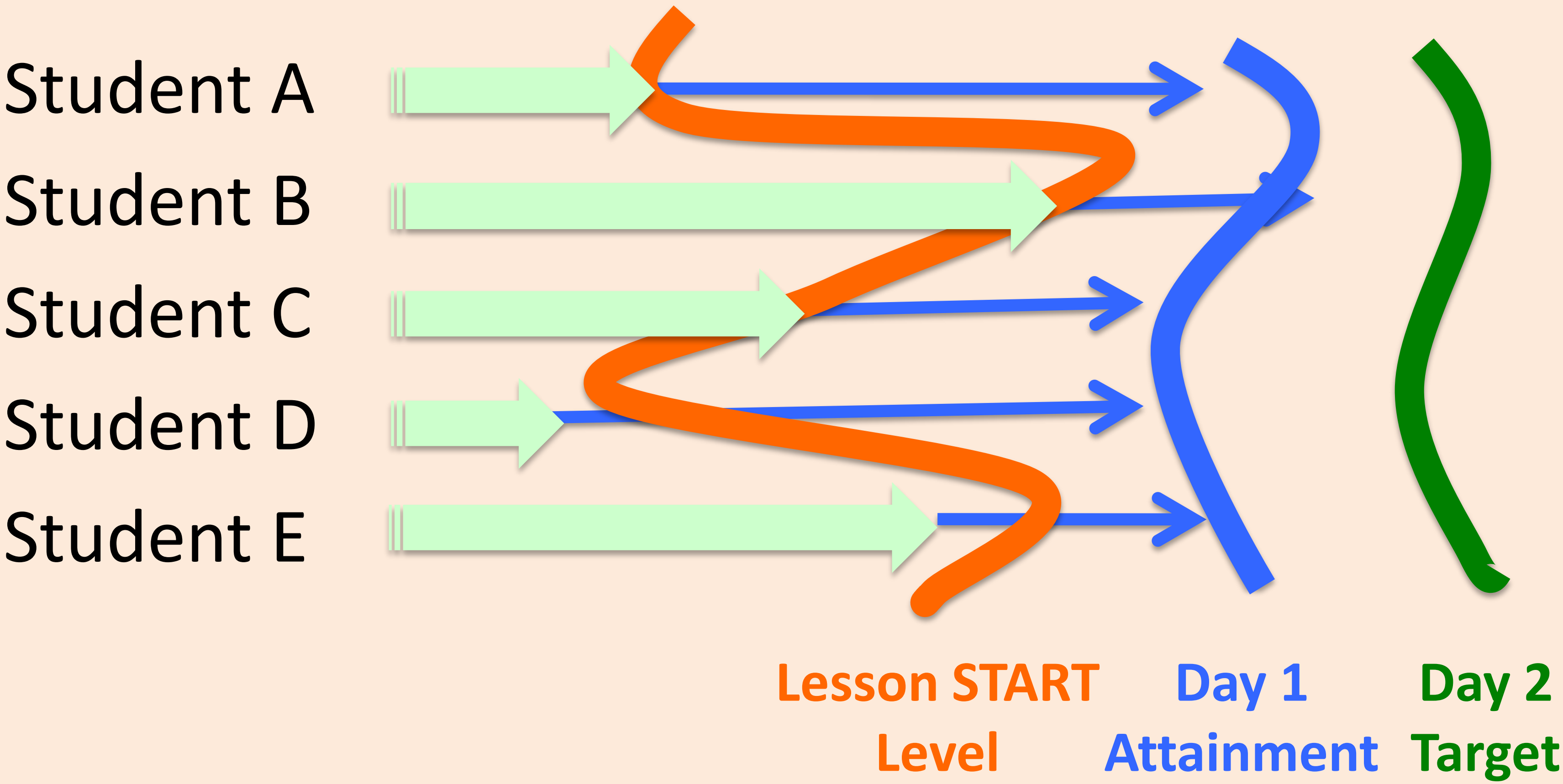


You - we – I designs better for content that depends on prior knowledge



Differences among students

- The first response, in the classroom: make different ways of thinking students' bring to the lesson visible to all
- Use 3 or 4 different ways of thinking that students bring as starting points for paths to grade level mathematics target
- All students travel all paths: robust, clarifying

Language, mathematics and EL

Language Differences and Content

- How knowledge, cognition and language are threads in a single fabric of learning,
  - Inadvertent ways system unravels this fabric: silos, assessment, classification of students, instruction
- Practices linked to discipline reasoning expressed in language and multiple representations
- Access to content courses
- Don’t leave out ELLs from Progression in text complexity and teaching for understanding

Discussions

- How can increased discussion from CCSS benefit Ells, rather than left out
- Communicative stamina needed, builds intellectual stamina:
- Video shown to kids
- How do we teach teachers to lead, manage discussions?

Imperfect

- Imperfect language is valuable and can express precise reasoning and ideas
- Progression through reality means progression through imperfections
- Not about waiting for the precise wording, but use of imperfect language to express reasoning and then making the language and reasoning more precise together
- Perfect teaching is unnecessary, imperfect works fine with stamina

Time

- Slow down for learning, thinking and language
  - The press of time against the scope and depth of curriculum
  - The press of time against the engagement, language processing and cognition of ELLs
  - The press of time against instruction in two languages
  - Time for teachers to learn, to think, to give feedback to students

Participants: where to find the time

- Some students need more time to learn than others, more feedback, more encouragement. Where can the more time come from? The more feedback? Encouragement?

Misconceptions

Where do they come from, and what to do about them

Misconceptions about misconceptions

- They weren’t listening when they were told
- They have been getting these kinds of problems wrong from day 1
- They forgot
- The other side in the math wars did this to the students on purpose

More misconceptions about the cause of misconceptions

- In the old days, students didn’t make these mistakes
- They were taught procedures
- They were taught rich problems
- Not enough practice

Maybe

- Teachers' misconceptions perpetuated to another generation (where did the teachers get the misconceptions? How far back does this go?)
- Mile wide inch deep curriculum causes haste and waste
- Some concepts are hard to learn

Whatever the Cause

- When students reach your class they are not blank slates
- They are full of knowledge
- Their knowledge will be flawed and faulty, half baked and immature; but to them it is knowledge
- This prior knowledge is an asset and an interference to new learning

Second grade

- When you add or subtract, line the numbers up on the right, like this:
- 23
- +9

- Not like this
-   23
- +9

Third Grade

- 3.24 + 2.1 = ?
- If you “Line the numbers up on the right “ like you spent all last year learning, you get this:

- 3.2 4
- <u>+ 2.1</u>
- <u>You get the wrong answer doing what you learned last year. You don’t know why.</u>
- <u>Teach: line up decimal point.</u>
- Continue developing place value concepts

progressions

- Every class has kids operating all over the progression, same bounces, normal, probably good,
- Teachers need to deal whole progression.
- Study group of teachers , book group
- progress to algebra one sheet

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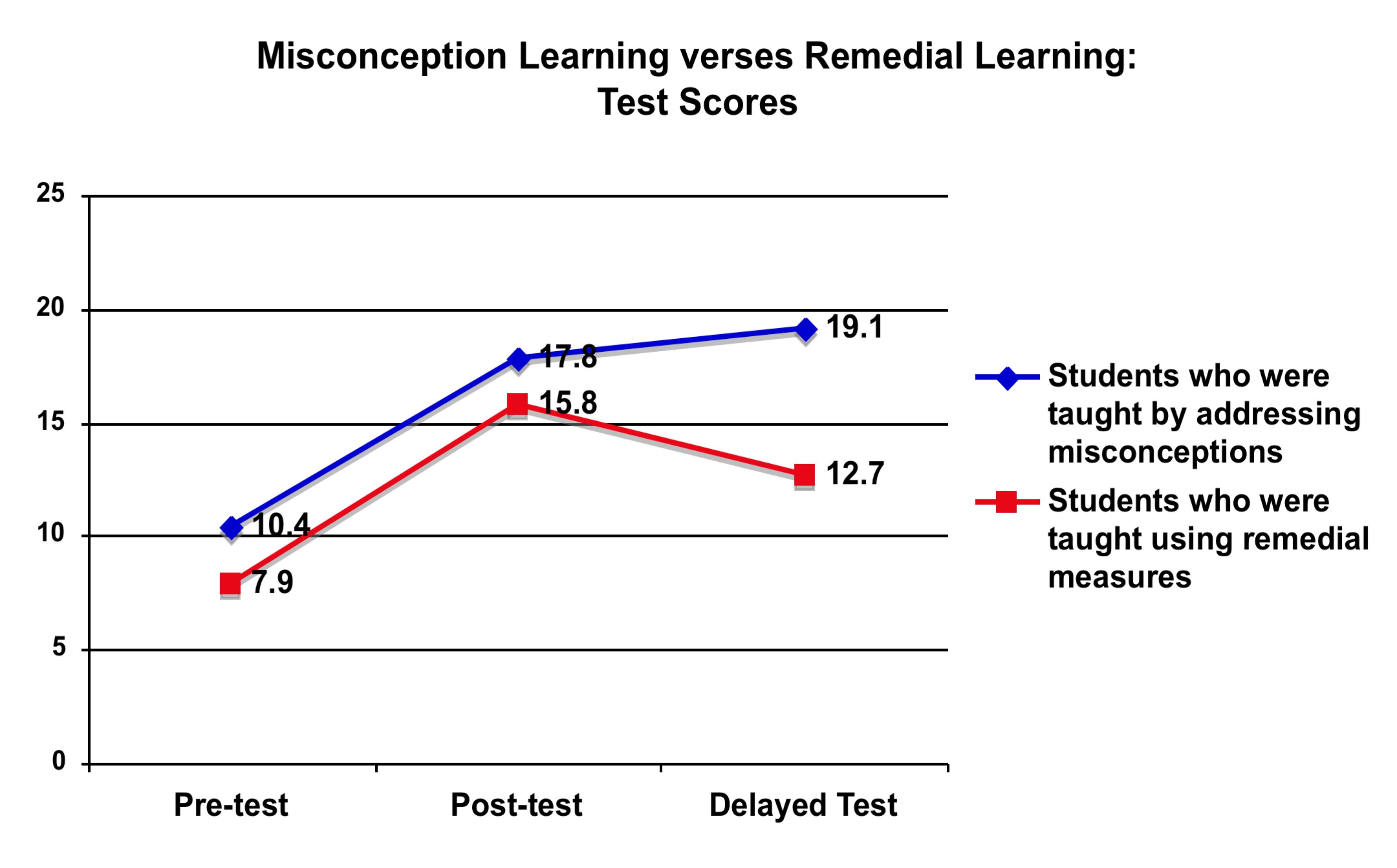
Chat: what misconceptions do students entering your school bring?

Research on Retention of Learning: Shell Center: Swan et al



Acceleration, catching up and moving on

Structure time in cycles

Cycles

- Problem by problem
- Daily
- Within unit
- Within semester
- Annual

Daily cycle

I. Social processes to learn other’s “ways of thinking” naturally travels progression from earlier ways of thinking to grade level ways of thinking.
II. Inside each grade level problem, a window back into the progression from earlier grades. Work through the window, don’t quit on the grade level.
III. Embedded tutoring
IV. Hints and scaffolding from teacher
V. Hints and scaffolding from program
VI. Homework help

Within Unit

I. Progression of problems
II. Progression of lessons
III. Rhythm from intuitively accessible contexts that scaffold thinking to mathematically precise, abstract and general. Learning to use mathematics as a reasoning tool.
IV. Small group "guided mathematics" for a day or 2 every week or 2, after a cycle of lessons. Students identified through the windshield of their actual work…finish what you start.

Beyond the classroom interventions

I. Most important and needed by most is help with the assigned work of the course. This includes homework help and study help. Should be available on much larger scale than we are used to. Open access to whomever wants it as well as assigned. Lower the social cost.

motivation

Mathematical practices develop character: the pluck and persistence needed to learn difficult content. We need a classroom culture that focuses on learning…a try, try again culture. We need a culture of patience while the children learn, not impatience for the right answer. Patience, not haste and hurry, is the character of mathematics and of learning.