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Abstract—This paper investigates the performance of grounding electrodes buried in the close vicinity of small soil heterogeneity, by using 2D and 3D finite element modeling. The proposed methodology enables consideration of electrode geometry effects which are not taken into account in standard analytical grounding system representation. The results are compared to those found in the literature or obtained by standard programs.

Index Terms—Grounding Systems, electrodes, finite element method, surface potential.

I. INTRODUCTION

The performance of grounding electrodes buried in the close vicinity of small soil heterogeneity has a considerable practical importance. As the behavior of small or concentrated grounding systems is not very different in high frequencies compared to low frequencies, it is usually sufficient to evaluate potentials produced by a DC current.

In many practical cases, analytical modeling of grounding systems is used based on thin wire approximation theory [1],[2]. Such a methodology uses closed form solutions and enables fast computation of grounding system performance in cases involving complex electrode structures [3],[4]. Although all typical electrodes practically satisfy the assumptions of thin wire approximation, such techniques may provide imperfect representation of certain aspects of the grounding system.

Specifically the grounding resistance is well approximated but the calculation of the surface potential of earth near the electrode may be inaccurate. In low frequency applications electrode external surface can be considered equipotential and thin wire theory introduces constant current density distribution. On the contrary, consideration of electrode geometry implies greater current density flow at the ends of the conductor.

This phenomenon has negligible effect on the global parameters as the grounding resistance but can influence considerably the local potential distribution. The discrepancy can be even more important in high frequency cases as the current distribution along the conductor is even more non-uniform.

The finite element method enables efficient modeling of both the electrode geometry and the surrounding soil heterogeneity, that is why it has been adopted for the grounding system representation.

This method has been applied to model a two dimensional grounding system geometry at low frequencies consisted of a vertical rod buried in a layered soil. The results obtained have been compared to those of a standard code based on thin wire approximation theory [7].

The case of a 3D grounding system geometry adopted usually when foundation grounding systems are used has been considered. The employed 3D finite element model results have been compared to those obtained by a commercial software [7].

II. MODELLING TECHNIQUES

Two different modeling procedures have been examined, and their results compared and discussed.

A. Thin Wire Approximation Method

When the shape of the grounding electrodes used is cylindrical while the length of the cylinder is much larger than its radius (D>>a) and its radius is much smaller than the wavelength (a<<\lambda) the effects of the cylinder dimensions can be neglected. Therefore the boundary conditions for a wire with infinite conductivity can be adopted involving zero total tangential field strength \(E\) on the surface of the cylinder as well as vanishing current at the ends of the cylinder. In practice a ratio of the length of the conductor segment to its radius of about 10 is satisfactory.

Moreover in the case examined uniform current distribution has been assumed. The solution giving potential values at any point in the surrounding soil can be given by integrating the contributions of each point current source that is placed along the length of the electrode.

In the case examined the potential, it is in the three hemispherical layers I,II and III can be easily expressed as follows:

\[
V_{r} = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} A_{mn} r^{n} P_{n}^{m}(\cos \theta) \cos m(\phi - \phi_{0})
\]

\[
V'_{g} = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \left( B_{mn} r^{n} + C_{mn} \frac{1}{r^{n+1}} \right) P_{n}^{m}(\cos \theta) \cos m(\phi - \phi_{0})
\]
where $P_n^m$ is Legendre function, and $A_{mn}, B_{mn}, C_{mn}, D_{mn}$ coefficients that can be determined from boundary conditions.

Extensive application of these formulae and discussion of the obtained results are given in [1].

B. Finite Element Method

Finite element modelling allows the calculation of complex grounding system geometries, and exact consideration of materials characteristics.

The soil constitution can be described in detail in every point in the region around the grounding system and nonlinearities can also be considered.

In the case of foundation grounding system arrangement considered hereafter the three dimensional configuration must be taken into account.

In the present paper both 2-dimensional axisymmetric and 3-dimensional configurations have been considered. In both cases the problem involves solution of the following Laplace equation:

$$ - \nabla \cdot (\sigma \nabla \Phi) = 0 $$  \hspace{1cm} (4)

where $\sigma$ is the electric conductivity and $\Phi$ the corresponding scalar potential.

By using standard weighting residual techniques the discrete problem can take the following form for the $i^{th}$ node:

$$ \sum_{m=1}^{n_{tot}} \sum_{j=1}^{j_{tot}} \int_{\Gamma_i} a_i (\sum_{m=1}^{n_{tot}} \nabla a_j \Phi_j) n \cdot d\Gamma = 0 $$  \hspace{1cm} (5)

where $a_i, a_j$ are shape functions used for the discretisation, $j_{tot}$ is the number of nodes of the $n^{th}$ element, $n_{tot}$ is the number of the elements sharing the node $i$, $\Gamma$ is the boundary of the solution domain $\Omega$, $n$ is the outside normal on $\Gamma$.

The boundary conditions used are natural Dirichlet conditions in the external boundaries of the solution domain, natural Neumann conditions in the soil-air interface and Dirichlet conditions on the electrode surface.

III. APPLICATION AND DISCUSSION

The method of 2D finite element analysis has been applied to model a grounding system at low frequencies consisted of a vertical rod buried in a layered soil as shown in Fig. 1. The soil model considered consists of three regions with resistivities $\rho_1=50 \ \Omega \ \text{m}$, $\rho_2=400 \ \Omega \ \text{m}$ and $\rho_3=1000 \ \Omega \ \text{m}$ respectively, separated by two concentric hemispheres. The vertical rod is 10m long its diameter is 5cm and is placed at a depth starting from $z=4m$ to $z=14m$ as shown in Fig. 1.
The obtained results have shown that the exact electrode dimension consideration involves voltage values at the surface of the earth considerably higher than those obtained when the thin wire approximation is adopted. This can be observed in Fig. 5, indicating that the equipotential lines in the soil area surrounding the electrode present higher density than those in Fig. 4.

Fig. 3b. Mesh used for the 2D problem analysis
(1700 nodes, 2600 first order triangular element)

Fig. 4. Equipotential lines distribution - thin wire approximation case
(0 < r < 30m)

Fig. 5. Equipotential lines distribution - precise electrode geometry representation case (0 < r < 30m)

Fig. 6. Geometry of the considered system of the two grids

Fig. 7. Detail of the 3D mesh used

Fig. 8. Mesh used for the problem analysis

Fig. 9. Surface potential obtained a) by the proposed 3D FEM model and b) by the method presented in [7]
A second example related to foundation grounding system structures has been considered. It is consists of two concentric square electrode configurations of side lengths of 5m and 10m respectively as in Fig. 6., placed in soil with low resistivity at a depth of 1m. The configuration of such a problem is three dimensional while, by considering its symmetries, the solution domain can be reduced to a triangular prism involving one octant of the whole geometry. The mesh used is extended at a distance of 300m from the electrodes and comprises 2500 nodes and 9240 first order tetrahedral elements (Figs 7 and 8). The calculated surface potential distribution by the 3D finite element model is compared to the one determined by a method involving almost closed form solutions [7] in Fig. 9. This figure illustrates the proposed model suitability for foundation grounding system analysis involving complex electrode configurations.

Moreover, the surrounding soil heterogeneity introduced in such cases by the concrete resistivity, consisting an important limitation of the analytical method [7], can be conveniently represented by the proposed technique. The effects of the soil resistivity in the case of a rectangular cross section concrete block embedding the grounding structure have been studied. The calculated results by the 3D FEM method of the surface potential variations along two different directions (x=0 and x=y) are shown in Figs 10 and 11 respectively. The cases of soil resistivity of 100 Ωm, 200 Ωm, 1000 Ωm and 2000 Ωm have been considered. These figures show the important impact of the surrounding soil resistivity on the distribution of the surface potential in the concrete near the electrodes.

IV. CONCLUSIONS

In this paper models based on two dimensional axisymmetric and three dimensional finite elements have been compared to standard methods used for the analysis of grounding systems including soil heterogeneities and complex electrode structures.

The proposed techniques enable both arbitrary soil heterogeneity consideration and exact electrode geometry representation, avoiding local surface potential discrepancies obtained by thin wire approximation theory.

V. REFERENCES