Influence of differentially heated horizontal walls on the streaming shape and velocity in a standing wave resonator

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**1. Introduction**

One of the important nonlinear phenomena in the field of acoustics is the acoustic streaming, a stationary fluid flow generated by sound. The interaction between the acoustic waves in a viscous fluid and the solid wall is responsible for this phenomenon. The classical streaming structure predicted by Rayleigh theory, is typically comprised of two streaming vortices per quarter-wavelength of the acoustic wave which are symmetric about the channel center line. The phenomenon of acoustic streaming has been extensively studied using the analytical [1], numerical [2,3,4] and experimental [5,6] methods.

The influence of the axial temperature gradient (i.e. the temperature gradient in the direction of the acoustic wave propagation) on acoustic streaming has been extensively studied analytically, numerically and experimentally. Rott [7] derived a simplified analytical formula for the acoustic streaming including the effect of the variable tube wall temperature. Aktas et al. [8] numerically investigated thermal convection in a 2-D resonator. In their model, the left and right sidewalls were kept at different temperatures, while the top and the bottom walls were kept insulated. They reported that the influence of mechanically-induced periodic oscillations on the heat transfer characteristics of the system is significant only in the presence of steady streaming flows. Thompson et al. [9] experimentally investigated the influence of axial temperature gradient on the behavior of the streaming flow. They observed that as the magnitude of the axial temperature gradient increases, the shape of streaming vortices becomes distorted.

Unlike the axial temperature gradient, the influence of transverse temperature gradient (i.e. the temperature gradient orthogonal to the direction of acoustic wave propagation) on acoustic streaming is scarcely investigated. Very recently, only one study has numerically investigated this behavior [10]. They considered the temperature dependent conductivity and viscosity in their model and conducted simulations at top-bottom wall temperature difference of 0, 20 and 60 °C. At 20 and 60 °C, they found that the classical two symmetric streaming vortices are distorted to one vortex. However, they did not study the transition behavior of streaming vortices due to the gradual increase in the temperature gradient. Furthermore, they did not provide any validation of their numerical model.

In the present study, we experimentally investigated the influence of transverse temperature gradient on the streaming patterns. The synchronized particle image velocimetry (PIV) technique is used to measure the streaming velocity fields for isothermal condition and differentially heated horizontal walls. To the best of authors' knowledge, this is the first experimental investigation of the streaming patterns in the presence of transverse temperature gradients.

**2. Experimental setup**

The experimental setup used in this study is shown in Fig. 1. The acoustic chamber is a Plexiglas channel of square cross-section. The channel is 105 cm long with the inner cross-section of 4 cm×4 cm. The walls of the channel are 10 mm thick. The two-dimensional streaming velocity fields inside the channel are measured using synchronized PIV technique. A New Wave Research 120 mJ Nd:YAG laser is used as a light source for the PIV measurements. A digital 2 Megapixel progressive scan CCD camera (JAI CV-M2) with the resolution of 1600 × 1200 pixels is used to image the flow. The camera is connected to a PC equipped with a frame grabber (DVR Express, IO Industries, London, ON, Canada) that acquired 8 bit images at a rate of 30 Hz.
four-channel digital delay generator (555-4C, Berkeley Nucleonics Corporation, San Rafael CA) is used to control the timing of the laser pulses. BIS(2-ETHYLHEXYL) SEBACATE mist with the mean diameter of 0.5 μm is used as the tracer particles. An aerosol generator (Lavision Inc., Ypsilanti MI) is used to generate the mist. The acoustic pressure is measured by a condenser microphone cartridge Model 377A10 PCB Piezotronics. A special loudspeaker driver is used to excite the acoustic standing wave inside the tube. The driver has the maximum power of 200 W. A function generator is used to generate the sinusoidal wave. The signal from the function generator is amplified by a 220-W amplifier. The loudspeaker is driven by this amplified signal (see Fig. 1).

The basic principal of the synchronized PIV technique is shown in Fig. 2. Consider the image taken at time $t_1$ in Fig. 2 as the first image and the image taken at time $t_2$ as the second image, with the time separation of $t_2 - t_1$. The cross-correlation of this image pair provides the acoustic velocity field at time $t_1$. Now, consider the image taken at time $t_1$ as the first image and the image taken at time $t_3$ as the second image, with the time separation of $t_3 - t_1$. Since the images acquired at $t_1$ and $t_3$ are exactly at the same phase, the acoustic velocity components at these times will be the same, therefore, the particle shift between these two images will only be due to streaming velocity. Thus, the cross-correlation of this image pair will provide the streaming velocity field at time $t_1$ (see Nabavi et al. [11] for more details).
details of the synchronized PIV technique). Trigger signal (TS) in Fig. 2 is used to trigger both laser and CCD camera of the PIV system. That is, both the laser and the camera are synchronized with the excitation signal via TS. The synchronization sequence for the excitation signal, laser pulses, camera frames and trigger signal is also shown in Fig. 2.

In the present experiments, the driver frequency is set equal to $f = 976 \text{ Hz}$ resulting in the formation of three complete standing waves inside the channel. The camera field of view is set equal to 10.5 cm in horizontal and 8 cm in vertical that allows to capture the streaming velocity field in the quarter-wavelength region. The maximum vibration displacement of the acoustic driver is set equal to 65 $\mu\text{m}$ and the corresponding maximum acoustic pressure is 775 Pa. Four different thermal boundary conditions are considered which are $\Delta T = 0$ (isothermal), $\Delta T = 0.3 \degree \text{C}$, $\Delta T = 0.8 \degree \text{C}$, and $\Delta T = 3 \degree \text{C}$, where $\Delta T$ is the temperature difference between the bottom and top walls (i.e. $\Delta T = T_{\text{bottom}} - T_{\text{top}}$).

To achieve isothermal boundary condition, the resonator is placed inside a large water tank of dimensions 50 × 50 × 90 cm. To achieve differentially heated horizontal walls, the resonator is placed on top of an aluminum plate (150 × 10 × 2 cm). The surface of the aluminum plate of the test section was coated black with high temperature paint to minimize the laser reflection. Two electric strip heaters are attached to the bottom of the aluminum plate. Constant temperature of the aluminum plate is maintained by using a PID controller. Five thermocouples were placed along the aluminum plate to act as a feedback to the PID controller.

Fig. 4. The streaming velocity fields correspond to the PIV images shown in Fig. 3. (a) $\Delta T = 0 \degree \text{C}$, (b) $\Delta T = 0.3 \degree \text{C}$, (c) $\Delta T = 0.8 \degree \text{C}$, and (d) $\Delta T = 3 \degree \text{C}$. Horizontal axis is the axial location in cm. Vertical axis is the transverse location in cm.
The characteristic response time of the seed particles is used as an estimate of the accuracy of the acoustic velocity measurements using PIV. The characteristic response time of the seed particles is computed by $T_p = \frac{u_t}{g}$, where $T_p$ is the particle response time, $u_t$ is the particle terminal velocity and $g$ is the acceleration due to gravity [12]. The terminal velocity is computed by $u_t = \left( \frac{\gamma - 1}{\rho_0 \nu_0} \right)^{1/2} D$, where $D$ is the diameter of the tracer particles, $\nu$ is the kinematic viscosity of the fluid and $\gamma$ is the ratio of the density of particle to the density of fluid [13]. Using the above equations, for $D = 0.5 \, \mu m$ and $u_t = 6.5 \, \mu m/s$, the particle response time is found to be $T_p = 0.67 \, \mu s$. For the driver frequency of 976 Hz, the particle response is more than 1500 times faster than the wave period. Thus, we conclude that the tracer particles accurately follow the flow.

For PIV cross-correlation, the size of the interrogation region is set equal to $32 \times 32$ pixels and the size of the search region is set equal to $64 \times 64$ pixels. A $50\%$ window overlap is used in order to increase the nominal resolution of the velocity field to $16 \times 16$ pixels. A three-point Gaussian sub-pixel fit scheme is used to obtain the correlation peak with sub-pixel accuracy. The spurious velocity vectors are detected and then corrected using a local median test.

### 3. Results and discussions

A sample PIV image for each of the four cases is shown in Fig. 3. The corresponding streaming flow patterns are shown in Fig. 4. The streamlines are also depicted in Fig. 4 for better flow visualization. For isothermal case (Fig. 4a), two streaming vortices per quarter-wavelength of the acoustic wave are observed which are symmetric about the channel center line. As the bottom wall is slightly heated ($\Delta T = 0.3 \, ^\circ C$, Fig. 4b), the bottom vortex vertically expands while the top vortex vertically contracts. As the temperature difference further increases ($\Delta T = 0.8 \, ^\circ C$, Fig. 4c), the top vortex completely disappears. Once the top vortex disappeared, a further increase in $\Delta T$ has no significant effect on the streaming flow structure. These results provide the first experimental evidence of the influence of transverse temperature gradient on the transitional behavior of the streaming flow structure. The results also indicate that the transition could occur at small values of $\Delta T$.

The influence of the transverse temperature gradient on the magnitude of the streaming velocity field is quantified in terms of the mean root-mean-square (RMS) axial streaming velocities. The results are plotted in Fig. 5 for all cases. The plot shows that as $\Delta T$ increases, the streaming velocity magnitude increases. The higher temperature at the bottom wall induces convective motion in the flow domain. Fig. 4 shows that once the transition occurred, the streaming patterns remain the same (for $\Delta T = 0.8 \, ^\circ C$ and $3 \, ^\circ C$), however, as shown in Fig. 5, the streaming velocity magnitude increases by $80\%$. These results indicate that the convective currents are aligned with the streaming flow patterns.

### 4. Conclusion

Influence of differentially heated horizontal walls on the shape and amplitude of acoustic streaming velocity field are investigated experimentally using the synchronized PIV technique. The results indicate that the temperature difference between the top and bottom walls deforms the symmetric streaming vortices to the asymmetric form. As the temperature difference increases, the amplitude of streaming velocity increases.

### Acknowledgements

This research is funded by the grants from Natural Science and Engineering Research Council of Canada (NSERC) and Concordia University.

### References