Black Hole Horizons and How They Begin

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ABSTRACT
Properties of black holes are examined that follow from the definition of the horizon as the boundary of the past of null infinity. The beginning of the black hole is defined as the set of spacetime events at which the null geodesics that generate the horizon enter the horizon. This set is spacelike and appears as a crease in the horizon because at least two generators cross at each of its points. The relation between the crease set and future null infinity is that between object and image produced by the gravitational lensing of the collapsing mass-energy. The crease set is not a manifold but has the structure of a tree. Near a vertex the horizon is a cone over a polyhedron.

1 Introduction
The horizon is what makes black holes both black and holes. We know the horizon as the infinite redshift (blackness) surface and as the point of no return (throat of the hole). Pictures show it, not incorrectly, as a round, black silhouette against a background of glowing gas or luminous stars, silent and massive, stolidly waiting to capture anything that gets in its throat. But it has not always been that way. Black holes generally are not eternal, but are created by gravitational collapse of matter. There was a time before the collapse when matter was spread out and there was no black hole. Then gravity caused matter to clump to sufficient mass and density to cause a horizon around the matter, and the horizon grew in size as more matter fell through it.

A word about nomenclature is in order. The entire history of a horizon is a three-dimensional hypersurface in four-dimensional spacetime. It will be called spacetime (abbreviated ST-) horizon, if it is necessary to distinguish it from the "snapshot" of the horizon at a particular time, that is, the intersection of the ST-horizon with a three dimensional spacelike surface $t = \text{const}$. The latter will be called space (abbreviated $S$-) horizon if it is not clear from the context which is meant. A similar distinction between black hole as the interior of either the ST-horizon or the $S$-horizon will be clear from the context.

How did a non-eternal horizon get started? Is there a stage of horizon formation in the life of a black hole? What came first, high density of mass and energy to cause a horizon, or a black hole horizon that matter had to cross to complete its irrevocable collapse? What seems like a chicken-and-egg problem does have an answer: the horizon came first and was ready for matter to fall through it when it reached sufficient concentration. This seems surprising because it goes against causality -- how did the horizon "know" that matter would ever reach the necessary concentration to support it in steady state? Moreover, when a black hole grows as more matter falls through its horizon, the horizon expands long before matter crosses.

The reason for this odd behavior lies in the very definition of a black hole horizon, which is based on the future of the spacetime in which it lives. Rather than being defined by initial conditions, like normal dynamical systems, the horizon satisfies a "teleological" final condition:
it must eventually hover in an unstable state between collapse and expansion. One cannot
exactly locate the horizon without knowing the entire future history of the spacetime: to be
sure that an event is outside the horizon, one must verify that it is possible to escape by a
timelike worldline from that point to arbitrarily large distances, taking an arbitrarily large time.
Such worldlines that escape wind up in a distant region where gravity is weak and the
spacetime is asymptotically flat.

In this paper I want to explore how a horizon starts. To do this we must first understand
how it ends, and go backwards from there.

2 The structure of infinity in Minkowski space.

Asymptotically flat means asymptotically like Minkowski space. What is infinity like in
Minkowski space? Like Euclidean space, Minkowski space has no place one can point to as the
location of infinity. These spaces are open sets, one can exhibit sequences that go to infinity,
but to reach it one has to equip them with a boundary -- and there are different ways to do
this, depending on how one maps the neighborhood of infinity into a finite region. The
mapping will distort spacetime, but in view of the importance of light rays (null lines) for
causality and black holes it is appropriate to demand that null lines not be distorted. Mappings
that accomplish this are the conformal transformations. They change the metric $ds^2$ by an
overall, position-dependent factor. Since $ds^2 = 0$ along null lines, the nullness does not
change after multiplication by a factor.

In Euclidean space a well-known conformal change is the inversion with respect to a unit
circle (in some units). This transformation maps a point at a distance $r$ from the origin of the
circle into one at a distance $R = 1/r$, leaving the angles of polar coordinates unchanged. In
two-dimensional polar coordinates the metric is

$$ds^2 = dr^2 + r^2 d\theta^2 = d(1/R)^2 + (1/R)^2 d\theta^2 = R^{-4}(dR^2 + R^2 d\theta^2)$$

Thus the conformal factor is $R^{-4}$, and the origin $R = 0$ of the inversion coordinates
corresponds to infinity in the original space. In this sense infinity has been added as a single
point to Euclidean space. Figure 1(a) shows how the inversion about the dotted unit circle
distorts space. Its outside becomes the inside after inversion, and vice versa. The dashed lines
at distance 1, 1.25, 1.5 become nested circles. The farther the line from the origin, the smaller
the circle. If the line moves to infinity to the right, the circle contracts to the point marked
``infinity''. The conformal map shows how the point at infinity is related to the finite points of
the original Euclidean space.

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1For example, in one boundary notion for Euclidean space all straight lines, continued indefinitely, reach the same
boundary point; in another notion they reach different boundary points, depending on their direction.
Conformal mappings (inversion). The inversion takes place about the dotted curves. The images of the dashed lines are the solid curves. (a) in Euclidean space, inversion about the dotted circle is a conformal (angle-preserving) map. The image of infinity is a point at the circle's origin. (b) in Minkowski space, inversion about a hyperbola maps lines (a, b, c) into hyperbolas, and infinity into a lightcone.

In Minkowski space a similar inversion, adapted to the indefinite metric, maps much of infinity into a finite region [1]. Here spacetime with coordinates $x, t$ is inverted about the unit hyperbola, $x^2 - t^2 = 1$, with new coordinates $X = x^2 - t^2, T = tx^2 - t^2$, and metric

$$ds^2 = dx^2 - dt^2 = (X^2 - T^2)^2(dX^2 - dT^2).$$

For finite $t$ and as $x \to \infty$, we get $(X, T) \to (0,0)$, so spacelike infinity is a single point as in the Euclidean case. Figure 1(b) shows the dotted hyperbola about which the inversion takes place, and the dashed lines a, b, c, which become correspondingly labeled hyperbolas after inversion. However, the entire lightcone $X^2 - T^2 = 0$ corresponds to infinite values of $(x, t)$:

Minkowski infinity consists of three parts, past null infinity, spacelike infinity, and future null infinity. (A fourth part, timelike infinity, has little relation to black holes.) The left quadrant of Figure 1(b) shows how a typical outgoing null ray, after inversion, becomes ingoing and ends at a point $P$ of future null infinity. This infinity can only be reached by null rays, but we may think of it as the location of observers far away from the center, who can observe everything that can be known about the exterior of the spacetime. (The worldline of an actual observer at a finite, large distance would be a line like the dashed lines, but far from the center, and in the inverted map it would be a timelike hyperbola very close to the infinity null line.)

What can be known are signals propagated along causal worldlines. The collection of all spacetime events that can send causal signals to any set of spacetime points is called the "past" of that set.

3 Definition of a black hole

We are now ready to tackle the definition of a black hole via its horizon. The spacetime must have an asymptotic region that is like Minkowski space and in particular has a future null infinity. We then define the (event) horizon as the boundary of the past of future null infinity.
infinity. This past may of course have no boundary, as in Minkowski space. But if it does, we have a black hole spacetime. We confine attention to the case when eventually the spacetime contains only one black hole, of spherical topology.

The black hole definition is useful for proving interesting and physically relevant properties of black holes, even though it has the above-mentioned lack of locality, and is not tied to large curvature or strong gravitational fields. In fact by an instructive modification a part of Minkowski space can become a black hole spacetime. In the conformal picture of Fig 1b, draw a horizontal line through point P and cut the line and everything to its future from the spacetime. Now future null infinity ends at P, its past is everything below the right-moving null ray and below future null infinity, and the horizon is this null ray itself. In the actual space the cut line is a spacelike hyperbola in the future of, and asymptotic to, the horizon. For an observer to experience the black hole properties of this horizon he has to come to point P for large times, for example by moving to the right and away from the origin with enough acceleration to stay always ahead of the horizon. Such observers, called Rindler observers [2], of course also outrun any signal that started from a point to the left of the horizon, so that region is for them the interior of the black hole, which they cannot see.

It is instructive to consider the four-dimensional space obtained by rotating this cut-up two-dimensional Minkowski space about the \( t \)-axis. The horizon becomes a double lightcone with tip where the two-dimensional null ray intersects the \( t \)-axis. Now infinity is likewise a light cone, and no single Rindler observer can see all of the past of this infinity. We must therefore think of observers everywhere near the infinity lightcone, and whatever any one of them can see is in the exterior of the black hole. This places the past part of the double lightcone into the exterior region, because each of its parts can be seen by some Rindler observer. The Rindler horizon is just a lightcone, it begins at a point on the \( t \)-axis and expands from there. Do all horizons start from a point?

4 The horizon's starting points

Two important properties of a horizon follow from its definition: it is a null hypersurface, and this surface is closed, without boundary. It cannot contain a timelike curve connecting to null infinity because this curve would then lie in the past of future null infinity, and not on the past's boundary. The only possibility therefore is that at each point of this three-dimensional hypersurface there is one null direction, with all other directions being spacelike. (There cannot be, for example, two null directions because their sum could be timelike.) Null hypersurfaces have the remarkable and useful property that the integral curves of their vector field of null directions are null geodesics. So the ST-horizon is generated by null geodesics, and we can think of the S-horizon as a wave front of the generators considered as wave vectors.

The ST-horizon has no boundary because it is itself a boundary [3]. Therefore it cannot "spring into being" at a finite size at some moment, for that initial figure would be its boundary in spacetime. It also cannot be smoothly bounded in the past because it would then have to be spacelike somewhere. The only possibility is that it starts not smoothly, with a crease [4]. The spacetime in which the horizon is embedded is however smooth, hence the null generators of the horizon can surely be continued backward in time beyond the crease. A beginning event
occurs when a generator enters into the horizon (Figure 2). Points before this event can be causally connected to infinity (escapes), points after this event cannot (does not escape); therefore the event itself must also lie on at least one other generator of the horizon, dividing those that escape from those that do not. Thus the crease set of beginning points is the intersection of the horizon with itself. As the intersection of three-dimensional null surfaces, this set is spacelike and has dimension 0, 1, or 2. Since a given horizon is a topological 2-sphere at late times, and the 2-sphere contracts to the beginning set at early times, this set is connected.

Beginning of a horizon. (a) Spacetime diagram in two dimensions. Points on the null geodesic after (above) a beginning event generate the horizon. No escape from the black hole is possible from those points. Points before the beginning must have at least one direction from which escape to infinity is possible. At the beginning point escape is marginally possible, so another generator crosses at the beginning point. (b) The horizon on successive two-dimensional spacelike slices, superimposed. The crease in the ST-horizon leaves an angular trace on a surface of constant time. The arrows indicate the direction of propagation of the horizon, the projection of the horizon's null direction. The crease point (tip of the angle) moves in a spacelike direction, at superluminal speed. As it slows down to light speed the crease disappears.

Though spacelike, the crease set does not necessarily occur all at one time. The 3-dimensional hypersurfaces of constant time (``time slices") will generally intersect the crease set in a lower-dimensional crease set (dimension 0 or 1), which lies on the S-horizon. The generators enter the horizon at the crease set, at least two at every crease point. Since they have nothing to do with the horizon before they enter we can pretend that they start at the crease point like rays from a flash that goes off at the crease point. At later times the wave front from all these flashes is the ST-horizon. Since the crease set is spacelike the flashes go off in a sequence that moves faster than the speed of light, and the S-horizon develops like a shock front with the characteristic Mach angle determined by the speed of the ``source" (the timing sequence for the flashes, Figure 2(a)).

The crease set is limited by the size of the final horizon, so it is reasonable that it stop somewhere. It cannot stop while the ``source" moves superluminally. Instead it slows down, and when it has reached the speed of light the Mach angle is $90^\circ$, the horizon has become
smooth and no further generators enter (Figure 2(b)). At this very end point the crease set has a null, future-pointing tangent. Such end points are the only caustic points of the family of generators that lie on the horizon. If the black hole is not eternal, the crease set is a finite, lower-dimensional beginning of the horizon that contains all the properties of the horizon, in the sense that the horizon develops causally from it. That is, the horizon is the boundary of the future of the crease set. In this sense the crease set is dual to future null infinity, a kind of geometrical optics image, as we shall see.

5  Is the horizon observable from the start?

The horizon is first and foremost a theoretical construct that depends on the behavior of spacetime in the distant future. But we are increasingly confident today that we observe black holes, or at least their effects, for example, at the centers of galaxies. Before long we may actually have pictures of a black hole as a dark disk in front of a luminous background [5]. That a mathematical construct should correspond closely to physical observations is not unusual. In this case the construct, the black hole, involves the distant future, whereas the observations of course do not; but when a black hole has been around for a while it is not unreasonable that we, the distant observers, have reached the distant future with respect to the black hole, and are observing it essentially from future null infinity.

The situation is rather different during the black hole’s formation epoch. The horizon generally exists before the collapse is complete, even in a finite, flat, Minkowski region as in Figure 5(a), where it is just an ordinary light cone. But in Minkowski space there are the same kind of light cones everywhere, so how can the one that is a horizon be distinguished from all the rest? How can an observer near the early horizon observe black hole properties, when his neighborhood that includes the black hole is just Minkowski space?

The black hole only appears to those that reach future null infinity. Distant observers can send a probe or a colleague near a black hole as it is forming, but he will only live to report his observations if he does not fall into the black hole. During the early formation period he must accelerate away from the horizon in order to avoid being caught up in the later collapse. During the early epoch he observes just Minkowski space, but as a Rindler observer. To this observer the Rindler black hole looks much like other black holes. The horizon can be observed at all stages of its life.

6  Relation of horizon dynamics to gravitational lensing.

The odd, non-causal behavior of the horizon becomes more reasonable if we read its history backwards in time. At late times when the black hole is fully formed, the S-horizon has a simple, spherical shape (if the black hole is not rotating it is a round 2-sphere). By moving along the null geodesic generators, a S-horizon can be continued into the past as well as into the future. That is, we think of the S-horizon as a wavefront, and propagate this wavefront forward
or backward in time by a Huygens' construction of geometrical optics. Small errors in the putative location of the horizon increase in the future, because a ray just outside the horizon will go off to infinity, but one just inside will collapse toward the center. For the same reason the development to the past is stable: a somewhat incorrect horizon surface approaches the true horizon when continued into the past. This property is used for ``horizon finding'' in numerically generated spacetimes. One starts with a good guess at the late-time horizon (for example, a surface of stationary area) and propagates backwards [6].

If gravity is attractive, the area of the horizon can be shown to be constant or increase to the future, and to be constant or decrease to the past. If the horizon is not eternal, its area has to decrease to the past. These changes are caused by real or effective mass-energy in the spacetime. Einstein's equations for the dynamics of spacetime and its matter content do not single out a time direction, they work the same way forward and backward in time. The process that makes the horizon more complicated at earlier times, with creases and topological changes, is the same as what in the forward time description is called gravitational lensing [7].

The basis of gravitational lensing is the bending of light rays by concentrations of mass and energy. When this was verified for the sun Einstein became world famous. Light rays that graze the sun are bent by 1.75 arcsec and brought to a ``focus'' at about 0.01 light years from the sun. But because gravity decreases with distance, light rays that pass at a greater distance are bent less, unlike the case of a real lens, where the outer rays are bent more than the central ones. Consequently a plane wave is not focused at a point but at a line, a kind of extreme spherical aberration. If the mass distribution itself is not axially symmetric about the line focus, effects akin to astigmatism spread the focus into a surface. Two (or more) rays, or horizon generators, intersect on the focal surface, so this surface is the crease set where the horizon starts. In a time sequence the S-horizon is a propagating wave front that has creases where it crosses a focal surface. The four-dimensional view would be like the ray diagram one draws in geometrical optics. In this backward time propagation the rays start at the object, future null infinity, and the crease set is its image, distorted from a point image by the imperfect gravitational lens of the collapsing matter.

Schematic picture of horizon at successive time slices for axially symmetric collapse of two centers, with the angular direction suppressed. The figure is read from the inside out: The
horizon starts as two points near the two centers, and expands to the two smallest football-shaped surfaces. They lose their points at the far ends by the process of Figure 2b and become spherical surfaces with a single conical point. They merge at these conical points and become one, more and more, spherical surface. From the outside in, the sequence is the development along the null normal in the negative time direction. Beyond the time of crossing the dotted part of the surface is not part of the horizon.

A typical, but only schematic, horizon history is shown in Figure 3 by the axially symmetric case of two mass concentrations that separately collapse to form two black holes, which later join to make a single black hole\(^2\) [8]. The spacelike surfaces chosen for the qualitative depiction in the figure are such that the collapsing matter and the later black holes remain at the same coordinate location, so that the collapse consists of the horizon coming up to meet the matter. The curves in the figure should be rotated about a horizontal axis to form closed surfaces, representing the S-horizon at successive times. They are to be read from the inside out, so the smaller surfaces occur earlier. The outermost surface should be surrounded by later, more spherical surfaces to represent the complete history of black hole formation.

If we read the figure backwards in time, from the outside in, the deformation of the outer spherical surface make sense in terms of the gravitational lensing and time delay caused by the matter concentrations (not shown in the figure) around the two centers. At times before the figure-8 type surface the surfaces intersect themselves, and the parts beyond the intersection points (shown dotted) are no longer part of the S-horizon. This is because what was the outward direction, allowing escape to infinity, is now the inside direction for the dotted surfaces, so there is escape from the inside. All points on the dotted curves are "snapshots" of null generators that eventually enter the horizon at a corner (crease) point. As we go further backwards in time, corners annihilate pairwise until the whole surface cab be drawn dotted and no more S-horizon is left (unless there are eternal black holes) -- the stage before the gravitational collapse took place.

7 Branching at a black hole's origin

The crease set at the beginning of a black hole's horizon is formed by the intersection of two parts of the horizon -- the parts propagating to the top and to the bottom, respectively, in Figure 3. In the axially symmetric case of the figure the crease set can only lie along a line on the axis of symmetry on the ST-horizon. (On the S-horizons shown in the figure, the crease points are conical singularities.) In general more than two horizon null surfaces can intersect on lower-dimensional sets; the typical case in four-dimensional spacetime is four null surfaces intersecting at a point. The crease set then consists of six two-dimensional spacelike surfaces on which the four null surfaces intersect pairwise. In a space and time description the S-horizon is the surface of a tetrahedron, and the crease set is the history of its six edges.

\(^2\)Because of the spacelike nature of the crease set there is no invariant meaning to the notion that one collapse occurs later than another: no matter how far two black holes are apart in space or time, if they will eventually collapse to one black hole, the crease set is a spacelike connection between them from the moment they are formed, and the separate black holes carry the creases from this connection at least until they merge.
Branching of a crease set in the local Lorentz frame at a branchpoint, in 3-dimensional spacetime. (a) The three heavy spacelike lines are the crease set that branches at the bottom (earliest) point. They span pairwise three null surfaces (the sides of the upside-down pyramid), which constitute the horizon. The null surfaces are tangent to the light cone, and the lines of tangency and lines parallel to it are the null generators (not shown). The intersection with a plane spacelike surface is a triangle (the base of the pyramid). (b) Because shortly after the branch point the horizon in the neighborhood of the branchpoint must be triangular, such a triangle must appear as a fourth component before three components of a horizon can merge.

Near the intersection point the geometry of the null surfaces can be described in a local Lorentz frame. Figure 4(a) shows the corresponding lower-dimensional picture. On a sequence of spacelike surfaces the horizon starts at the intersection point and expands to form a triangle; in four dimensional spacetime it would be a tetrahedron. The trace of the crease set on the S-horizon also consists of creases, in this case the corners of the triangle. In four dimensions it would be the corners and edges of the tetrahedron.

Since self-intersections of the horizon can occur at more than one point, the crease set in general has the structure of a spacelike, two-dimensional tree, and the ST-horizon consists of 3-dimensional null surfaces spanned by the branches of the tree. Because the null surfaces start, rather than end, at the crease set, it is always possible to find a time slicing in which a component of the horizon starts at the intersection point and expands as a tetrahedron (or higher polyhedron, if more than four null surfaces meet at the intersection point). An example is the way horizons merge in a spacetime with polyhedral symmetry. Consider a stage where there are already $n$ black holes, either eternal or due to previous collapse, at what would be the $n$ corners of a polyhedron. Each of the $n$ S-horizons has creases, in anticipation of later joining with the others. The conical points cannot come together directly, as the two horizons of Figure 3 do. Instead, another horizon component starts at the center of symmetry, expands to a polyhedron, and meets and "annihilates" with the $n$ separate horizons. Figure 4(b) shows the analogous case in a lower dimension. This sort of behavior is actually found by the horizon finder in numerical spacetimes [9].

8 Examples by exact solutions

The simplest case that is completely known is the collapse of a spherical shell of mass
The horizon starts at a time $R/c$ before the moment when the size of the shell has reached the Schwarzschild radius $R = 2GM/c^2$, and until that moment it expands as a light cone in the flat, Minkowski space that is the interior of the shell. After that moment the horizon radius remains constant at $R$ (Figure 5(a)). The spherically symmetrical collapse appears to be the only case when an exact horizon history is known in four dimensional spacetime.

In a lower, three-dimensional spacetimes instructive examples of null surfaces can be constructed, but Einstein’s equations forbid black holes unless there is a negative cosmological constant $\Lambda$ [10]. If the size of the black hole is small (compared to the scale $1/\sqrt{\Lambda}$ of the cosmological constant), and the interest is in the neighborhood of the black hole, one can set $\Lambda = 0$ approximately and operate in locally Minkowskian spacetime. The characteristic funnel shape of a black hole’s spacelike surface is replaced by the cylindrical shape that the funnel has near its throat. The spacelike 1-horizon is a circle around the cylinder that moves at the speed of light. In spacetime the S-horizon is a cylinder as well, but the axis direction is lightlike. For a collapsing ring of mass-energy Figure 5(a) applies as drawn in the Minkowski region, with the usual convention that the time direction is upward and null directions have a $45^\circ$ slope. In the Schwarzschild region the collapsing matter (dotted region in the figure) has caused the null direction to tip inward, to vertical at the horizon.

The horizon associated with the collapse of various matter configurations. (a) Collapse of a spherical shell, with one angular direction suppressed. The dotted area represents the collapsing matter (which should be world lines rather than points; it also should be rotated all around the figure). When this matter crosses, the horizon stops expanding and becomes cylindrical. All the generators (not shown) enter at the tip of the light cone in the flat Minkowski space in the interior of the shell. (b) Horizon for spherical shell collapse followed by infall of a point particle. To construct a solid corresponding to the picture (a), glue together the surfaces joined by the double-headed arrows as well as the long lines on the left and right, and the short roof-shaped lines at the bottom. The top part of the resulting figure is cylindrical (constant circumference), the null generators are parallel to the axis of this cylinder, or to the long lines on the left and right. A few or them are shown, most originate at the tip of the bottom cone, others along the heavier-drawn crease line along the middle of the figure. The conical singularity caused by gluing together the V shape near the top is the crossing point of the point
particle. (c) A discrete version of part (a), where six point particles cross the horizon. Only six generators (not shown) enter at the bottom tip of the upside-down pyramid, the rest enter along the spacelike edges of the pyramid. The heavier-drawn null line is an example of the latter.

Three-dimensional spacetime admits point particles, represented by conical points (a location of angle deficit). The horizon for a collapsing ring, followed by an in-falling point particle, is shown in Figure 5(b). Since the horizon is flat except where the ring matter and the particle cross, it can be slit open and flattened out on the plane. The double arrows show what points should be identified to sew up the slits. The section of a circle at the bottom represents the light cone as in figure 5(a). The left and right edges of the top part diverge because, in anticipation of further mass entering later, the horizon is not a constant-radius cylinder after the ring collapse, but an expanding cone. The point particle causes an angle deficit on the horizon, drawn here as a wedge cut out with edges identified. After this wedge is cut out the spatial circumference of the upper part of the figure is constant -- the horizon has reached its final size. The horizon generators are parallel to the identified edges, and cross in the region between the conical tip and the particle, along the heavy black line. This is the crease where part of the horizon begins. The remaining, large part begins at the bottom tip of the cone, where most of the generators enter.

The entry of all generators at one point, as for the spherical collapse of figure 5(a), is very special and atypical. This can be shown by the 2+1-dimensional example of figure 5(c). Consider the situation where the ring collapse is replaced by \( n \) individual point particles symmetrically arranged in a circle. The crease set must have the same symmetry, so it branches out as an \( n \)-pointed star from the center. Since the three-dimensional spacetime in this model is flat, the horizon subtended by the arms of the star consists of null planes, the sides of the upside-down pyramid. In this case only \( n \) generators enter at the tip, and an infinite number of others enter at the star's branches. The heavy black line is an example of a generator that enters at the edge of the pyramid. The 1-horizon is a regular \( n \)-gon that expands until it reaches the entry points of the point particles. Passing over these conical points straightens out the angles at the corners, and the \( n \)-gon becomes the final smooth circular horizon.

9 Summary and Conclusions

The beginning of a black hole is understood as the beginning of its horizon, and that in turn is the locus in spacetime where the horizon generators enter the horizon. On this locus the horizon has a crease characterized by a delta-function infinity in its extrinsic curvature, and that crease set represents the horizon’s beginning. The crease set is spacelike and generally not a submanifold, but has the structure of a tree. The horizon is simultaneously the boundary of the past of null infinity, and of the future of the crease set.

The horizon due to spherically symmetrical collapse, which starts as a light cone from one point, is very atypical. Generally one cannot find a smooth spacelike surface that contains the whole crease set, so the beginning does not occur all at one time. In the history according to a smooth time function, the entry of generators continues for an extended time and can follow a complex pattern as different parts of the crease set flash into the picture of an evolving time slice. On a given time slice the horizon has edges and corners where generators enter. The
edges and corners propagate at a speed exceeding that of light. As they propagate outward to less highly curved regions of spacetime, their speed decreases and their angle becomes less sharp until they disappear and the horizon becomes smooth. If the horizon initially has several components, the creased parts propagating toward each other become smooth as they merge.

References


Jörg Frauendiener "Conformal Infinity"


