Biding for Ad Spaces

Q4.1 How exactly does an eBay auction work?

Illustration 12 shows a standard eBay auction:

- *Start price*: At the beginning, the seller announces a start price, and can also choose to set a secret reserve price. If the end result of the auction is less than this, he can choose not to sell the item.

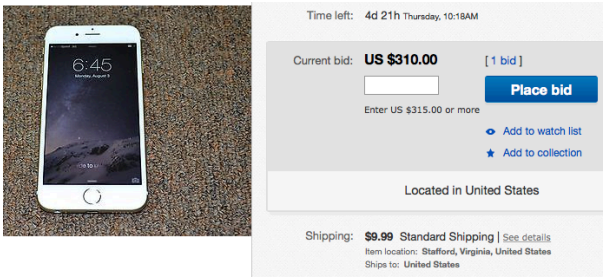

Illustration 12: An eBay auction for an iPhone 6 in August 2015. The "Current bid" is the ask price, and the "Time left" indicates the hard close time of the bid.

- *Duration*: eBay has a publicly announced time horizon, after which the auction ends. In Illustration 12, the "Time left" gives the hard close time of the iPhone auction.
- *Minimum increment*: There is a minimum increment between successive bids, so as to prevent inefficiencies in the auction. For example, in Illustration 12, we would not want $310.01 to be the next bid, since that would only be an increase of one cent.
- *Final sale*: At the conclusion of the auction the winner pays the smaller of the highest bid and second highest bid plus the increment, *i.e.*, the minimum of `Bid1` and `Bid2` + `increment`.
- *Ask price*: Some information about the current bid is continuously released to the public, but not necessarily the current highest bid. eBay displays an ask price, which is the lowest value that will be accepted for the next bid. To determine this, eBay takes the price that the highest bidder would have to pay if the auction ended now (*i.e.*, the minimum of `Bid1` and `Bid2` + `increment`),

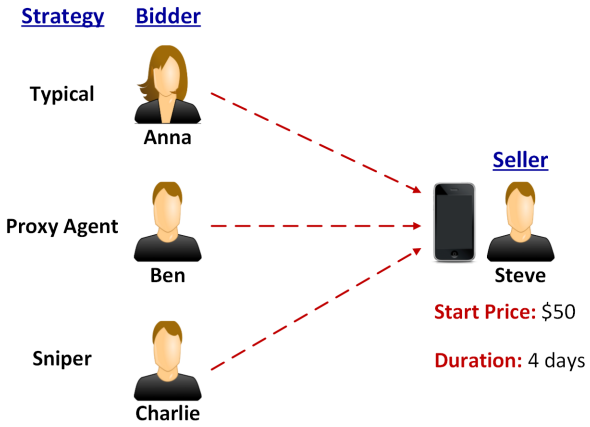

Illustration 13: Scenario we use for our eBay auction example. There are three bidders, each of whom uses a different strategy.

and adds `increment` to that. Bidders get a sense of where the current bids lie from the ask price, but cannot be certain whether it is derived from `Bid1` or `Bid2`. In Illustration 12, the "Current bid" represents the ask price.

Q4.2 Can we walk through an example of an eBay auction?

Suppose there are three bidders, Anna, Ben, and Charlie, who are each interested in an iPhone, as shown in Illustration 13. At the start, Steve, the seller, lists a used iPhone on eBay, with a start price of $50 and a duration of 4 days. We'll use a minimum increment of $5.

The evolution of bids is summarized in Illustration 14. On Day 1, Anna enters the eBay auction, with her first bid at the ask price of $50. What will eBay change the ask price to now? Adding the bid and the minimum increment, we get $50 + $5 = $55.

On Day 2, Ben decides that he also wants an iPhone, but doesn't have the time to sit and watch the auction. He elects instead to let a "proxy agent" do the work for him. A proxy agent is an automated bidding mechanism offered by eBay, which allows a user to set a maximum bid to which the proxy will keep bidding as necessary (*i.e.*, until the ask price is too high). Ben sets the proxy's maximum bid at $80. Since

	Day 1	Day 2	Day 3	Day 4	Day 5
Anna	**$50**	$50	$70	**$95**	–
Ben	–	**$55**	**$75**	–	–
Charlie	–	–	–	–	**$110**
Ask price	$55	$60	$80	$85	$105

Illustration 14: The bidding history and ask price evolution in our eBay auction example. The highest price at each stage is bolded.

this is larger than the ask price, the proxy starts by bidding the ask price of $55.

So what does the ask price change to now? Well, the highest bid is now $55, and the second highest is Anna's original $50. Since `Bid1` = $55 and `Bid2` + `increment` = $50 + $5 = $55 are the same, the new ask price is $55 + $5 = $60.

On Day 3, Anna sees that she's no longer the highest bidder. Rather than the ask price, let's say she bids ten dollars higher, at $70. What is the new ask price? Well, now $70 is the high bid and $55 the second highest. Since $55 + $5 = $60 is less than $70, the new ask price is $60 + $5 = $65. Ben's proxy agent immediately bids $65, since it is lower than his maximum. But Anna is still the highest, at $70. Since $65 + $5 is the same as $70, the new ask price is $70 + $5 = $75.

Is the proxy done? Not quite. It can see that Ben is still not the highest bidder, so it will take the $75. Now, Ben has the highest bid at $75, and Anna's $70 is the second highest. Hence, the ask price becomes $75 + $5 = $80.

On Day 4, in a fit of frustration, Anna decides to enter $95 as her bid. The ask price then updates to $80 + $5 = $85. What will Ben's proxy agent do? Well, start by updating the ask price: now $95 is the highest bid and $75 is second. Since $75 + $5 = $80 is less than $95, the new ask price is $80 + $5 = $85. But $85 is higher than the proxy's maximum bid of $80, so it will not continue bidding.

On Day 5, Anna sees she is the highest bidder and gets confident. But on the last day of the auction, Charlie, who has been watching all along and also really wants an iPhone, bids $110. Now, $110 is the highest and $95 is second. Since $110 is larger than $95 + $5 = $100, the ask price becomes $100 + $5 = $105.

In the end, Anna doesn't have time to react to Charlie's bid and loses

the auction. As the winner, what will Charlie end up paying for the phone? Recall that the price is whichever of his bid, `Bid1`, or the second highest plus the increment, `Bid2` + `increment`, is lower. Since \$110 is more than \$95 + \$5 = \$100, Charlie will end up paying \$100 for the phone.

This type of behavior on Charlie's part of submitting a high bid near the end of the auction is known as sniper bidding. The sniper's objective is to obtain the item at the current bid level by coming in at the last minute and surprising everyone with a higher bid.

Q4.3 What assumptions did we make in describing the different types of auctions?

- We assumed that the observed number of clicks per hour for an ad space would be the same as the estimated click-through rate. In reality, the estimated rate would be obtained a long term average, and the hourly observations would deviate from this simplified average, thereby affecting the price paid.
- We also assumed that the click-through rate was independent of the actual content of the ad placed in the space. In reality, some ads are naturally more attractive than others, and would tend to obtain more clicks in the same space as a result.
- We assumed that the average revenue generated from each click was independent of the particular space. So whether you have Google's first ad space or their last, we expected that you would make the same money *each time you receive a click*. This is a much more reasonable assumption than the previous two.
- We assumed that each bidder has an *intrinsic* valuation of the item, which they all know. In reality, bidders may not know precisely how to quantify their own valuation.
- We also assumed that each bidder's valuation was *private*. A **private valuation** means that nobody knows it except the bidder herself. This is not true in general: sometimes people make their valuations public so everyone can get an idea of how much an item is worth.
- Finally, we assumed that each bidder's valuation was *independent*. An **independent valuation** means that it will not change depending on what other peoples' are. This is also not true in

general: if you are thinking of buying a house, your real-estate agent's valuation of the property will likely be able to influence your own.