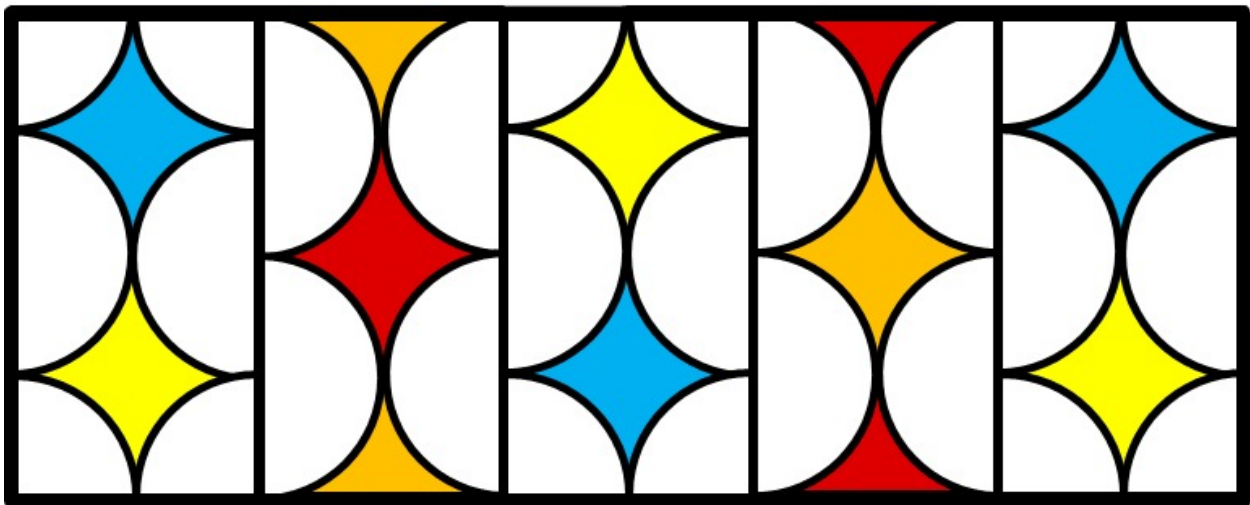


7.G Stained Glass

Task

The students in Mr. Rivera's art class are designing a stained-glass window to hang in the school entryway. The window will be 2 feet tall and 5 feet wide. They have drawn the design below:



They have raised \$100 for the materials for the project. The colored glass costs \$5 per square foot and the clear glass costs \$3 per square foot. The materials they need to join the pieces of glass together costs 10 cents per foot and the frame costs \$4 per foot.

Do they have enough money to cover the costs of the materials they will need to make the window?

IM Commentary

The purpose of this task is for students to find the area and perimeter of geometric figures whose boundaries are segments and fractions of circles and to combine that information to calculate the cost of a project. The shape of the regions in the stained glass window are left purposefully unspecified, as one component skill of modeling with mathematics (MP4) is for students to make simplifying assumptions themselves. Given the precision needed for these estimates, assuming the curves in the design are arcs of a circle is not only reasonable, it is the most expedient assumption to make as well. What is important is that students recognize they are making this assumption and are explicit about it.

The question of whether the students have to pay for the scraps of glass left over from cutting out the shapes can be dealt with in different ways. In reality, if they had to buy the glass at a store, the glass would likely come in square or rectangular sheets and they would need to buy more than they were going to use. But exactly how much extra material they would have to buy depends on how the raw materials are sold, so without additional information, it would be hard to determine that without doing some research into how stained glass is sold. Alternatively, the art teacher might already have the materials and just wants his students to stay within a certain budget for the materials they use, knowing that the scraps can be used for future student projects. In any case, this task can provide the springboard for a good classroom discussion around issues that students need to think about when modeling with mathematics.

Solution

There are many ways to do this. Here is one:

Assume that the students only have to pay for the glass they use and not the scraps that they would cut away. That means we need to figure out the area of the colored glass and the area of the clear glass as well as the total length of the seams between the panels of glass.

First, we need to find the area of the clear glass and the area of the colored glass.

The entire rectangle is 2 feet by 5 feet. Assuming that the curves are all parts of a circle with a 1 foot diameter, there are five 1 by 2 foot rectangles with either 4 half-circles or 2 half circles and 4 quarter circles of clear glass. That means there are 2 full circles of clear glass in each 1 by 2 foot rectangle. Thus, there are 10 complete circles of clear glass, each with a 1 foot diameter (or a $\frac{1}{2}$ foot radius). Then area of the entire window is

10 square feet, and the area of the clear glass is

$$10 \times \pi \left(\frac{1}{2}\right)^2 = \frac{5}{2}\pi$$

or approximately 7.9 square feet. That means the area of colored glass is approximately $10 - 7.9 = 2.1$ square feet.

Now we need to find the total length of the "seams" between the pieces of glass.

Again, there are 10 circles. Their total circumference is

$$10 \times \pi \times 1$$

which is about 31.4 feet. There are also four 2-foot straight "seams." So all together there are about 39.4 feet of "seams."

The frame is $2+2+5+5 = 14$ feet.

The cost for the clear glass is $7.9 \times 3 = 23.70$ dollars.

The cost for the colored glass is $2.1 \times 5 = 10.50$ dollars.

The cost for the materials for the seams is $39.4 \times 0.10 = 3.94$ dollars.

The cost of the frame is $14 \times 4 = 56$ dollars.

The total cost of the materials is $23.70 + 10.50 + 3.94 + 56 \approx 94$ dollars. So if these assumptions are accurate, they have just enough money to buy the materials. If they need to pay for the scraps or if they break pieces as they go, they don't have much wiggle room.



