

A-SSE Profit of a company, assessment variation

Task

The profit, P (in thousands of dollars), that a company makes selling an item is a quadratic function of the price, x (in dollars), that they charge for the item. The following expressions for $P(x)$ are equivalent:

$$P(x) = -2x^2 + 24x - 54$$

$$P(x) = -2(x - 3)(x - 9)$$

$$P(x) = -2(x - 6)^2 + 18$$

1. Which of the equivalent expressions for $P(x)$ reveals *the price which gives a profit of zero* without changing the form of the expression?

2. Find a price which gives a profit of zero.

3. Which of the equivalent expressions for $P(x)$ reveals *the profit when the price is zero* without changing the form of the expression?

4. Find the profit when the price is zero.

5. Which of the equivalent expressions for $P(x)$ reveals *the price which produces the highest possible profit* without changing the form of the expression?

6. Find the price which gives the highest possible profit.

For each of parts (a), (c), and (e), students choose which form of the function is displayed, and then use that to answer the corresponding questions in parts (b), (d), and (f) respectively. Students are assessed both on selecting the correct form of the function and on their numerical answer for each part.

IM Commentary

This task is part of a joint project between [Student Achievement Partners](#) and Illustrative Mathematics to develop prototype machine-scorable assessment items that test a range of mathematical knowledge and skills described in the CCSSM and begin to signal the focus and coherence of the standards.

Task Purpose

The primary purpose of this task is to assess students' knowledge of certain aspects of the mathematics described in the High School domain A-SSE: Seeing Structure in Expressions. Specifically, standard A-SSE.3 reads

A-SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines

This task emphasizes the *choice* aspect of the full standard, giving students the option of several forms of a quadratic expression to choose from when answering questions about a real-world scenario governed by that expression. The task avoids having the students do the algebraic manipulations themselves, focusing instead on using the structure of the expressions.

Cognitive Complexity

Mathematical Content

Students will likely have encountered several "complete the square" problems which focus on the algebraic manipulation of quadratic expressions. By stripping this procedural aspect of the content away, students are assessed on a more conceptual understanding of quadratic expressions.

Mathematical Practices: The task, and the standard A-SSE.3 in general, ties closely to MP 7, Look for and make use of structure.

At the high school level, structure of the type mentioned in MP 7 manifests itself in the form of algebraic expressions, and the current task concerns itself quite explicitly with exploiting the structure to solve problems. For example, students need to recognize that the structure of the factored form (along with the zero-product property for real numbers) allows them to much more easily find a price that leads to a profit of zero than would, say, the standard form of the quadratic.

Linguistic Demand:

The language used to describe the business context is moderate for use in high school, though not entirely trivial for non-native speakers.

Stimulus Material:

Students are presented with a block of text to read and process.

Response Mode:

For each of three questions, students choose one of the three forms from a drop-down menu, and enter their final answer into a text box.

Solution

For each of the three questions, choosing the correct form of the expression allows the answering of the question without doing any computations. Specifically:

a. $P(x) = -2(x - 3)(x - 9)$ shows the values of x that make the profit zero without any computation because a quadratic function in factored form $P(x) = a(x - r_1)(x - r_2)$

has roots r_1 and r_2 and the profit is zero when $P(x) = 0$.

b. The profit is zero when $-2(x - 3)(x - 9) = 0$, which occurs when $x = 3$ or $x = 9$. The company breaks even if the price charged for the product is \$3 or \$9.

c. $P(x) = -2x^2 + 24x - 54$ shows the profit when the price is zero without any computation because the value of a quadratic function in standard form $P(x) = ax^2 + bx + c$ when $x = 0$ is c .

d. In this case $c = -54$, so the profit is -54 (in thousands of dollars) when the price is zero. If the company gives the product away for free, it loses \$54,000.

e. $P(x) = -2(x - 6)^2 + 18$ gives us the price which maximizes profit without any computation because the maximum value of a quadratic function in vertex form $P(x) = a(x - h)^2 + k$ is k and occurs at $x = h$.

f. From $P(x) = -2(x - 6)^2 + 18$, we see that the maximum profit is 18 thousand dollars, and it occurs when $x = 6$. The company should charge a price of \$6 for this product in order to maximize its profit.

This is a 3-point item.



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