S-MD.2 Bob's Bagel Shop

Task

Bob's Bagel Shop has estimated the following probabilities for the number of bagels a randomly-selected customer buys when he or she enters the shop.

No. of Bagels	Probability
1	.5
2	.1
3	.1
6	.1
12	.2

Bob sells bagels for \$0.80 each, or \$9.00 for a dozen. If $X =$ the amount of money Bob collects from a randomly-selected customer, find and interpret the expected value of X.

IM Commentary

The purpose of this task is to assess a student's ability to compute and interpret an expected value. Notice that interpreting expected value requires thinking in terms of a long-run average.

A common student error is to find the expected number of bagels sold and then

multiply this by 0.8. But because a customer buying 12 bagels receives a discount, this approach does not lead to a correct answer.

The probability distribution used in this task is artificial and a bit unrealistic. For example, there are no customers who purchase 0 bagels (someone who just purchases a beverage) or who purchases 4 bagels. You might want to have the class develop a different probability distribution that includes possible values of 0, 1, 2, ..., 12. Students could then explore several possible assignments of probabilities and assess the impact on the expected value.

You could also have students explore different pricing strategies and evaluate the effect on expected money collected per customer.

Solution

First we need the probability distribution of X:

No. of Bagels	X	Probability
1	0.80	.5
2	1.60	.1
3	2.40	.1
6	4.80	.1
12	9.0	.2

Then the expected value of X is:

$$E(X) = \Sigma X \cdot P(X) = 0.8 \cdot 0.5 + 1.6 \cdot 0.1 + 2.4 \cdot 0.1 + 4.8 \cdot 0.1 + 9.0 \cdot 0.2 = 3.08$$

The mean amount of money Bob can expect to make per customer in the long run is $3.08.

Illustrative
Mathematics