A-CED Clea on an Escalator

Task

It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

IM Commentary

This task has students create equations to model a physical scenario, and then reason with those equations to come up with a solution. All three components -- understanding the physical model, creating the appropriate equations, and then solving the equations -- require students to engage in several Standards for Mathematical Practice. In particular, we note the task could be used as an illustration of MP1 (making sense of problems), MP2 (abstract and quantitive reasoning), and 4 (mathematical modeling).

The modeling aspect of the task has students reason about rates, and in particular, requires the use of

$$\text{distance} = \text{rate} \times \text{time}.$$

This formula assumes that Clea is walking at a constant rate (and the escalator is moving at a constant rate as well), so the teacher may wish to direct students to make these assumptions, and/or hold a discussion on the reasonableness of this assumption.

An additional complicating factor is that while we are given units for time (seconds) no units for distance are given. Many students may opt not to put in distance units because this does not impact the answer and the teacher may wish to address the

reason why. The vital issue in solving this problem is that the units for the two rates are determined by the units chosen for distance and time in each case. . In this situation, feet or meters seem like the most reasonable choice for distance

As the AMC test results show, a large number of students took the difference of the two times, 60 seconds - 24 seconds, as an answer. The teacher may wish to specifically address why this is incorrect.

This task was adapted from problem #13 on the 2012 American Mathematics Competition (AMC) 10B Test. On this test, which was taken by 35086 students, the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers
(A)	36	24.62
(B)*	40	42.60
(C)	42	12.12
(D)	48	4.81
(E)	52	1.02
Omit	--	14.81

Of the 35,086 students: 17,169, or 49%, were in 10th grade; 9,928 or 28%, were in 9th grade; and the remainder were below than 9th grade.

Solution

Let d denote the distance of the trip down the escalator (in feet for example). Then we have the formula

$$\text{distance} = \text{rate} \times \text{time}.$$

Our time has been given to us in seconds and so if we use feet for the distance then

the rate will be measured in feet per second. Let r be the rate that the escalator is travelling and s the rate that Clea is walking. We know that it takes 60 seconds for Clea to descend the escalator while walking when the escalator is not moving. This means

$$d = 60 \times s.$$

The second piece of information tells us that it takes Clea 24 seconds to descend if she walks while the escalator is moving. This is modeled by the equation

$$d = 24 \times (r + s).$$

The two rates are added because both Clea and the escalator are moving downward. If we set the two equations equal to one another we get

$$60s = 24r + 24s.$$

We can solve this and find that $r = \frac{3}{2}s$. So the escalator is traveling $\frac{3}{2}$ as fast as Clea. Since it took Clea 60 seconds to descend when the escalator was not moving it will take the escalator $\frac{2}{3} \times 60$ seconds or 40 seconds to descend.