

4.NF Making 22 Seventeenths in Different Ways

Task

Which of the following sums are equal to $\frac{22}{17}$?

a. $\frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17}$

b. $\frac{3}{17} + \frac{8}{17} + \frac{3}{17} + \frac{10}{17}$

c. $\frac{6}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$

d. $\frac{12}{17} + \frac{10}{17}$

e. $\frac{1}{17} + \frac{1}{17} + \frac{9}{17} + \frac{3}{17}$

Find another way to write $\frac{22}{17}$ as a sum of fractions.

IM Commentary

This task is a straightforward task related to adding fractions with the same denominator. The main purpose is to emphasize that there are many ways to decompose a fraction as a sum of fractions, similar to decompositions of whole numbers that students should have seen in earlier grades (see e.g. K.OA.3). It is suitable for assessment or, with opportunity for classroom discussion, it could also be useful in instruction.

A common stumbling block with adding fractions is that students will incorrectly add

the numerators together and the denominators together like so:

$\frac{2}{17} + \frac{3}{17} = \frac{2+3}{17+17} = \frac{5}{34}$. This task could be used to determine whether a student is likely to make this error.

Solutions

Solution: Solution 1

First we must add the fractions together for each part

$$\text{a. } \frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17} = \frac{(5+4+3+10)}{17} = \frac{22}{17}$$

$$\text{b. } \frac{3}{17} + \frac{8}{17} + \frac{3}{17} + \frac{10}{17} = \frac{(3+8+3+10)}{17} = \frac{24}{17}$$

$$\text{c. } \frac{6}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17} = \frac{(6+4+3+5+2+2)}{17} = \frac{22}{17}$$

$$\text{d. } \frac{12}{17} + \frac{10}{17} = \frac{(10+12)}{17} = \frac{22}{17}$$

$$\text{e. } \frac{1}{17} + \frac{1}{17} + \frac{9}{17} + \frac{3}{17} = \frac{(1+1+9+3)}{17} = \frac{14}{17}$$

Now we can see that parts a, c, and d all add to $\frac{22}{17}$.

For the second question, any decomposition similar to the ones above that adds to $\frac{22}{17}$ will work. Take for example: $\frac{1}{17} + \frac{21}{17}$ or $\frac{2}{17} + \frac{20}{17}$.

Solution: Solution 2

Instead of adding all the fractions together to form a single fraction we can re-write the sum to make it obvious which is the same and which is not. Take part a and part b for example:

$$\text{a. } \frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17}$$

$$\text{b. } \frac{3}{17} + \frac{8}{17} + \frac{3}{17} + \frac{10}{17}$$

$$= \frac{3}{17} + \frac{(2+6)}{17} + \frac{3}{17} + \frac{10}{17}$$

$$= \frac{3}{17} + \frac{2}{17} + \frac{6}{17} + \frac{3}{17} + \frac{10}{17}$$

$$= \frac{(3+2)}{17} + \frac{6}{17} + \frac{3}{17} + \frac{10}{17}$$

$$= \frac{5}{17} + \frac{6}{17} + \frac{3}{17} + \frac{10}{17}$$

Now comparing part a to part b, we can see that since $\frac{4}{17} < \frac{6}{17}$ and all other fractions in the sum are the same

$$\frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17} < \frac{5}{17} + \frac{6}{17} + \frac{3}{17} + \frac{10}{17}$$

And as a result they can not be the same fraction.

Now looking at part c:

$$c. \frac{6}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$$

$$= \frac{(5+1)}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$$

$$= \frac{5}{17} + \frac{1}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17}$$

Then changing the order of addition:

$$= \frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{5}{17} + \frac{2}{17} + \frac{2}{17} + \frac{1}{17}$$

$$= \frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{(5+2+2+1)}{17}$$

$$= \frac{5}{17} + \frac{4}{17} + \frac{3}{17} + \frac{10}{17}$$

And now by comparing part a and the re-written version of part c we can see that they represent the same fraction.

Parts d and e are done in a similar way.



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