8.EE Ants versus humans

Task

The average mass of an adult human is about 65 kilograms while the average mass of an ant is approximately 4×10^{-3} grams. The total human population in the world is approximately 6.84 billion, and it is estimated there are currently about 10,000 trillion ants alive.**1**

Based on these values, how does the total the total mass of all living ants compare to the total mass of all living humans?

1: Holldobler, B., E. Wilson, *Journey to the Ants: A Story of Scientific Exploration* (London, England: The Belknap Press of Harvard University, 1994).

IM Commentary

This task requires students to work with very large and small values expressed both in scientific notation and in decimal notation (standard form). In addition, students need to convert units of mass. The solution below converts the mass of humans into grams; however, we could just as easily converted the mass of ants into kilograms. Students are unable to go directly to a calculator without taking into account all of the considerations mentioned above. Even after converting units and decimals to scientific notation, students should be encouraged to use the structure of scientific notation to regroup the products by extending the properties of operations and then use the properties of exponents to more fluently perform the calculations involved rather than rely heavily on a calculator. In particular, teachers could encourage students to estimate the ratios of the two masses before resorting to calculator use.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, the commentary will spotlight one practice connection in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task helps illustrate Mathematical Practice Standard 6, “Attend to precision.” This task provides an opportunity to discuss the accuracy of these calculations and an appropriate level of precision. For example, it is obviously not possible to count the total number of ants in the world at any given moment, nor is it feasible to weigh every adult human. Similarly, the numbers in the task statement make heavy use of "averages" (implicit and unstated is that these averages are mean values, bringing an opportunity to discuss precision in mathematical terminology), a process which inherently raises questions -- were children included in the average human mass calculation? How would their exclusion affect the conclusion? Questions of this form abound, and their analysis fosters critical thinking skills.

Students engage in MP.6 by communicating precisely to others using scientific and/or decimal notation, accurate metric labeling, and mathematical terminology. First, students determine the unit of measure that would be most accurate and appropriate to compare the total mass of living ants to the total mass of all living humans, grams or kilograms. Once they determine the appropriate unit of measure, then they use scientific notation to find the approximate total mass of all ants and all humans. This process emphasizes precision once again by centering on the accuracy of these calculations and on the appropriate level of precision that is required in this context.

Solution

We are told the total number of ants in the world is about 10,000 trillion or $10,000 \times 10^{12} = 10^4 \times 10^{12} = 10^{16}$ ants. In addition, the average mass of a single ant is 4×10^{-3} grams. Thus, the approximate total mass of all ants in the world is

$$\left(4 \times 10^{-3} \text{g}\right)\left(10^{16}\right) = 4 \times 10^{-3+16} \text{g} = 4 \times 10^{13} \text{g}.$$

The mass for humans is given in kilograms while the mass for ants is in grams. We convert the unit of mass for a human to grams as follows,

$$(65\text{kg})\left(\frac{10^3 \text{g}}{1\text{kg}}\right) = \left(65 \times 10^3\right) \text{g} = \left(6.5 \times 10^1\right)\left(10^3\right) \text{g} = 6.5 \times 10^{1+3} \text{g} = 6.5 \times 10^4 \text{g}.$$

Since there are 6.84 billion humans on earth, the total mass of all humans on earth can be approximated as

$$\begin{aligned}\left(6.5 \times 10^4 \text{g}\right)(6.84 \text{ billion}) &= \left(6.5 \times 10^4 \text{g}\right)\left(6.84 \times 10^9\right) \\ &= (6.5 \times 6.84)\left(10^4 \times 10^9\right) \text{g} \\ &= 44.46 \times 10^{4+9} \text{g} \\ &= \left(4.446 \times 10^1\right) \times 10^{13} \text{g} \\ &= 4.446 \times 10^{14} \text{g}.\end{aligned}$$

Thus, the total mass of all humans in the world is greater than the total mass of all ants in the world. In fact, the calculations above show the total mass of all humans in the world is about 10 times the total mass of all ants.

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