8.G Congruent Segments

Task

Line segments AB and CD have the same length. Describe a sequence of reflections that exhibits a congruence between them.

IM Commentary

Students' first experience with transformations is likely to be with specific shapes like triangles, quadrilaterals, circles, and figures with symmetry. Exhibiting a sequence of transformations that shows that two generic line segments of the same length are congruent is a good way for students to begin thinking about transformations in greater generality.

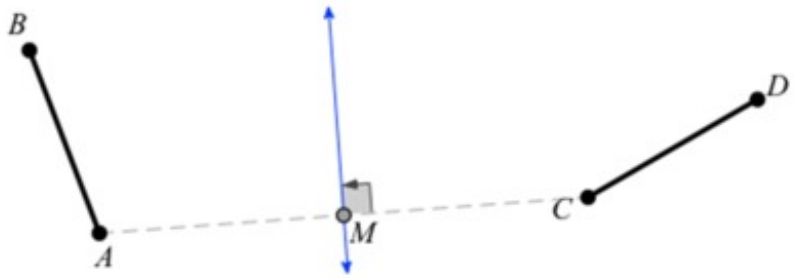
Solution

Recall the definition of a reflection: Given a fixed line L, a point P is sent to P' so that

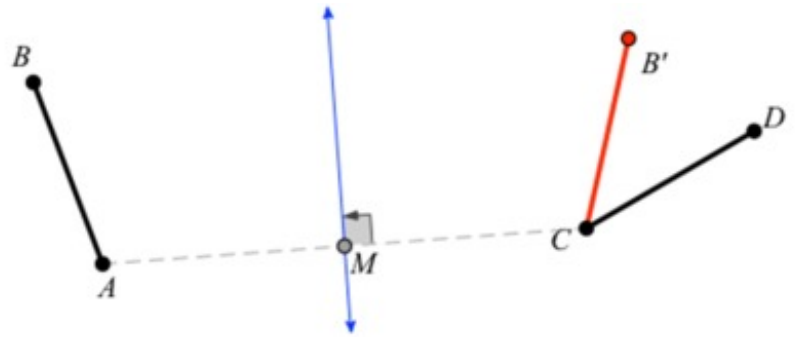
- The line through P and P' is perpendicular to L, and
- The distance from P to L is equal to the distance from to P' to L.

So a reflection that sends P to P' must have a line of reflection that is the perpendicular bisector of the segment PP'.

Create the midpoint M of segment AC and draw the line through M and perpendicular to AC.



Reflect AB across this line. The reflection sends A to C and B to a point we will call B'



If B' happens to land on D, then we are done. If not, create the midpoint of the segment B'D and the line through this midpoint and point C. This line is the perpendicular bisector of B'D (in High School, this should be proved, but for Grade 8 it can be observed. It follows from SSS congruence). Reflect CB' across this line. Since C is on the line it is fixed, and B' is sent to D.

This shows that for any two segments of the same length, one can be mapped to the other by a sequence of at most two reflections.