8.F Battery Charging

Task

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?

b. How much time would Sam need to charge the battery 100% on both devices?

IM Commentary

This task has students engaging in a simple modeling exercise, taking verbal and numerical descriptions of battery life as a function of time and writing down linear models for these quantities. To draw conclusions about the quantities, students have to find a common way of describing them. There are three solution techniques

presented below:

a. Finding equations for both functions.

b. Using tables of values.

c. Using graphs.

There are also ample opportunities to talk about the role of modeling here, touching on mathematical practice standard MP4. How reasonable is it that the output units are reported as percents? Does the model hold for all time? In particular, note that the model predicts that the percent charged grows linearly for all time, even beyond 100%!

If the task is done in small groups, different groups would likely use different representations in their solutions. Having groups present their answers could lead to a rich discussion on connecting different representations of functions.

Solutions

Solution: Finding and using equations

a. The battery charge of both devices can be modeled with linear functions. The wording describing the MP3 player suggests a linear function since it uses a constant rate of change. The table of values for the video game player shows a constant rate of change for the first 30 minutes. It is a reasonable assumption that the battery will continue to charge at the same rate. However, it is an assumption on our part. (Another possibility would be that as the battery charge approaches 100%, the rate of change decreases, but that would be much harder to model.)

The MP3 player charges at a rate of 12 percentage points every 15 minutes, which is equal to 0.8 percentage points per minute. If we let y be battery charge of the device (in percentage points) we have:

$$y = 0.8t + 40,$$

where t is measured in minutes.

We know that the video game player is initially 20% charged and from the table we see that the charge increases by an additional 12 percentage points every 10 minutes, or

1.2 percentage points per minute. So for this function we get:

$$y = 1.2t + 20.$$

Sam's family is planning to leave the house 60 minutes after Sam started charging his devices. We are looking for the charge when $t = 60$:

MP3 player: $y = 0.8 \cdot 60 + 40 = 88$ % charged

video game player: $y = 1.2 \cdot 60 + 20 = 92$ % charged

b. To answer this question, we need to find the values of t for which each function has output value 100.

MP3 player: Solving $100 = 0.8t + 40$ for t we have, $t = 75$ minutes.

video game player: Solving $100 = 1.2t + 20$ for t we have $t = 67$ minutes.

So if Sam's family could wait just 15 more minutes, Sam could have both devices fully charged for the car trip.

Solution: Using tables

a. Since the video game player's battery charge is given in a table, we can extend the table and see what value it will give after 60 minutes. Note that the rate of change of the data in the table is constant: For every 10 minutes the charge increases by 12 percentage points. Assuming that this pattern continues, we have:

time charging (minutes)	0	10	20	30	40	50	60
video game player battery charge (%)	20	32	44	56	68	80	92

We can make a similar table for the MP3 player:

time charging (minutes)	0	15	30	45	60

MP3 player battery charge (%)	40	52	64	76	88

So after 60 minutes, the MP3 player's battery would be 88% charged and the video game player will be 92% charged.

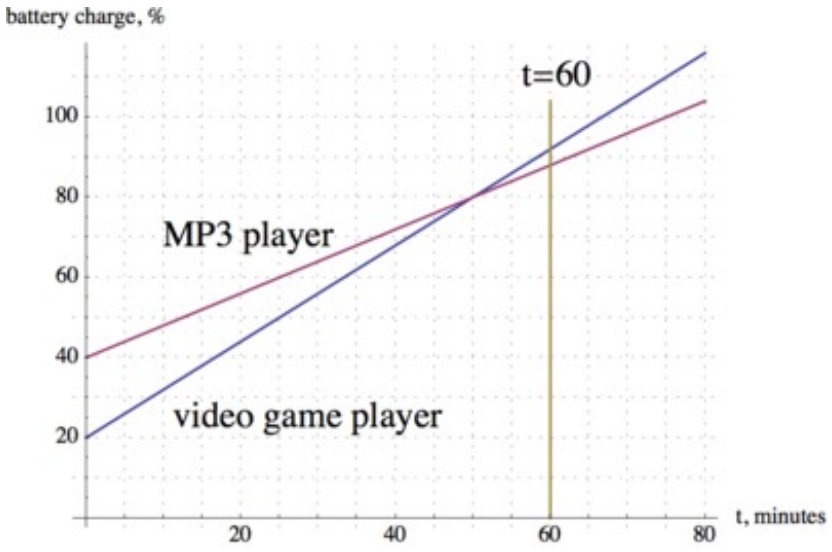
b. We can see from the table above that the MP3 player would be fully charged in another 15 minutes, we just have to add one more column to the table to find that answer.

The video game player will need less than 10 minutes to fully charge, since we are only missing 8 percentage points after 60 minutes. To be exact, using the rate of increase, we will need 2/3 of 10 minutes, which is just under 7 minutes.

Solution: Using graphs

a. With the given information, it is quite straight-forward to graph the functions for both devices. For the MP3 player we have a starting value (i.e. vertical intercept) of 40% and a rate of change (i.e. slope) of 12/15 = 0.8 percentage points per minute.

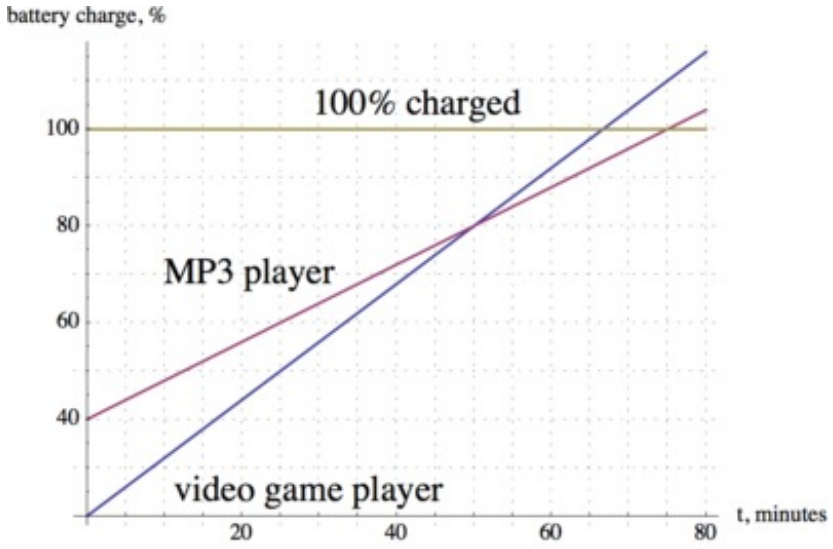
For the video game player we have a starting value of 20% and the rate of change for the data in the table is constant at 12/10 = 1.2 percentage points per minute. Below are the two graphs.



We can estimate from the graph that after 60 minutes the MP3 player has a battery

charge of just under 90% and the video game player has a battery charge of just over 90%. Zooming in on a graphing calculator or other graphing device would give us better estimates.

b. To find out how long it will take until both batteries are fully charged, we need to find values of t for which the output value is 100% for both functions.



From the graph we see that the MP3 player will take the longest to charge and it will take about 75 minutes total. So if Sam's family can wait an extra 15 minutes before they leave, Sam would have both devices fully charged.