6.G Christo's Building

Task

Christo and Jeanne Claude are artists who wrap large things – bridges, buildings, even islands. (See www.christojeanneclaude.net for images.)

Imagine that Jean Claude and Christo are preparing for their next large project – that of wrapping a large building. They are building a scale model in order to plan out the project.

The model building will measure $11\frac{1}{2}$ inches high, and $7\frac{1}{2}$ inches wide, and 6 inches long.

They are considering building the model with blocks that measure $\frac{1}{2}$ in on each side.

a. What is the volume of the building in cubic inches?

b. How many of the blocks that measure $\frac{1}{2}$ in on each side would Jeanne Claude and Christo need to make the model building?

c. Draw a picture of the model building and determine how much fabric Jean Claude and Christo will need to cover the model building.

d. The way Jean Claude and Christo have made their model, they know that they the number of square feet needed to cover the real building is 144 times the number of square inches needed to cover the model. How much fabric will Jean Claude and Christo will need to cover the real building?

IM Commentary

This task is primarily about volume and surface area, although it also gives students an early look at converting between measurements in scale models and the real objects they correspond to. Students are not required to understand the general principles for converting between measurements in scale drawings (or models) and measurements in their referents until 7th grade (see 7.G.1), but here they are given the conversion explicitly. This kind of task is more appropriate in an instructional setting rather than as an assessment item since it is laying the groundwork for work in seventh grade rather than representing a mature competency in 6th grade. (Thanks to Esther Koontz, a teacher who reviewed this task, for suggesting to include part (d).)

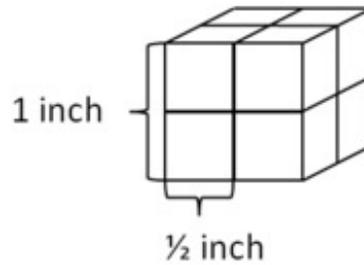Submitted by Karen Gerow from the University of Georgia to the fourth Illustrative Mathematics contest.

Solution

a.
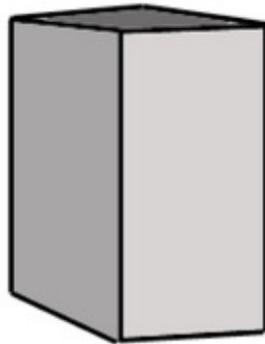
$$l \times w \times h = \text{Volume}$$

$$11\frac{1}{2} \text{ in} \times 7\frac{1}{2} \text{ in} \times 6 \text{ in} =$$

$$\frac{23}{2} \text{ in} \times \frac{15}{2} \text{ in} \times 6 \text{ in} =$$

$$= 517\frac{1}{2} \text{ in}^3$$

b. One way to do this is to figure out how many cubes that are $\frac{1}{2}$ inch on each side it takes to make a cubic inch. The picture below shows that it takes $2 \times 2 \times 2 = 8$ of the blocks to make a cubic inch.

1 inch

½ inch

Every cubic inch of volume requires 8 cubes, so we can think of a cubic inch as a group of 8 smaller cubes). Since the volume is given in cubic inches, we multiply the volume by 8:

$$517.5 \times 8 = 4140$$

So they would need 4140 of the blocks to make the model.

Another way to do it: we can figure out how many blocks long, how many blocks wide, and how many blocks tall the model will be if we divide each of the dimensions by $\frac{1}{2}$.

$$6 \div \frac{1}{2} = 12 \text{ blocks long}$$

$$7\frac{1}{2} \div \frac{1}{2} = 15 \text{ blocks wide}$$

$$11\frac{1}{2} \div \frac{1}{2} = 23 \text{ blocks tall}$$

So the base will be 12 blocks long and 15 blocks wide.

$$12 \times 15 = 180$$

so 180 blocks will make up the base. The building will be 23 blocks tall, and each layer will contain 180 blocks.

$$23 \times 180 = 4140 \text{ blocks}$$

Again we see that they would need 4140 of the blocks to make the model, which might take a long time to glue together. Perhaps Jean Claude and Christo might want to take another approach to building their model.

c. The building needs to have the four sides and the top covered; here is a net for the faces of the building that will be covered by fabric:
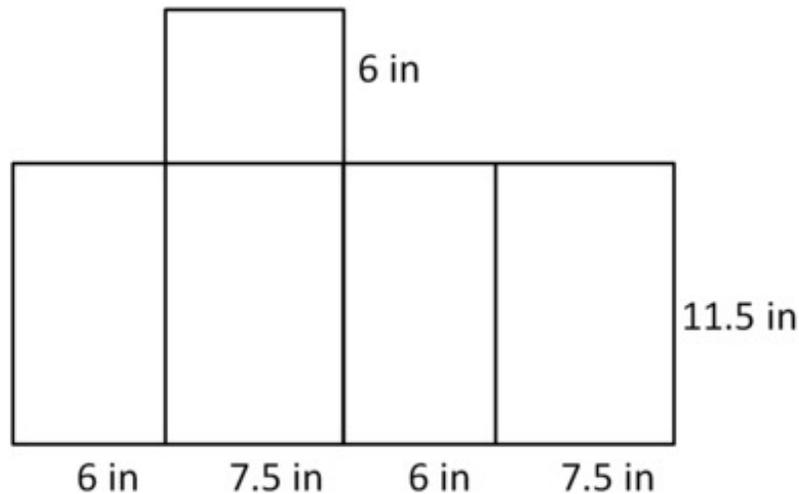
It has:

- two sides that are 11.5 inches by 6 inches,
- two sides that are 11.5 inches by 7.5, and
- the top of the building which is 6 inches by 7.5 inches.

We need to add the areas of all the faces:

$$[2 \times (11.5 \times 6)] + [2 \times (11.5 \times 7.5)] + (6 \times 7.5) = 138 + 172.5 + 45$$
$$= 355.5$$

So the area of the fabric needed to cover the model building is 355.5 in^2.

d. The way Jean Claude and Christo have made their model, they know that they the number of square feet needed for the real building is 144 times the number of square inches needed for the model. In part (c) we calculated that number to be 355.5, so they need $144 \times 355.5 = 51,192$ square feet of fabric for the real building. That is a lot of fabric!