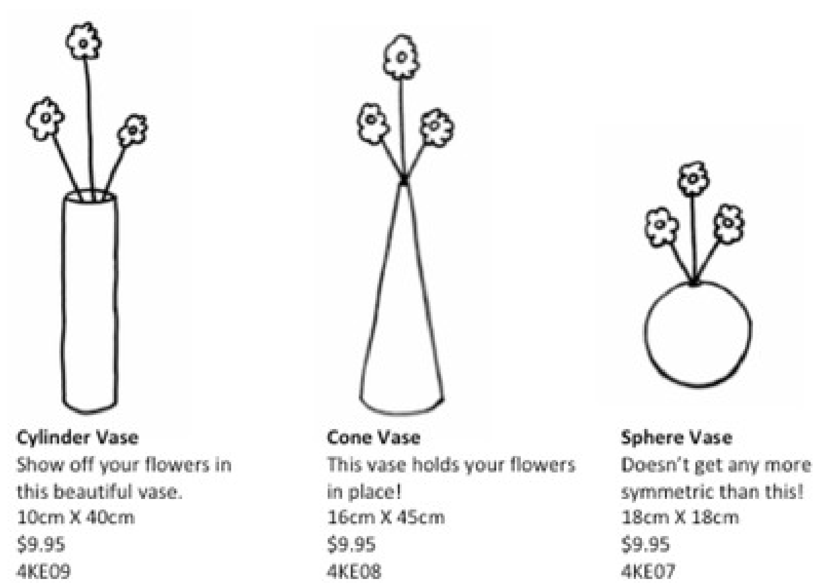
Illustrative Mathematics

8.G Flower Vases

Task

My sister's birthday is in a few weeks and I would like to buy her a new vase to keep fresh flowers in her house. She often forgets to water her flowers and needs a vase that holds a lot of water. In a catalog there are three vases available and I want to purchase the one that holds the most water. The first vase is a cylinder with diameter 10 cm and height 40 cm. The second vase is a cone with base diameter 16 cm and height 45 cm. The third vase is a sphere with diameter 18 cm.

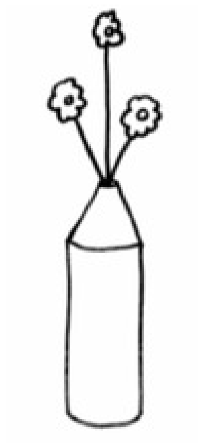


a. Which vase should I purchase?

b. How much more water does the largest vase hold than the smallest vase?

c. Suppose the diameter of each vase decreases by 2 cm. Which vase would hold the most water?

d. The vase company designs a new vase that is shaped like a cylinder on bottom and a cone on top. The catalog states that the width is 12 cm and the total height is 42 cm. What would the height of the cylinder part have to be in order for the total volume to be $1224\pi \text{ cm}^3$?



Pencil Vase
The perfect gift for your math teacher!
12cm X 42cm
$9.95
4KE06

e. Design your own vase with composite shapes, determine the volume, and write an ad for the catalog.

IM Commentary

The purpose of this task is to give students practice working the formulas for the volume of cylinders, cones and spheres, in an engaging context that provides and opportunity to attach meaning to the answers.

When used in a classroom setting, the task could be supplemented by questions that ask students to thinking about the relationship between volume and liquid capacity. For example, after part (b), the teacher could ask the students for other ways to determine which vase holds the most water, with the expectation that students might respond with the idea of pouring water from one vase into another.

Submitted by Nora Oswald to the fourth Illustrative Mathematics Task Writing Contest 2012/02/13.

Solution

a. You should purchase the cylinder vase. If r is the radius and h is the height, then, using the fact that the radius is half the diameter, we get

$$\begin{aligned}\text{Cylinder Volume} &= \pi r^2 h\\ &= \pi(5)^2(40)\text{ cm}^3\\ &= 1000\pi\text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Cone Volume} &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3}\pi(8)^2(45)\text{ cm}^3\\ &= 960\pi\text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Sphere Volume} &= \frac{4}{3}\pi r^3\\ &= \frac{4}{3}\pi(9)^3\text{ cm}^3\\ &= 972\pi\text{ cm}^3.\end{aligned}$$

b. $\text{Cylinder Volume} - \text{Cone Volume} = 1000\pi\text{ cm}^3 - 960\pi\text{ cm}^3 = 40\pi\text{ cm}^3$

c. If the diameter decreases by 2 cm, then the radius decreases by 1 cm. Now the cone holds more water:

$$\begin{aligned}\text{Cylinder Volume} &= \pi r^2 h\\ &= \pi(4)^2(40)\text{ cm}^3\\ &= 640\pi\text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Cone Volume} &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3}\pi(7)^2(45)\text{ cm}^3\\ &= 735\pi\text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Sphere volume} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(8)^3 \text{ cm}^3 \\ &= 682\frac{2}{3}\pi \text{ cm}^3.\end{aligned}$$

d. The total volume is the volume of the cylinder plus the volume of the cone. If the cylinder has height x cm then the cone has height $42 - x$ cm, since the total height is 42 cm. So the volume of the cylinder plus the volume of the cone is

$$\begin{aligned}\pi r^2 x + \frac{1}{3}\pi r^2(42 - x) &= \pi(6)^2(x) + \frac{1}{3}\pi(6)^2(42 - x) \\ &= 36\pi x + 12\pi(42 - x) \\ &= 36\pi x + 504\pi - 12\pi x \\ &= 24\pi x + 504\pi.\end{aligned}$$

So to find x we must solve the equation

$$\begin{aligned}1224\pi &= 24\pi x + 504\pi \\ 720\pi &= 24\pi x \\ x &= 30 \text{ cm}\end{aligned}$$

e. Answers will vary.